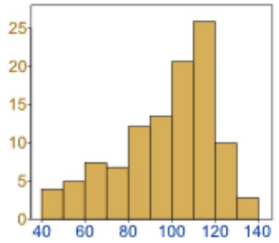
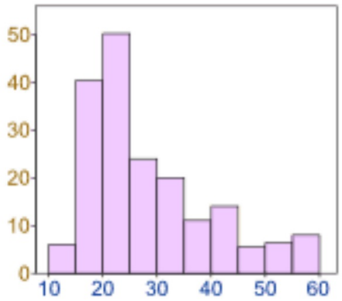
Section 12-7: Normal Distributions

Data can be "distributed" (spread out) in different ways.

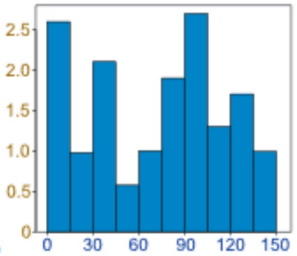
The data could be pushed to the right



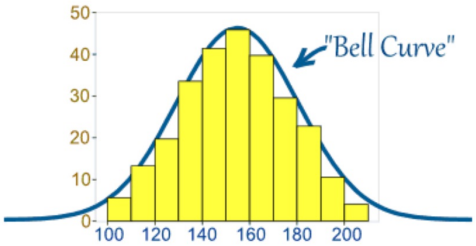
.... or to the left



Or it can be all jumbled up



But there are many cases where the data tends to be around a central value with no bias left or right.



A Normal Distribution

http://www.mathsisfun.com/data/quincunx.html

Many things closely follow a Normal Distribution:

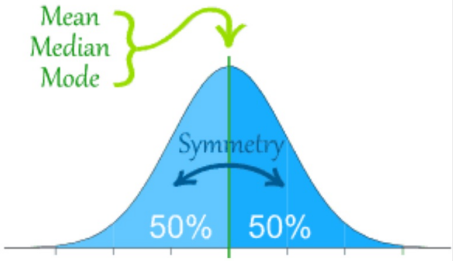
If the sample size is large enough

- heights of people
- size of things produced by machines
- errors in measurements
- blood pressure
- marks on a test

We say the data is "normally distributed".

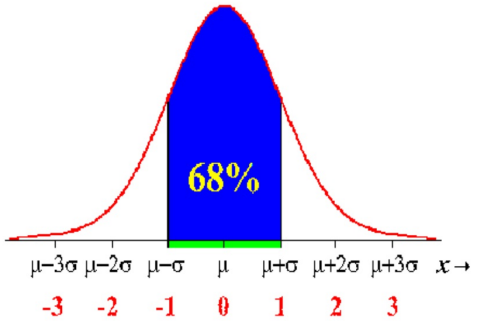
The Normal Distribution has:

- mean = median = mode
- symmetry about the center
- 50% of values less than the mean and 50% greater than the mean



The 68-95-100 Rule For Normal Distributions

Approximately 68% of the observations fall within 1 standard deviation of the mean



$\bar{x}$ = Sample Mean

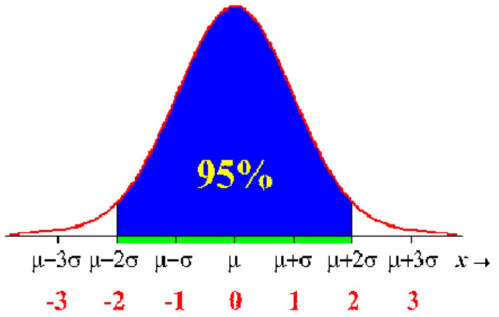
"x bar"

μ = Population Mean

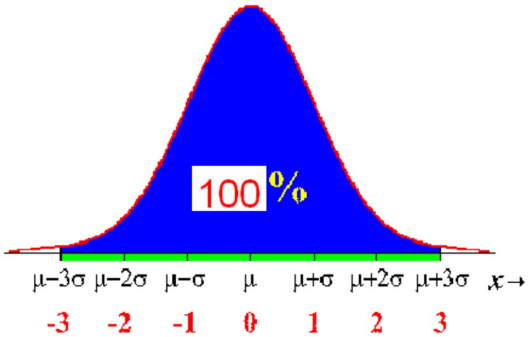
Greek letter "Mu"

We will almost always be using the entire set of data (population) so for our sake we can use these two interchangeably

