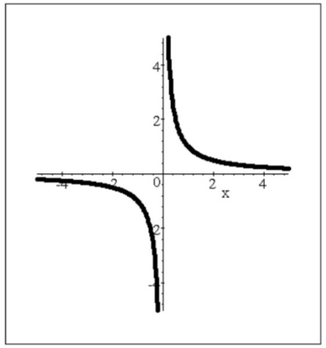
Horizontal Asymptotes:

The horizontal line that a graph approaches as you move farther and farther to the left and right. (END BEHAVIOR)

The value of y that the function approaches as x gets larger and larger (pos and neg).

The graph might come down to the HA at the end or it might rise up to the HA at the end



This is the graph of the Parent Reciprocal Function:

$y = 1/x$ the Horizontal Asymptote is $y = 0$

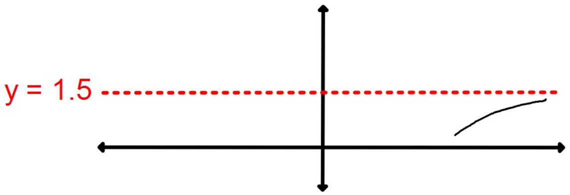
The left-end approaches the HA from BELOW

The right-end approaches the HA from ABOVE

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

The HA of this rational function is $y = 1.5$

Does the right-end approach the HA from above or below?



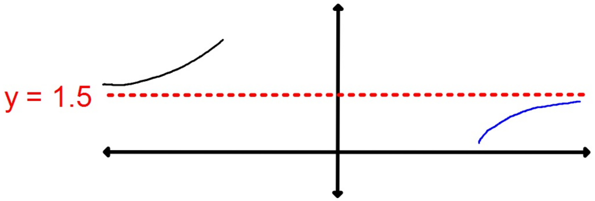
Right-end

x	y
10	1.2207
100	1.479
1000	1.498
10000	1.4998

this shows that the function is just below the HA (<1.5) on the right-end.

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

Does the left-end approach the HA from above or below?



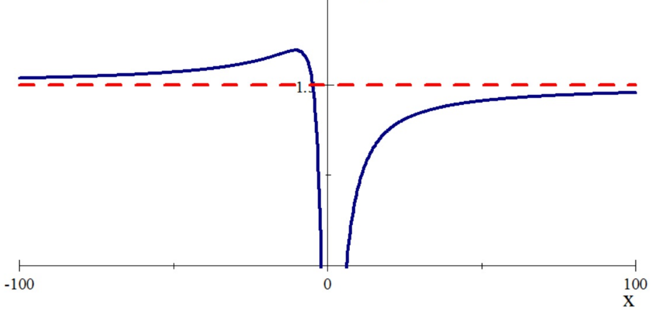
Left-end

x	y
-10	1.5962
-100	1.519
-1000	1.502
-10000	1.5002

this shows that the function is just above the HA (>1.5) on the left-end.

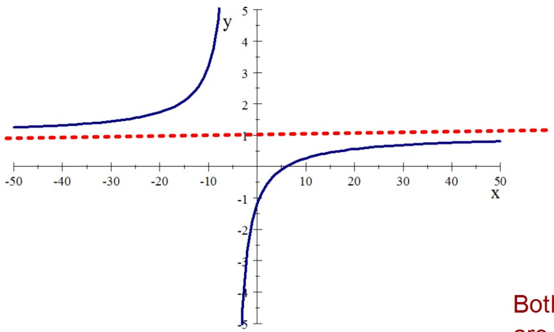
$y = \frac{3x^2 - 4x}{13 + 2x^2}$

this is the actual graph of the function and it shows that the left-end approaches the HA from above and the right-end approaches the HA from below.



