Linear Programming:

A technique that identifies the minimum or maximum value of some quantity that is modeled with some Objective Function that meets a set of constraints.

Notes

Our book calls this the Vertex Principle

The Corner-Point Principle:
Any maximum or minimum value of a linear combination of variables will occur at one of the vertices of the feasible region (shaded region).

A farmer wants to plant some acres of soybeans and wheat this season.

W = # acres of wheat

S = # of acres of soy

- The farmer has up to 240 acres of land to use for these crops.
- The farmer has only enough seed for at most 180 acres of wheat.

Define variables and write four inequalities to model the constraints in this situation.

$S + W \le 240$

$W \le 180$

$W \ge 0$

$S \ge 0$

S = # acres of Soybean

W = # acres of Wheat

$S \ge 0$ $W \ge 0$

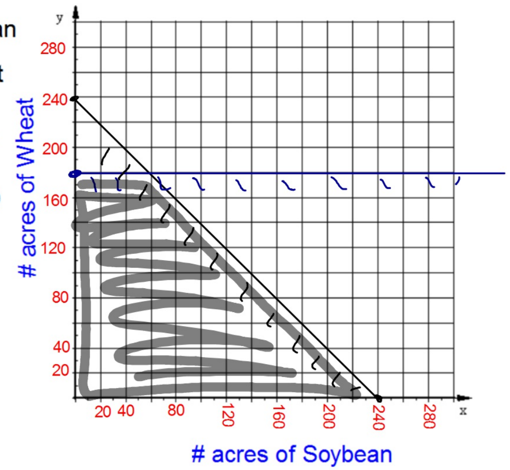
$S + W \le 240$ $W \le 180$

(0,0)

(60,180)

(0,180)

(240,0)



Suppose that the farmer can sell the Soybeans for $150 an acre and the Wheat for $200 and acre.

Write an equation that models this statement

$150s + 200w = T$

This equation is called the Objective Function

S = # acres of Soybean

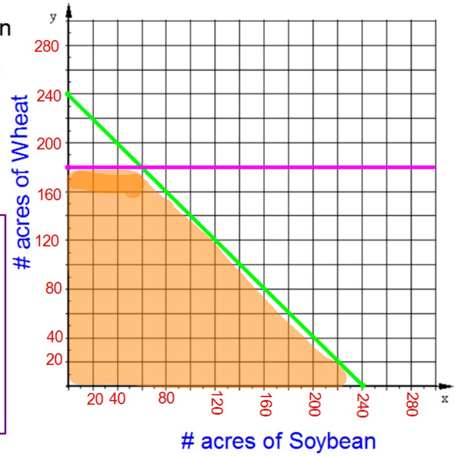
W = # acres of Wheat

$S \geq 0$ $W \geq 0$

$S + W \leq 240$ $W \leq 180$

Find the coordinates of all corners of the feasible region:

(0,0)
(240,0)
(60,180)
(0,180)



How many acres of each should be planted in order to maximize the income?

Corners of feasible region (S,W)	Objective Function 150S + 200W = Income
(0,0)	0
(240,0)	$36,000
(60,180)	$45,000
(0,180)	36,000

The farmer can maximize their income of $45,000 by planting 60 acres of Soybean and 180 acres of Wheat.

You want to sell some paintings and sculptures at a craft show. You spend 12 hours on each painting and 18 hours on each sculpture but only have 72 hours to work before the show. Each painting costs you $24 to make and each sculpture costs you $12 to make and you only have $96 to spend.

Notes

Write and graph a system of four inequalities to model the constraints in this situation.

$12p + 18s \leq 72$

$24p + 12s \leq 96$

$p \geq 0$

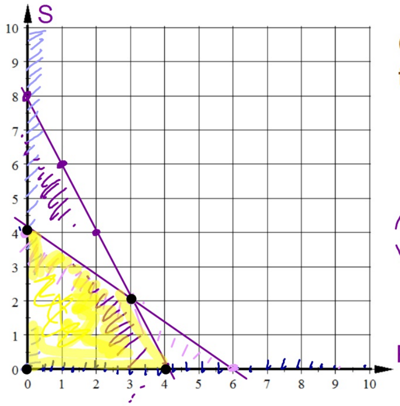
$s \geq 0$

$12p + 18s \leq 72$

$24p + 12s \leq 96$

$p \geq 0$

$s \geq 0$



Coordinates of the feasible region:

(0, 0)

(0,4)

(4,0)

(3,2)

If you sell paintings for $45 each and sculptures for $70 each how many of each should you make and sell in order to maximize your income?

Write an Objective Function.

P = # paintings
S = # sculptures
I = Income

$45p + 70s = I$

How many of each type of artwork should the artist make in order to maximize income?

Objective Function:

$45p + 70s = I$

(p,s)	45p + 70s = I
(0,0)	0
(0,4)	280
(4,0)	180
(3,2)	275

The artist should create 4 sculptures to maximize their income of $280

Hwk #15 Sec 3-4

This is #16 on the assignment sheet

Pages 142-143
Problems 2, 3, 5, 6, 11, 20
(use graph paper for 5, 6, 11, 20)

A carpenter in his spare time makes birdhouses and mailboxes. He wants to make and sell some in a few weeks at a craft fair.

It costs him \$36 to make each birdhouse and \$20 to make each mailbox. He wants to spend no more than \$540 on materials. He's been pretty busy with his regular job so he's planning on making no more than 19 items for the craft fair.

He plans on selling each birdhouse for \$75 and each mailbox for \$45.

Write and graph a system of inequalities to find out how many of each he should make and sell in order to maximize his income.

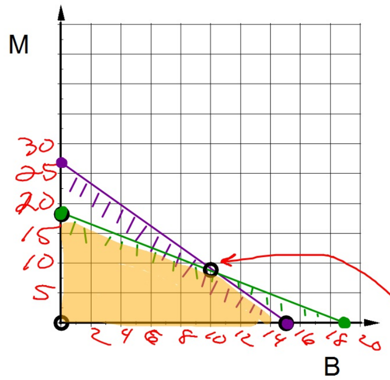
B = # birdhouses
M = # mailboxes

$B \geq 0$

$M \geq 0$

$36B + 20M \leq 540$

$B + M \leq 19$



$B \geq 0$ $M \geq 0$

These two inequalities define the 1st quadrant

$36B + 20M \leq 540$

B-intercept = 540/36 = 15
M-intercept = 540/20 = 27

Shade below this line

$B + M \leq 19$

B-intercept = 19
M-intercept =19

Shade below this line

Corners of the feasible region:

(0,0) (0,19) (15,0) (10,9)

This point is found by finding the point of intersection of the lines B+M=19 and 36B+20M=540 using Substitution, Elimination, or Matrices.

Objective Equation:

$75B + 45M = T$

Corners of the feasible region
(0,0) (0,19) (15,0) (10,9)

B	M	75B + 45M = T
0	0	0
0	19	855
15	0	1125
10	9	1155

The carpenter makes a maximum amount of money (\$1155) if he makes and sells 10 birdhouses and 9 mailboxes.