Outcomes, Probability, and Statistics

You want to order a 1-topping pizza for dinner.

When you call they ask you what size of pizza, what kind of crust, and which topping you want. You have the following to choose from:

3 Sizes
7 different toppings
4 different kinds of crust.

$3 \cdot 7 \cdot 4$

How many different 1-topping pizzas are possible? = 84 different pizzas

Use the Multplication Counting Principle

You have 6 different pictures you want to hang in a row on your wall. How many different ways could you arrange the paintings on your wall?

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ = 720

Factorial: $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$6! = 720$

Factorial is usually used if you are arranging ALL of the available items.

Find each:

1. $7! = 5040$

2. $10! = 3{,}628{,}800$

3. $\frac{8!}{5!} = 336$

There are 12 people on a basketball team and only 12 uniform numbers to pass out.

1. How many different ways can all 12 uniform numbers be passed out to the players?

$12! = 479{,}001{,}600$

2. If there were 12 uniforms but only 10 players, how many ways could the uniforms be passed out?

$12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = \frac{12!}{2!}$

$= 239{,}500{,}800$

How many different four digit sequences can you create using the digits from 0 to 9 if digits can repeat?

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$

How many different four digit sequences can you create using the digits from 0 to 9 if digits CAN'T repeat?

$10 \cdot 9 \cdot 8 \cdot 7 = 5040$

In a certain state lisence plates consist of 3 letters then 3 digits.

How many different lisence plates are possible if letters and digits CAN repeat?

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

$= 17{,}576{,}000$

How many different lisence plates are possible if letters and digits CAN'T repeat?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 11{,}232{,}000$

How many different lisence plates are possible if letters CAN repeat but digits CAN'T repeat?

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 9 \cdot 8 = 12{,}654{,}720$

You open a new account and need to create a password.

Passwords must be 5 characters long and must contain the following:
3 Letters(case sensitive) followed by 2 digits (0-9).
You can repeat letters but can't repeat digits.

How many different passwords are possible?

$52 \cdot 52 \cdot 52 \cdot 10 \cdot 9 =$ 12,654,720

Multiplication Counting Principle:
The number of outcomes is the product of the number of choices for each step.

Factorial:
Is mostly used when you are using ALL of a given amount of items.

Permutation:
The number of outcomes when order DOES matter.

There are 5 people running a race.
What if prizes are only awarded to the top three finishers?
In other words, how many ways can 1st, 2nd, and 3rd places be awarded to 5 people running in the race?

Multiplication Counting Principle:

Permutation: An arrangement of items when order DOES matter.

Permutation Formula: When order DOES matter

Ways to arrange n items r at a time.

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

n= total # items

r = # arranging at a time

${}_5P_3 = 60$

There are 5 people running a race.

What if prizes are only awarded to the top three finishers?

In other words, how many ways can 1st, 2nd, and 3rd places be awarded to 5 people running in the race?

Multiplication Counting Principle: $5 \cdot 4 \cdot 3 = 60$

OR

Permutation: Arrangment of 5 things 3 at a time $_5P_3 = 60$

You have 8 new books that just arrived and you want to display them a shelf in the book store.

1. How many ways can you arrange these 8 books on the shelf?

$8! = 40{,}320$ or $_8P_8 = 40{,}320$

2. There is only room for 5 books on the shelf. How many ways can you arrange 5 of these books on the shelf? $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = {}_8P_5 = 6720$

You are playing Scrabble. You choose 7 tiles from the pile.

1. How many ways can you arrange all 7 in front of you? $7!$ or $_7P_7 = 5040$

2. If you can only play 3 at a time, find the number of ways you can arrange 3 of the 7 tiles on the board.

$7 \cdot 6 \cdot 5$ or $_7P_3 = 210$

There are 12 members of a club. The club needs to select a President and a Treasurer.

How many different ways can a President and a Treasurer be selected from amongst the 12 members?

$12 \cdot 11$ or $_{12}P_2 = 132$

You want to order a two topping pizza. If there are only 4 toppings to choose from, how many different 2 topping pizzas are possible?

Toppings:

- Pepperoni
- Mushrooms
- Onions
- Green Peppers

Create a systematic list to help answer this question.

PM MO OG
PO MG
PG

6 different 2-topping pizzas

You want to order a two topping pizza. If there are only 4 toppings to choose from, how many different 2 topping pizzas are possible?

Toppings:

- Pepperoni
- Mushrooms
- Onions
- Green Peppers

Does order matter in this situation?

No

Combination:

Selecting a number of items when order DOESN'T matter.

You order a shake at a shop. There are 7 ingrediants to choose from. You buy a shake that contains 4 ingredients. How many different shakes are possible?

Does this situation represent a Combination or a Permuation?

Combination.

Combination Formula: When order DOESN'T matter

Ways to choose n items r at a time.

$$ {}_nC_r = \frac{n!}{r!\,(n-r)!} $$

n= total # items

r = # selecting at a time