Horizontal Asymptote Exploration:

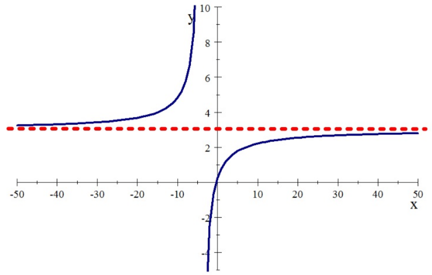
1. $y = \frac{x - 6}{x + 5}$ HA: y = 1



x	y
100	.89524
1000	.98905
10000	.9989
-100	1.1158
-1000	1.0111
-10000	1.0011

Both the right and left ends are approaching y = 1

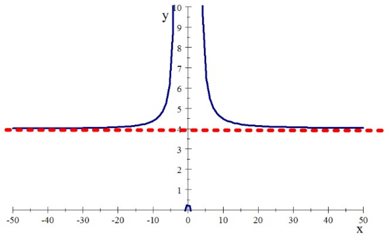
2. $y = \frac{3x + 1}{x + 4}$ HA: y = 3



x	y
100	2.8942
1000	2.989
10000	2.9989
-100	3.1146
-1000	3.011
-10000	3.0011

Both the right and left ends are approaching y = 3

3. $y = \frac{8x^2 + x - 6}{2x^2 - 21}$ HA: y = 4



x	y
100	4.0089
1000	4.0005
10000	4.0001
-100	3.9989
-1000	3.9995
-10000	3.99995

Both the right and left ends are approaching y = 4

What do you notice in the equations that would give you the HA?

1. $y = \frac{x - 6}{x + 5}$ HA: y = 1 = 1/1

2. $y = \frac{3x + 1}{x + 4}$ HA: y = 3 = 3/1

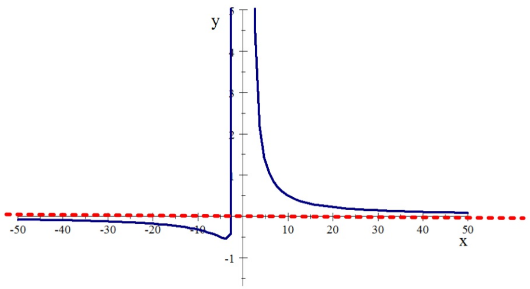
3. $y = \frac{8x^2 + x - 6}{2x^2 - 21}$ HA: y = 4 = 8/2

What do these three equations have in common?

The degree of the numerator and denominator are the same.

When this is true the HA is the ratio of the leading coefficients.

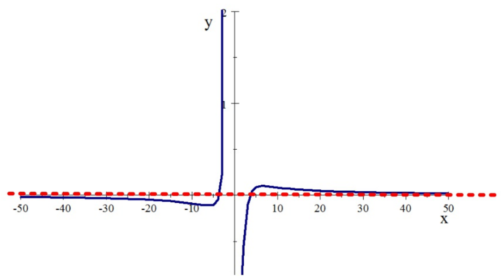
4. $y = \frac{4x + 9}{x^2 - 3}$ HA: y = 0



x	y
100	.04091
1000	.00401
10000	.0004
-100	-.0391
-1000	-.004
-10000	-.0004

Both the right and left ends are approaching y = 0

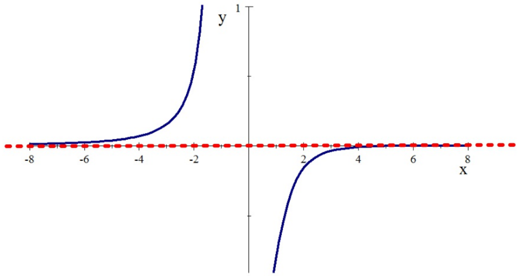
5. $y = \frac{x^2 - 13}{x^3 + 7}$ HA: y = 0



x	y
100	.00999
1000	..001
10000	.0001
-100	-.01
-1000	-.001
-10000	-.0001

Both the right and left ends are approaching y = 0

6. $y = \frac{x-5}{2x^3+3}$ HA: Y=0



x	y
100	.000047
1000	.0000005
10000	.000000005
-100	-.000053
-1000	-.0000005
-10000	-.000000005

Both the right and left ends are approaching y = 0

What do you notice in the equations that would give you the HA?

