

Horizontal Line Test: a visual test to determine if the inverse relation will be a function.

If any horizontal line can intersect a graph more than once then the graph of the inverse is NOT a function

One-to-One Functions:

Each y value is produced from exactly one x value.

If horizontal lines can touch a graph at most one time.

Inverses ARE functions



Many-to-One Functions:

Each y value may be produced from more than one x value.

If a horizontal line can touch a graph "many" times (more than once)

Inverses are NOT functions



Original Relation

Inverse Relation

$f(x)$ ————— Becomes ————— $f^{-1}(x)$

The point (a,b) ————— Becomes ————— The point (b,a)

Domain of $f(x)$ ————— Becomes ————— Range of $f^{-1}(x)$

Range of $f(x)$ ————— Becomes ————— Domain of $f^{-1}(x)$

Graph of $f(x)$ ————— Reflect over $y=x$ ————— Graph of $f^{-1}(x)$
Becomes

Find the equation of the Inverse Relation.

1. $f(x) = 2x - 3$ $f^{-1}(x) =$

$$\frac{x+3}{2} = y$$

2. $f(x) = (x+5)^3 - 7$

$$f^{-1}(x) = \sqrt[3]{x+7} - 5$$

1. $f(x) = 2x - 3$ $f^{-1}(x) = \frac{x+3}{2}$

Composites of Inverses

Find each of the following:

A. $f(f^{-1}(x)) = x$

$$2\left(\frac{x+3}{2}\right) - 3 = x+3-3 = x$$

B. $f^{-1}(f(x)) = x$

$$\frac{(2x-3)+3}{2} = \frac{2x}{2} = x$$

2. $f(x) = (x+5)^3 - 7$ $f^{-1}(x) = \sqrt[3]{x+7} - 5$

A. $f(f^{-1}(x)) = x$

$$\begin{aligned} & \left(\sqrt[3]{x+7} - 5 + 5\right)^3 - 7 \\ & \left(\sqrt[3]{x+7}\right)^3 - 7 \\ & x+7-7 = x \end{aligned}$$

B. $f^{-1}(f(x)) = x$

$$\begin{aligned} & \sqrt[3]{\left((x+5)^3 - 7 + 7\right) - 5} - 5 \\ & \sqrt[3]{(x+5)^3 - 5} - 5 \\ & x+5-5 = x \end{aligned}$$

If $f(x)$ and $f^{-1}(x)$ are inverses, then

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x$$

Find the equation of the inverse.

1. $x = \frac{\sqrt[4]{2y-9}}{2} + 8$

$$\frac{(2(x-8))^4 + 9}{2} = y$$

Find the equation of the inverse.

2. $y = \frac{(4x + 7)^3 - 2}{3}$

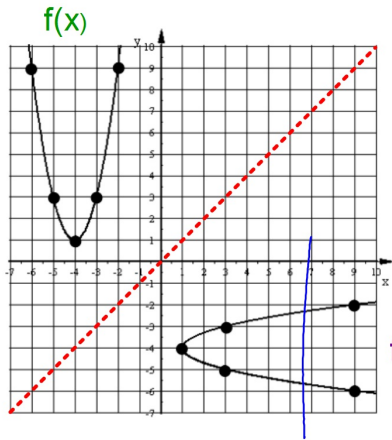
Find the equation of the inverse.

3. $y = 2\left(\frac{\sqrt{6x + 1}}{11}\right)^5 - 3$

Find the equation of the inverse.

4. $y = 9\sqrt[3]{\frac{x + 7}{2}} - 5 + 4$

You can now finish Hwk #19.



The inverse relation for the Quadratic:

$f(x) = 2(x+4)^2 + 1$
is a "sideways parabola"

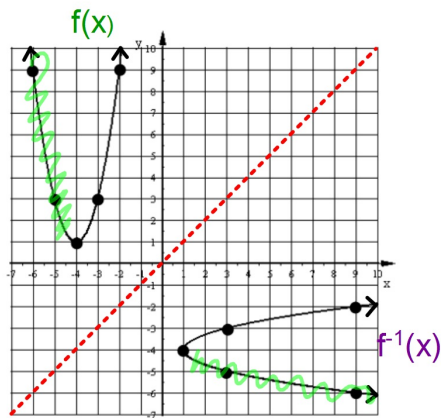
Is $f^{-1}(x)$ a function?

NO

Write the equation for the inverse relation.

$$f(x) = 2(x+4)^2 + 1$$

$$f^{-1}(x) = \pm \sqrt{\frac{x-1}{2}} - 4$$



To make $f^{-1}(x)$ a function we must "cut off" part of $f(x)$.

If we cut off the left side of $f(x)$ what does $f^{-1}(x)$ look like?

The top half of a sideways parabola

What is the domain and range of this new $f(x)$?

$$x \geq -4$$

$$y \geq 1$$

What is the domain and range of this new $f^{-1}(x)$?

$$y \geq -4$$

$$x \geq 1$$

Write the Inverse Relation of this:

$$f(x) = 3\sqrt{x-1} - 4$$

$$f^{-1}(x) = \left(\frac{x+4}{3}\right)^2 + 1$$

What does the graph of the inverse look like?

$$f^{-1}(x) = \left(\frac{x+4}{3}\right)^2 + 1$$

a parabola that shifted left and up
and is wider

What does the graph of the original relation look like?

$$f(x) = 3\sqrt{x-1} - 4$$

The top half of a "sideways" parabola

$$f(x) = 3\sqrt{x-1} - 4$$

Why is the graph of the above only
"half a sideways parabola"?

- If it were both halves then it wouldn't be a function.
- Without a sign in front of the radical it means the Principal Square Root (positive root).

Describe the transformations to the parent
function the following equation represents.

$$y = -4(x - 7)^2 + 11$$

- x-axis reflection (upside down)
- Vertical Stretch Factor of 4
- Shift 7 units right
- Shift 11 units up

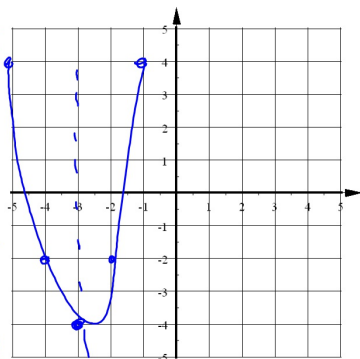
$y = -4(x - 7)^2 + 11$ State the Vertex of this
parabola.

$(7, 11)$

Graph this parabola:

$$y = 2(x+3)^2 - 4$$

$$\sqrt{1} \rightarrow \sqrt{2}$$
$$2\sqrt{4} \rightarrow \sqrt{8}$$



x	y
-2	-1

Graph of the Parent Function:

$$y = \sqrt{x}$$

x	y
0	0
1	1
4	2

