$$\frac{x^3 - 5x^2 + 8x - 11}{x + 3} = x^2 - 8x + 32 - \frac{107}{x+3}$$

What is the remainder? -107

What is the zero of the divisor? -3

Evaluate the dividend using the zero of the divisor $(-3)^3 - 5(-3)^2 + 8(-3) - 11 = -107$

Do you notice anything?

The remainder is the same as if you replace x in the dividend with the zero of the divisor!

Theorem **Remainder Theorem**

If a polynomial $P(x)$ of degree $n \geq 1$ is divided by $(x - a)$, where a is a constant, then the remainder is $P(a)$.

If you want just the remainder evaluate the dividend using the zero of the divisor.

Find the remainder to this quotient: $\frac{2x^3 + x^2 - 7x - 10}{x - 2}$ $\rightarrow R = -4$

$2(2)^3 + (2)^2 - 7(2) - 10 = -4$

What is the remainder of this quotient?

$$\frac{6x^2 + 5x - 2}{x - 4}$$

remainder = 114

evaluate the dividend using the zero of the divisor: +4

$6(4)^2 + 5(4) - 2 = 114$

Is $x - 3$ a factor of $2x^3 - 12x^2 + 21x - 9$?

$2(3)^3 - 12(3)^2 + 21(3) - 9 = 0$

$R = 0$

The divisor is a factor of the dividend...
Only if the remainder is zero!

Since the remainder is zero, x-3 IS a factor.

Is $x+2$ a factor of $x^3+7x^2+3x-21$?

$f(-2) = (-2)^3 + 7(-2)^2 + 3(-2) - 21 = -7$

Since the remainder ISN'T zero
x+2 is NOT a factor.

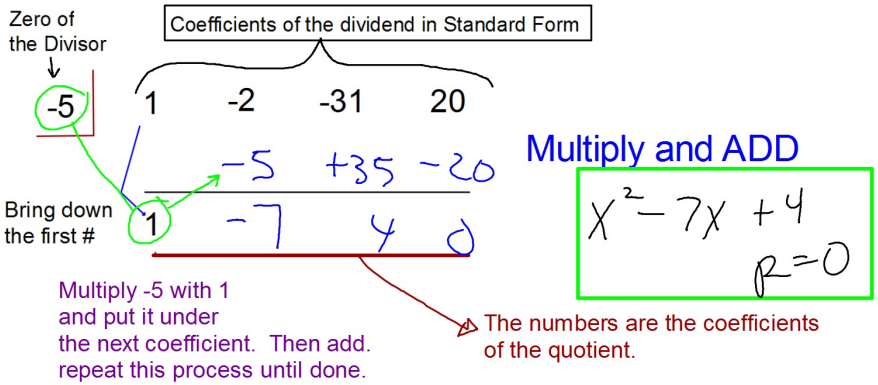
Synthetic Division

Uses the zero of the divisor.
By reversing the sign of the divisor you can ADD throughout the process instead of subtracting.

Works only when the leading coefficient of the divisor is 1.

Meaning either ÷(x + a) or ÷(x - a)

$$\frac{x^3-2x^2-31x+20}{x+5}$$



Multiply and ADD

$x^2 - 7x + 4$
$R = 0$

Find each quotient using Synthetic Division

1. $\frac{4x^3 - 6x^2 - 7x - 33}{x - 3}$

3 | 4 -6 -7 -33
12 18 33
4 6 11 0

$= 4x^2 + 6x + 11$ $R = 0$

2. $\frac{2x^4 + 18x^3 + 34x^2 + 43x + 10}{x + 7}$

-7 | 2 18 34 43 10
-14 -28 -42 -7
2 4 6 1 3

$= 2x^3 + 4x^2 + 6x + 1$
$R = 3$

Find this quotient using Synthetic Division.

$$\frac{4x^3 - x + 9}{x - 3}$$

3|

	4	0	−1	9
		12	36	105
	4	12	35	114

$4x^2 + 12x + 35$ R 114

Is x+7 a factor of $x^3 - 2x^2 + 10x - 21$?

Using the Remainder Theorem we find the remainder isn't zero so x+7 is not a factor.

$(-7)^3 - 2(-7)^2 + 10(-7) - 21 = -532$

Doing Synthetic Division we find the remainder isn't zero so x+7 is not a factor

−7	1	−2	10	−21
		−7	63	−511
	1	−9	73	−532

Given $f(x) = 3x^4 - 5x^3 + 8x^2 - 7x + 10$

Find f(2)

$f(2) = 3(2)^4 - 5(2)^3 + 8(2)^2 - 7(2) + 10 = 36$

Given $f(x) = 3x^4 - 5x^3 + 8x^2 - 7x + 10$

How could you use Synthetic Division to find f(2)?

f(2) gives the remainder of the quotient when the divisor is (x - 2)

f(2) is the remainder of this $\frac{3x^4 - 5x^3 + 8x^2 - 7x + 10}{x - 2}$

2	3	−5	8	−7	10
		6	2	20	26
	3	1	10	13	36

f(2) = 36

Find f(-3) for the function $f(x) = 5x^3 + 11x^2 - 2x + 7$

$f(-3) = 5(-3)^3 + 11(-3)^2 - 2(-3) + 7 = -23$

Or you could do Synthetic Division with -3 and the answer will be the remainder:

$$\begin{array}{r|rrrr} -3 & 5 & 11 & -2 & 7 \\ & & -15 & 12 & -30 \\ \hline & 5 & -4 & 10 & -23 \end{array}$$

Given $x - 5$ is a factor of $2x^3 - 11x^2 - 16x + 105$

Use synthetic division to help find the other two factors.

$$\begin{array}{r|rrrr} 5 & 2 & -11 & -16 & +105 \\ & & 10 & -5 & -105 \\ \hline & 2 & -1 & -21 & 0 \end{array}$$

$2x^2 - x - 21$

-42, -7, $+6$, -1

	x	$+3$
$2x$	$2x^2$	$+6x$
-7	$-7x$	-21

$2x^3 - 11x^2 - 16x + 105$
$=(x-5)(2x-7)(x+3)$

You can now finish Hwk #31: Sec 6-3

Pages 324 - 325

Problems 4, 9, 14, 15, 24, 37, 41

(this was originally #32 on the assignment sheet)