4. $y = \frac{4x+9x}{x^2-3}$ HA: y = 0

5. $y = \frac{x^2-13}{x^3+7}$ HA: y = 0

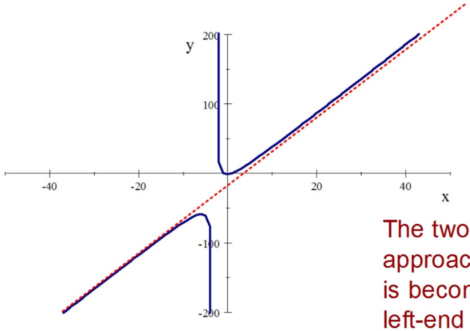
6. $y = \frac{x-5}{2x^3+3}$ HA: y = 0

What do these three equations have in common?

The degree of the denominator is greater than the degree of the numerator.

When this is the case the HA is always y = 0

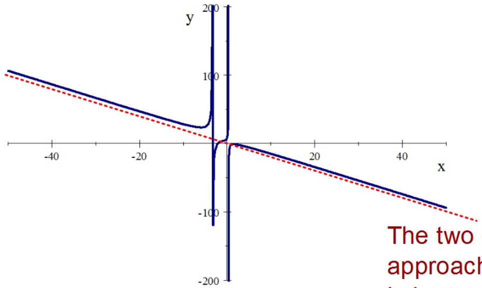
7. $y = \frac{5x^2-4}{x+3}$ HA: NONE



x	y
100	485.4
1000	4985
10000	49985
-100	-515.4
-1000	-5015
-10000	-50015

The two ends of this function aren't approaching the same number. The right-end is becoming larger and larger positive and the left-end is becoming larger and larger negative. This function has NO HA.

8. $y = \frac{-2x^3+5x-8}{x^2+3x-1}$ HA: NONE



x	y
100	-194.1
1000	-1994
10000	-19994
-100	206.15
-1000	2006
-10000	20006

The two ends of this function aren't approaching the same number. The right-end is becoming larger and larger negative and the left-end is becoming larger and larger positive. This function has NO HA.

What do you notice in the equations that would tell you that there is no HA?

7. $y = \frac{5x^2 - 4}{x + 3}$ HA: None

8. $y = \frac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA: None

What do these three equations have in common?

The degree of the numerator is greater than the degree of the denominator.

When this is the case the function has NO HA.

Without using a table or a graph how could you tell from the equation what the horizontal asymptote is or if it even has one?

Predict the Horizontal Asymptote for each of the rational functions below, if any.

a. $y = \frac{10x + 7}{5x - 3}$ HA: $y = 2$

b. $y = \frac{6x^2 - 5}{2x + 3}$ HA: NONE

c. $y = \frac{12x - 11}{3x^2 - 1}$ HA: $y = 0$

Horizontal Asymptotes: Depends on

Determine by the equation the Horizontal Aysmptote for each rational function, if it has one.

1. $y = \frac{x^3 + 4x^2 - 9}{2x^2 + 6}$

Deg of Num > Deg of Denom

No HA

2. $y = \frac{15x^2 - 2x + 10}{3x^2 + 5}$

Deg of Num = Deg of Denom

HA: y = 15/3 = 5

3. $y = \frac{20x + 13}{4x^2 + 9}$

Deg of Denom > Deg Num

HA: y = 0

x and y intercepts of Rational Functions:

Y-Intercepts: the result of replacing x with zero.

Find the y-intercepts of each function

1. $y = \dfrac{x^2 - 9x + 20}{x^2 + 7x + 10}$

y-int = 20/10 = 2

2. $y = \dfrac{4x^2 + 3x}{2x^2 - 7x + 1}$

y-int = 0/1 = 0

3. $y = \dfrac{x^2 - 4}{2x^2 + 6x}$

y-int = -4/0 which is undefined so, there is NO y-int

In general, the y-intercepts of a Rational Function is the:

Ratio of the Constants

A graph can have at most ONE y-intercept.

X-Intercepts: the result of replacing y with zero.

This means you are setting the ratio equal to zero and solving for x.

The only way a fraction equals zero is if the NUMERATOR equals zero.

In general, the x-intercepts of a Rational Function are the:

Zeros of the numerator (as long as they don't match zeros of the denominator)

A graph can have multiple x-intercepts.