Approximately 95% of the observations fall within 2 standard deviations of the mean



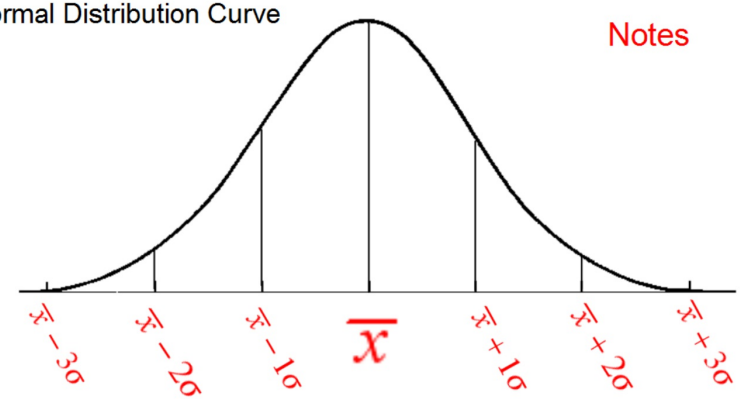
Approximately 100 % of the observations fall within 3 standard deviations of the mean

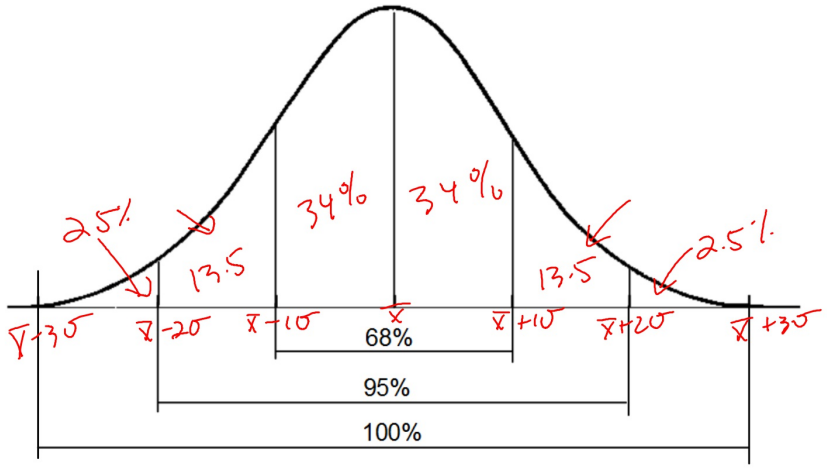


$$(\mu - \sigma \leq x \leq \mu + \sigma) \approx 0.6827$$
$$(\mu - 2\sigma \leq x \leq \mu + 2\sigma) \approx 0.9545$$
$$(\mu - 3\sigma \leq x \leq \mu + 3\sigma) \approx 0.9973$$

Normal Distribution Curve

Notes





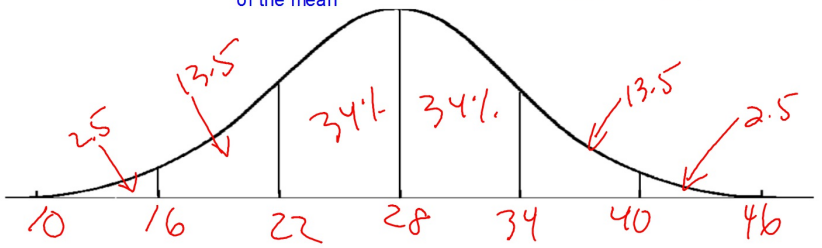
http://www.mathsisfun.com/data/standard-normal-distribution-table.html

An employer surveyed its employees about commuting time to work. The mean commuting time was 28 minutes with a standard deviation of 6 minutes.

1. Within what range of times do 68% of the workers spend time commuting?

within one standard deviation of the mean

22 – 34 min



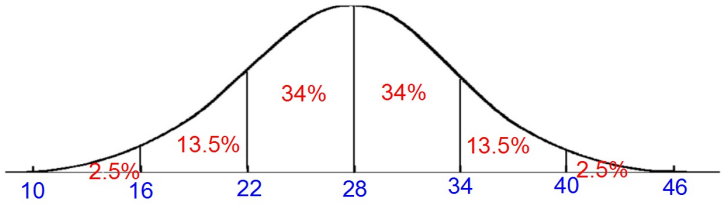
2. Within what range of times do 95% of the workers spend time commuting?

