Solving a system of Linear Equations using matrices.

Matrix: Rows and Columns of data

$$\begin{bmatrix} 5 & 6 & -1 \\ 0 & -4 & 8 \end{bmatrix}$$

Dimensions of a Matrix:

the matrix at the left has the following dimensions:

2 x 3 "two by three"

Each number in a matrix is called an element

Matrices are named using a capital letter

Entering matrices on a calculator:

$$A \begin{bmatrix} 5 & 6 & -1 \\ 0 & -4 & 8 \end{bmatrix}$$

1. press 2ND then X^{-1}
2. Arrow key to EDIT and press ENTER
3. Enter the dimensions you want (Rows x Columns)
4. Enter the data.

$$A \begin{bmatrix} 5 & 6 & -1 \\ 0 & -4 & 8 \end{bmatrix} \quad 2 \times 3$$

$$B \begin{bmatrix} -9 & -1 & 2 \\ 7 & 3 & 0 \end{bmatrix} \quad 2 \times 3$$

$$C \begin{bmatrix} 4 & -3 \\ 2 & 10 \\ -6 & 5 \end{bmatrix} \quad 3 \times 2$$

Which two matrices can be:

1. Added A and B: either A+B or B+A
2. Subtracted A and B: either A-B or B-A
3. Multiplied A and C or B and C
4. Divided You can't divide matrices!

To Add and Subtract two matrices they must have the exact same dimensions.

To multiply two matrices the second matrix must have the same number of rows as the number of columns in the first matrix Their middle numbers must match:

A*C = 3 x 2 * 2 x 3 or C*B = 2 x 3 * 3 x 2

the dimensions of the answer are the first and last numbers of the two matrices being multiplied.

The dimensions of the answer matrix when you multiply two matrices:

C*B = 3 x 2 * 2 x 3 = 3 x 3

When you can multiply you get the dimensions of the resulting matrix by taking the very first number and the very last number. 3 x 3

If the middle numbers don't match you CAN'T multiply

State the dimensions of the answer matrix when you do the following matrix multiplications with the given matrices, if possible:

A (3x1) $\begin{bmatrix} 4 \\ 7 \\ 8 \end{bmatrix}$ B (2x3) $\begin{bmatrix} -1 & 0 & 2 \\ 5 & 6 & 11 \end{bmatrix}$ C (1x4) $\begin{bmatrix} 9 & -4 & 6 & 8 \end{bmatrix}$ D (3x2) $\begin{bmatrix} 0 & 3 \\ 4 & 13 \\ -5 & 2 \end{bmatrix}$

A*C = 3x1 • 1x4
yes, these can be multiplied and answer matrix will be 3x4

D*A = 3x2 • 3x1
No, these can't be multiplied

D*B = 3x2 • 2x3
yes, these can be multiplied and answer matrix will be 3x3

A*D = 3x1 • 3x2
No, these can't be multiplied

Solving a system of equations using matrices.

3.

$9C + 12D = 21$
$10C + 4D = -14$

Coefficient Matrix - A

$\begin{bmatrix} 9 & 12 \\ 10 & 4 \end{bmatrix}$ 2x2

Constant Matrix - B
(Answer Matrix)

$\begin{bmatrix} 21 \\ -14 \end{bmatrix}$ 2x1

Matrix Equation

$$A\begin{bmatrix} X \\ Y \end{bmatrix} = B$$

To solve for $\begin{bmatrix} X \\ Y \end{bmatrix}$

you would normally

divide by matrix A.

But we can't divide matrices so instead we multiply both sides by the inverse of matrix A → A^{-1}

Solving a Matrix Equation

$$A\begin{bmatrix} X \\ Y \end{bmatrix} = B$$

Solve for x and y by moving matrix A to the other side by mulitplying by the inverse of A → A^{-1}

$$\begin{bmatrix} X \\ Y \end{bmatrix} = B \cancel{A^{-1}}$$

2x1 2x2

OR

$$\begin{bmatrix} X \\ Y \end{bmatrix} = A^{-1} \cdot B$$

2x2 2x1

$4x + 3y = 23$

$2x - 4y = 6$

Dimensions of matrix A (coefficient matrix) 2 x 2

Dimensions of matrix A^{-1} 2 x 2

Dimensions of matrix B (constant matrix) 2 x 1

A^{-1} • B

2 x 2 • 2 x 1

or

B • A^{-1}

2 x 1 • 2 x 2

because of the dimensions of matrices A^{-1} and B you can only perform A^{-1}•B

$$\begin{bmatrix} X \\ Y \end{bmatrix} = A^{-1} \cdot B$$

Solving a system of linear equations using matrices.

1.

$9C + 12D = 21$

$10C + 4D = -14$

Coefficient Matrix: A

$\begin{bmatrix} 9 & 12 \\ 10 & 4 \end{bmatrix}$

Constant Matrix: B

(Answer Matrix)

$\begin{bmatrix} 21 \\ -14 \end{bmatrix}$

$A^{-1} \cdot B = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

this means C = -3
and D = 4

or Sol: (-3,4)

2.

$3c - 7d = -31$

$4c + 7d = -25$

A

$\begin{bmatrix} 3 & -7 \\ 4 & 7 \end{bmatrix}$

B

$\begin{bmatrix} -31 \\ -25 \end{bmatrix} = \begin{bmatrix} -8 \\ 1 \end{bmatrix}$ c d

$(-8, 1)$