

Write an equation for the translation of $y = \frac{3}{x}$ that has the given asymptotes.

1. $y = 4$ and $x = -3$

$$y = \frac{3}{x+3} + 4$$

3. $y = -5$ and $x = 0$

$$y = \frac{3}{x} - 5$$

2. $y = 0$ and $x = 9$

$$y = \frac{3}{x-9}$$

Write the equation of each transformation of the reciprocal function:

$$y = \frac{1}{x}$$

1. 5 units left, twice as tall, branches are in quadrants I and III

$$y = \frac{2}{x+5}$$

2. 8 units up, half as tall, branches are in quadrants II and IV

$$y = \frac{-0.5}{x} + 8$$

3. 3 units right, 2 units down, branches are in quadrants II and IV

$$y = \frac{-1}{x-3} - 2$$

You can now finish Hwk #2

Sec 9-2

Due tomorrow

Practice Sheet.

When the denominator of a rational function is zero the function is undefined.

Because this value of x can never be used this leads to a break in the graph (it's not continuous)

These breaks in the graph are one of two types:

Vertical Asymptotes

Holes

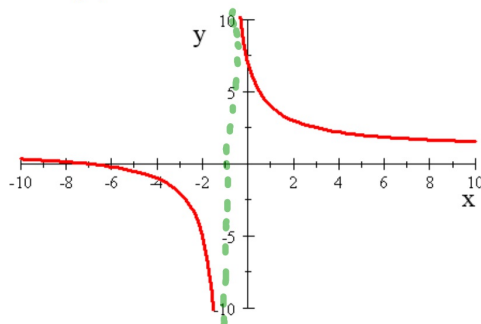
Graph the rational function $f(x)$ in a standard window.

$$f(x) = \frac{x+7}{x+1}$$

There is a break in the graph at $x = -1$

This kind of break in the graph is called a

Vertical Asymptote

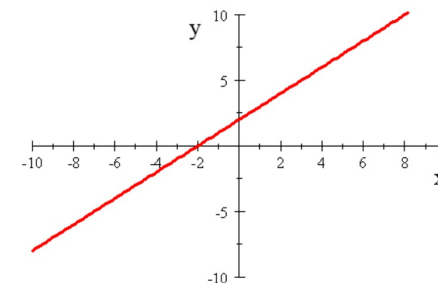


Graph the rational function $f(x)$ in a standard window.

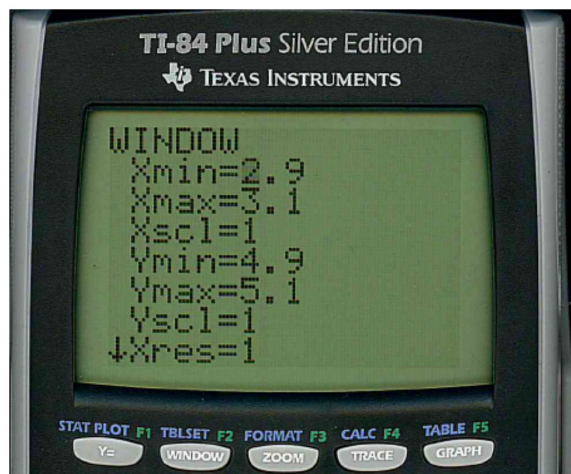
$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

Do you see a vertical asymptote?

Why do you think that there isn't a vertical asymptote at $x = 3$?

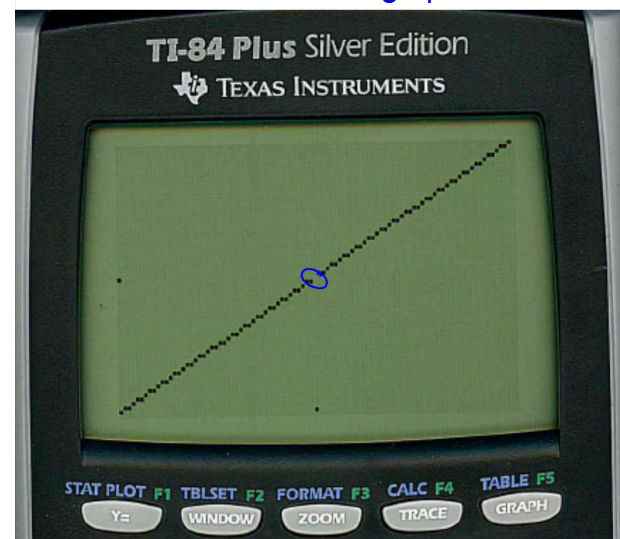


Look at the graph in the following window:



What do you see?

