

Sec 7-2: Rationalizing Denominators of Radical Expressions

To rationalize a denominator means to remove any irrational number from the denominator.

Rationalize each denominator and simplify.

$$\frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$$

$\sqrt{169} \rightarrow 13$

What follows are three ways you might rationalize

Multiply numerator and denominator by the same square root that's in the denominator.

$$\frac{20}{\sqrt{18}}$$

$$\begin{aligned} \frac{20}{\sqrt{18}} \cdot \frac{\sqrt{18}}{\sqrt{18}} &= \frac{20\sqrt{18}}{18} = \frac{10\sqrt{18}}{9} \leftarrow \sqrt{18} \\ &= \frac{10 \cdot 3\sqrt{2}}{9} \\ &= \boxed{\frac{10\sqrt{2}}{3}} \end{aligned}$$

Multiply numerator and denominator by the smallest square root that makes the denominator a perfect square.

$$\begin{aligned} \frac{20}{\sqrt{18}} \cdot \frac{\sqrt{2}}{\sqrt{2}} &= \frac{20\sqrt{2}}{\sqrt{36}} = \frac{20\sqrt{2}}{6} \\ &= \boxed{\frac{10\sqrt{2}}{3}} \end{aligned}$$

Simplify radical first
then rationalize

$$\frac{20}{\sqrt{18}} = \frac{20}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{6} = \boxed{\frac{10\sqrt{2}}{3}}$$

Rationalize each denominator and simplify.

$$\frac{7}{\sqrt[3]{11}} \cdot \frac{\sqrt[3]{11^2}}{\sqrt[3]{11^2}} = \frac{7\sqrt[3]{11^2}}{\sqrt[3]{11^3}}$$

What follows is four ways that you might rationalize $\frac{3}{\sqrt[3]{16}}$

$$\frac{3}{\sqrt[3]{16}} \cdot \frac{\sqrt[3]{16^2}}{\sqrt[3]{16^2}} = \frac{3 \cdot \sqrt[3]{16^2}}{\sqrt[3]{16^3}} = \frac{3 \cdot \sqrt[3]{16^2}}{16} = \frac{3 \cdot 4 \sqrt[3]{4}}{16} = \frac{3\sqrt[3]{4}}{4}$$

$$\frac{3}{\sqrt[3]{16}} = \frac{3}{\sqrt[3]{4^2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{3\sqrt[3]{4}}{\sqrt[3]{4^3}} = \frac{3\sqrt[3]{4}}{4}$$

$$\frac{3}{\sqrt[3]{16}} = \frac{3}{2\sqrt[3]{8}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{3\sqrt[3]{2^2}}{2\sqrt[3]{2^3}} = \frac{3\sqrt[3]{2^2}}{2 \cdot 2} = \frac{3\sqrt[3]{4}}{4}$$

$$\frac{3}{\sqrt[3]{16}} = \frac{3}{\sqrt[3]{2^4}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{3\sqrt[3]{2^2}}{\sqrt[3]{2^6}} = \frac{3\sqrt[3]{4}}{4}$$

Rationalize each denominator and simplify. Assume all variables are positive.

$$\frac{5}{\sqrt{m}} \cdot \frac{\sqrt{m}}{\sqrt{m}} = \boxed{\frac{5\sqrt{m}}{m}}$$

$$\frac{12}{\sqrt{m^3}} \cdot \frac{\sqrt{m}}{\sqrt{m}} = \frac{12\sqrt{m}}{\sqrt{m^4}} = \boxed{\frac{12\sqrt{m}}{m^2}}$$

$$\frac{2}{\sqrt[3]{m^{11}}} \cdot \frac{\sqrt{m}}{\sqrt{m}} = \boxed{\frac{2\sqrt{m}}{m^6}}$$

What would you multiply numerator and denominator by to rationalize each?

$$\frac{1}{\sqrt[3]{p}} \cdot \frac{\sqrt[3]{p^2}}{\sqrt[3]{p^2}}$$

$$\frac{7}{\sqrt[5]{d^2}} \cdot \frac{\sqrt[5]{d^3}}{\sqrt[5]{d^3}}$$

$$\frac{3}{\sqrt[6]{a^{15}}} \cdot \frac{\sqrt[6]{a^3}}{\sqrt[6]{a^3}}$$

What would you multiply numerator and denominator by to rationalize each?

$$\frac{10}{\sqrt{c^5 d^9}} \cdot \frac{\sqrt{cd}}{\sqrt{cd}}$$

$$\frac{6}{\sqrt[3]{rp^2}} \cdot \frac{\sqrt[3]{r^2 p}}{\sqrt[3]{r^2 p}}$$

