Y-intercepts of a Rational Function:

Replace x with zero

Ratio of the Constants

X-intercepts of a Rational Function:

Replace y with zero

The zeros of the numerator
(unless they are also zeros of the denominator)

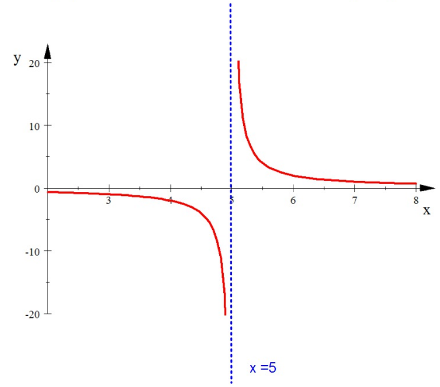
What a graph does when it approaches a Vertical Asymptote:

$$y = \frac{2}{x - 5}$$

VA: x = 5

the graph either increases or decreases without bound as you approach a VA.

the y-value either becomes larger and larger positive or larger and larger negative.



Let's put this all together.
Graph the following Rational Function showing:

- All asymptotes as dashed lines
- X & Y-intercepts, if any
- Correct behavior around each asymptote.

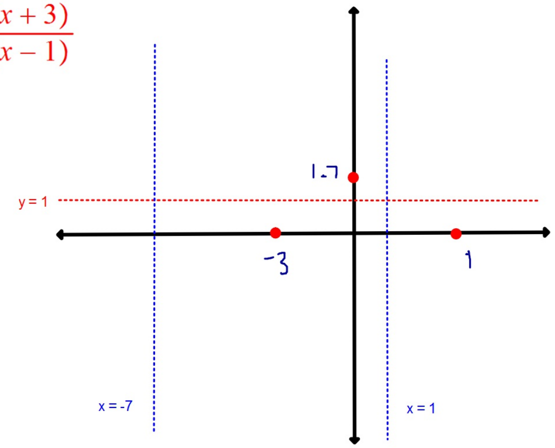
$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

x-int: 4, -3

y-int: $\frac{-12}{-7} = 1.7$

VA: X = -7, 1

HA: y = 1



What does the graph do on the left and right sides of the two VA?

$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

Left of X = -7

x	y
-7.1	$\frac{-\,-}{-\,-}$

pos ↓ up

Right of X = -7

x	y
-6.9	$\frac{-\,-}{+\,-}$

= NEG Down

Left of X = 1

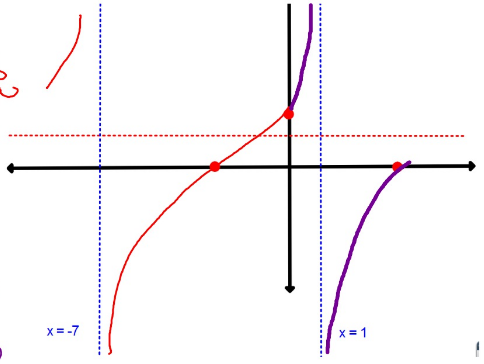
x	y
0.9	$\frac{-\,+}{+\,-}$

= Pos ↓ up

Right of X = 1

x	y
1.1	$\frac{-\,+}{+\,+}$

= NEG ↓ DowN



Does the graph approach the HA y=1 from above or below on the:

$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7}$$

Left-end

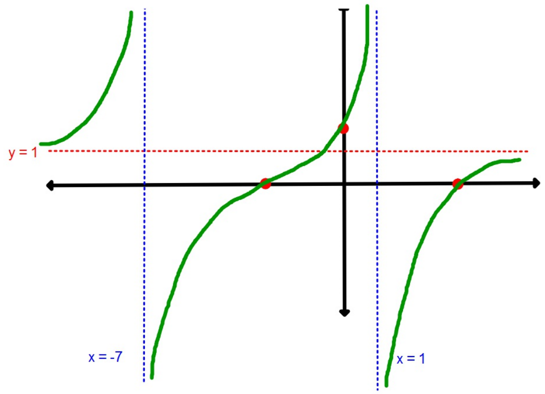
x	y
-100	1.074
-1000	1.007
-10000	1.0007

approaches the HA from above on the left

Right-end

x	y
100	0.93345
1000	0.99304
10000	0.9993

approaches the HA from below on the right



Graph this rational function:

$$y = \frac{x-8}{x^2 + 3x - 4} = \frac{x-8}{(x+4)(x-1)}$$

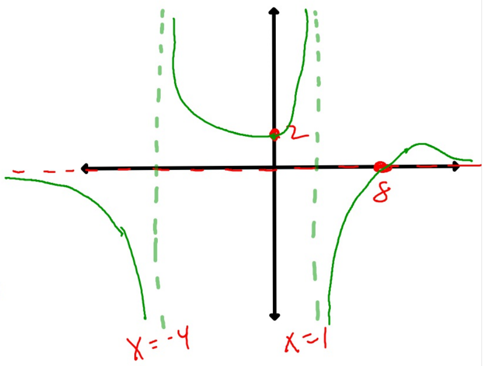
y-int: 2

x-int: 8

HA: y=0

VA: X=-4, 1

Typically if a graph goes up on one side of a VA it tends to go down on the other side.



Graph this rational function:

$$y = \frac{x^2 - 25}{x^2 - 6x + 9} = \frac{(x+5)(x-5)}{(x-3)^2}$$

y-int: $-\frac{25}{9} = -2.7$

x-int: ±5

HA: y=1

VA: X=3

Because of the double factor in the denominator $(x - 3)^2$ the graph does the same thing on both sides of the VA x=3 (goes down)