within 2 standard deviations of the mean

16 min to 40 min

3. What % of workers spend between 10 and 22 minutes commuting?

2.5 + 13.5 = 16%



4. What % of workers spend between 28 and 40 minutes commuting?

34% + 13.5% = 47.5%

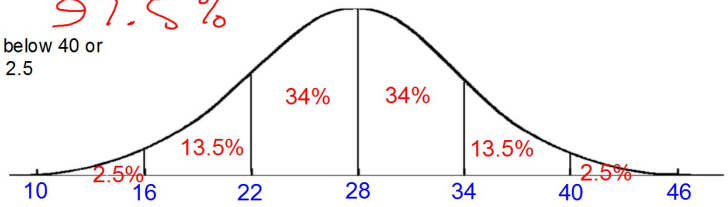
5. What % of workers spend less than 28 minutes commuting?

50%

6. What % of workers spend less than 40 minutes commuting?

97.5%

All % below 40 or
100 - 2.5



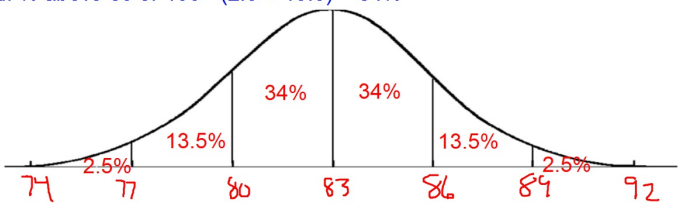
A standardized test had the following statistics:
Mean = 83 Std Dev = 3

1. Your score was 89.

What % of the scores were less than yours?
All % less than 89 or 100 - 2.5 = 97.5%

2. Your score was 80.

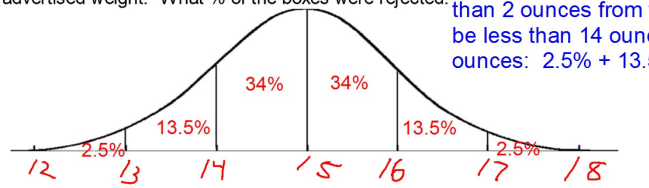
What % of the scores were better than yours?
All % above 80 or 100 - (2.5 + 13.5) = 84%



A cereal manufacturer samples some boxes of cereal which are supposed to weight 16 ounces.
The mean weight of the boxes sampled was 15 with a standard deviation of 1

1. What % of the boxes weighed less than 17 ounces?
add up all % below 17 or 100 - 2.5 = 97.%
2. What interval contained 95% of the boxes?
Within two standard deviations of the mean: 13 to 17
3. What % of the boxes contained from 13 to 15 ounces?
13.5 + 34 = 47.5%
4. A box is rejected if it is more than 2 ounces from the advertised weight. What % of the boxes were rejected.

The advertised weight is 16 ounces. More than 2 ounces from this weight would be less than 14 ounces or greater than 18 ounces: 2.5% + 13.5% = 16% of the boxes were rejected.



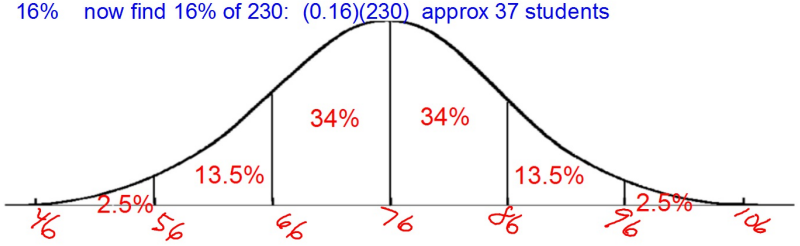
Scores on an exam are normally distributed with a mean of 76 and a standard deviation of 10. There were 230 tests taken.

1. How many students scored above 96?

2.5% now find 2.5% of 230: (0.025)(230) approx 6 students

2. How many students scored below 66?

16% now find 16% of 230: (0.16)(230) approx 37 students



Scores on an exam are normally distributed with a mean of 76 and a standard deviation of 10. There were 230 tests taken.

3. How many students scored between 56 and 86?

13.5 + 34 + 34 = 81.5% Now find 81.5% of 230: (0.815)(230) approx 187 students

4. You select an exam at random. What is the probability that the score is between 76 and 96?

34 + 13.5 = 47.5%