c 7-2: Multiplying and Dividing Radical Expressions.

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real #'s, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

The reverse is also true:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

When n is even both a and b must be positive

Simplify each. Assume that all variables are positive.

1. $\sqrt{3w^3} \cdot \sqrt{8w^6}$

$\sqrt{24 w^9}$

$4 \cdot 6$

$= 2w^4 \sqrt{6w}$

2. $\sqrt{5m^7n^4} \cdot \sqrt{15mn^5}$

$5 \cdot 3$

$\sqrt{5^2 \cdot 3\, m^8 n^9}$

$= 5m^4n^4\sqrt{3n}$

3. $\sqrt{30g^7h^{12}} \cdot \sqrt{18g^4h^8}$

$6 \cdot 5$ $\quad$ $6 \cdot 3$

$\sqrt{6^2 \cdot 15\, g^{11} h^{20}}$

$= 6g^5h^{10}\sqrt{15g}$

4. $\sqrt{7a} \cdot \sqrt[3]{7a}$

$(7a)^{1/2} \cdot (7a)^{1/3}$

$= (7a)^{1/2+1/3}$

$= (7a)^{5/6}$ or $\left(\sqrt[5]{7a}\right)^6$

5. $\sqrt[3]{9Q^4R^2} \cdot \sqrt[3]{6Q^5R^8}$

$3 \cdot 3 \qquad 3 \cdot 2$

$\sqrt[3]{3^3 \cdot 2 Q^9 R^{10}}$

$= 3Q^3R^3\sqrt[3]{2R}$

6. $\sqrt[4]{18m^6n^3} \cdot \sqrt[4]{45m^9n^{13}}$

$2 \cdot 3 \cdot 3 \qquad 5 \cdot 3 \cdot 3$

$\sqrt[4]{3^4 \cdot 10\, m^{15} n^{16}}$

$= 3m^3n^4\sqrt[4]{10m^3}$

7. $\sqrt[5]{8c^6d^{13}} \cdot \sqrt[5]{12c^7d^3}$

$2 \cdot 2 \cdot 2 \qquad 2 \cdot 2 \cdot 3$

$\sqrt[5]{2^5 \cdot 3\, c^{13} d^{16}}$

$= 2c^2d^3\sqrt[5]{3c^3d}$

Simplify each. Assume that all variables are positive.

1. $\sqrt{63g^4h^7} \cdot \sqrt{15g^6h^4}$

2. $\sqrt[3]{49m^5n^{-13}} \cdot \sqrt[3]{21m^7n^4}$

3. $\sqrt[5]{12c^4d} \cdot \sqrt[5]{14c^7d^3} \cdot \sqrt[5]{20c^2d^8}$

Dividing radical expressions:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real #'s and $b \neq 0$,
then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

The reverse is also true:

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ if a and b are positive

Simplify each. Assume that all variables are positive.

1. $\dfrac{\sqrt{24x^5y^{13}}}{\sqrt{3xy^4}} = \sqrt{\dfrac{24x^5y^{13}}{3xy^4}} = \sqrt{8x^4y^9}$

$= 2x^2y^4\sqrt{2y}$

2. $\sqrt{\dfrac{50a^5b^4}{9a^4b^{12}}} = \dfrac{\sqrt{50a^5b^4}}{\sqrt{9a^4b^{12}}} = \dfrac{5a^2b^2\sqrt{2b}}{3a^2b^6}$

$= \dfrac{5\sqrt{2b}}{3b^4}$

Simplify each. Assume that all variables are positive.

3. $\dfrac{\sqrt{48x^9y^8}}{\sqrt{2x^6y^3}} = \sqrt{\dfrac{48x^9y^8}{2x^6y^3}} = \sqrt{24x^3y^5}$ (4.6)

$= 2xy^2\sqrt{6xy}$

4. $\dfrac{\sqrt[3]{2x^{16}y^5}}{\sqrt[3]{54x^3y^{11}}} = \sqrt[3]{\dfrac{2x^{16}y^5}{54x^3y^{11}}} = \sqrt[3]{\dfrac{x^{13}}{27y^6}}$

$= \dfrac{x^4\sqrt[3]{x}}{3y^2}$