

Completing the Square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Diagram illustrating the steps to complete the square:

- 1st**: Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
- 2nd**: Factor the left side as a perfect square trinomial.

Complete the square for each.

$$1. \ x^2 + 14x + 49 = (x + 7)^2$$

$$2. \ x^2 - 24x + 144 = (x - 12)^2$$

$$3. \ x^2 + 56x + 784 = (x + 28)^2$$

$$4. \ 2x^2 + 8x + 6 = (x + 2)^2$$

Solve by completing the square.

$$x^2 + 22x = 5$$

$$x^2 + 22x + 121 = 5 + 121$$

$$\sqrt{(x + 11)^2} = \sqrt{126}$$

14.9

Now solve using Square Roots

$$x + 11 = \pm 3\sqrt{14}$$

$$x = -11 \pm 3\sqrt{14}$$

$$x^2 - 14x + 11 = 0$$

Rewrite into: $ax^2 + bx = c$ Form

$$x^2 - 14x = -11$$

$$x^2 - 14x + 49 = -11 + 49$$

$$x^2 - 14x + 49 = 38$$

$$\sqrt{(x - 7)^2} = \sqrt{38}$$

Now solve using Square Roots

$$x - 7 = \pm \sqrt{38} \rightarrow x = 7 \pm \sqrt{38}$$

Solve by completing the square:

$$x^2 - 10x + 7 = 0$$

$$x^2 - 10x + 25 = -7 + 25$$

$$\sqrt{(x-5)^2} = \sqrt{18}$$

$$x-5 = \pm 3\sqrt{2}$$

$$x = 5 \pm 3\sqrt{2}$$

Solve by completing the square:

$$x^2 + 6x - 16 = 0$$

$$+16 \quad +16$$

$$x^2 + 6x = 16$$

$$x^2 + 6x + 9 = 16 + 9$$

$$(x+3)^2 = 25$$

$$\sqrt{(x+3)^2} = \sqrt{25}$$

$$x+3 = \pm 5$$

$$x = -3 + 5$$

$$x = -3 - 5$$

$$x = 2$$

$$x = -8$$

Solve by completing the square:

$$x^2 + 4x + 27 = 0$$

$$x^2 + 4x + 4 = -27 + 4$$

$$\sqrt{(x+2)^2} = \sqrt{-23}$$

NO REAL SOL

Find the coordinates of the vertex of each quadratic.

Vertex Form

$$y = (x+3)^2 - 7$$

$$(-3, -7)$$

Standard Form

$$y = x^2 + 6x - 11 \quad \frac{-6}{2} = -3$$

$$(-3)^2 + 6(-3) - 11$$

$$9 - 18 - 11$$

$$-9 - 11 = -20$$

$$(-3, -20)$$

Completing the Square to write equation in Vertex Form.

$$y = x^2 + 6x - 11$$

Rearrange so that one side is only $x^2 + bx$

$$y + 11 = x^2 + 6x + 9$$

Now complete the square

$$y + 20 = (x + 3)^2$$

Move the 20 over to the right side.

$$y = (x + 3)^2 - 20$$

Vertex $(-3, -20)$

Write each equation into Vertex Form

1. $y = x^2 + 8x + 3$

2. $y = (x^2 + 12x - 19)(-1)$

$$y - 3 = x^2 + 8x + 16$$

$$-y = x^2 - 12x + 19$$

$$y + 13 = (x + 4)^2$$

$$-y - 19 = x^2 - 12x + 36$$

$$y = (x + 4)^2 - 13$$

$$-y + 17 = (x - 6)^2$$

$$-1(-y) = ((x - 6)^2 - 17) - 1$$

$$y = -(x - 6)^2 + 17$$

Solve by completing the square:

$$x^2 + 3x - 10 = 0$$

$$x^2 + 3x + \frac{9}{4} = 10 + \frac{9}{4}$$

$$\sqrt{(x + \frac{3}{2})^2} = \sqrt{\frac{49}{4}}$$

$$x + \frac{3}{2} = \pm \frac{7}{2}$$

$$+\frac{7}{2} - \frac{3}{2} = \frac{4}{2} = 2$$

$$-\frac{7}{2} - \frac{3}{2} = \frac{-10}{2} = -5$$

Why can't you solve this equation by completing the square, the way it's written.

$$2x^2 + 16x - 26 = 0$$

$$ax^2 + bx + c$$

a must be 1

$$\frac{2x^2 + 16x}{2} = \frac{26}{2}$$

$$x^2 + 8x + 16 = 13 + 16$$

$$\sqrt{(x + 4)^2} = \sqrt{29}$$

$$x + 4 = \pm \sqrt{29}$$

$$x = -4 \pm \sqrt{29}$$

Therefore, you would have to factor out the 5 or, if it's an equation, divide both sides by 5.