This kind of break in the graph is called a **Hole**



There is a hole in the graph when $x=3$ because it makes the function undefined.

Why did this graph have a Vertical Asymptote at $x = -1$

$$f(x) = \frac{x+7}{x+1}$$

and

this graph have a hole at $x = 3$?

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

$$f(x) = x+2$$

The original equation simplifies to the line $y=x+2$. This is why the graph looks like a line but has a hole when $x=3$.

Breaks in the graph are caused by zeros of the denominator and are called:

Points of Discontinuity

Holes

or

Vertical Asymptotes

Occur at values of x that are zeros of

both the denominator

AND numerator

Occur at values of x that are zeros of

the denominator ONLY.

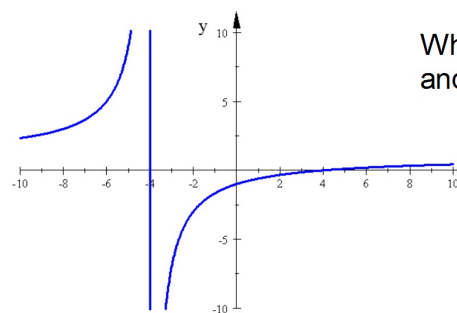
An exception to this rule:

difference of perfect squares

$$y = \frac{x^2 - 16}{x^2 + 8x + 16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$

Handwritten diagram showing the difference of perfect squares: $x^2 - 16$ is factored into $(x+4)(x-4)$. The denominator $x^2 + 8x + 16$ is factored into $(x+4)(x+4)$. A handwritten '16' is crossed out with a blue 'X' and the number '8' is written below it, indicating the process of factoring the denominator as a perfect square trinomial.

$$y = \frac{x^2 - 16}{x^2 + 8x + 16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$



Why is there a VA at $x=-4$ and not a hole?

Even though the factors $(x+4)$ are common to the numerator and denominator, when you cancel them there is still $(x+4)$ left in the denominator.

Properties**Vertical Asymptotes**

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a , then there is a hole in the graph or a vertical asymptote at $x = a$.

Find any points of discontinuity and classify them as Vertical Asymptotes or Holes.

$$1. y = \frac{2x(x+1)(x-6)}{(x-1)(x-6)}$$

Pts of Discontinuity: $x = 1, 6$
zeros of the denominator

VA: $x = 1$

Holes: $x = 6$

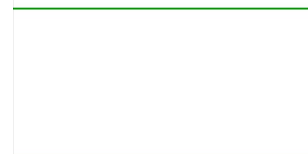
$$2. y = \frac{3x^2 - 6}{x^2 - 4}$$

$3(x^2 - 2)$
 $(x+2)(x-2)$

Pts of Discontinuity: ± 2
zeros of the denominator

VA: $x = 2, -2$

Holes: NONE



$$3. y = \frac{(x-4)(x+3)}{x^2 - 16}$$

$(x+4)(x-4)$

Pts of Discontinuity:
 $x = -4, 4$

VA: $x = -4$

Holes: $x = 4$

$$4. y = \frac{(x-5)(x-5)}{x^2 - 10x + 25}$$

$(x-5)(x+1)$

Pts of Discontinuity:
 $x = -1, 5$

VA: $x = -1$

Holes: $x = 5$

$$5. y = \frac{2x^2}{x^2 + 3}$$

VA:

There are no points of discontinuity because the denominator has no real zeros!

Holes: