

Use this function: $y = \frac{3x + 40x^2 - 4}{13x - 34 + 2x^2}$

The HA of this function is $y=20$

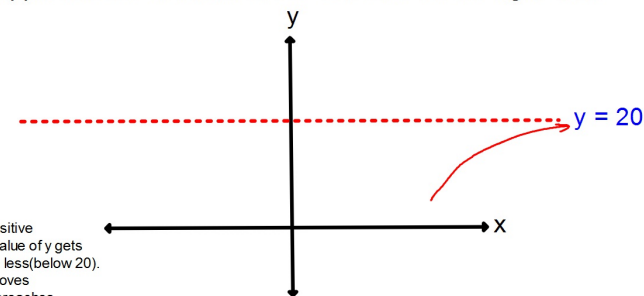
Use the TABLE function on the graphing calc to find out:

1. Does the graph approach the HA from above or below on the right end.
2. Does the graph approach the HA from above or below on the left end.

$$y = \frac{3x + 40x^2 - 4}{13x - 34 + 2x^2}$$

1. Does the graph approach the HA from above or below on the right end.

X	Y
100	18.823
1000	19.873
10000	19.987
100000	19.999

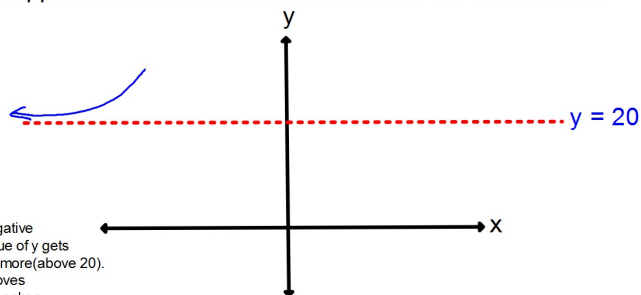


As the value of x gets bigger positive (moves farther to the right) the value of y gets closer to 20 but is always a little less (below 20). This means that as the graph moves farther to the right the graph approaches $y=20$ from below.

$$y = \frac{3x + 40x^2 - 4}{13x - 34 + 2x^2}$$

2. Does the graph approach the HA from above or below on the left end.

X	Y
-100	21.4131
-1000	20.1296
-10000	20.0129
-100000	20.0013



As the value of x gets bigger negative (moves farther to the left) the value of y gets closer to 20 but is always a little more (above 20). This means that as the graph moves farther to the left the graph approaches $y=20$ from above.

Vertical Asymptote:

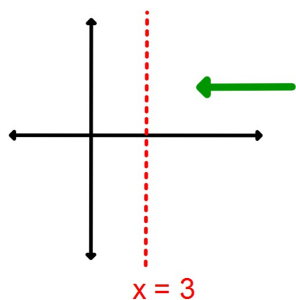
A break in the graph created by a zero of the denominator.

Behavior of a graph near a VA:

Near a VA the graph either increases or decreases without bound because as you plug in values for x that get closer and closer to the VA the denominator gets smaller and smaller thus making the fraction get bigger and bigger.

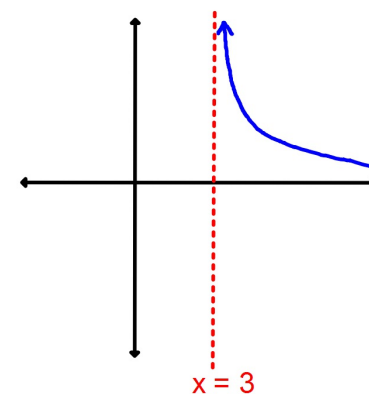
As the graph approaches a VA only one of two things will happen:

For example: As you approach $x = 3$ from the right the graph might:

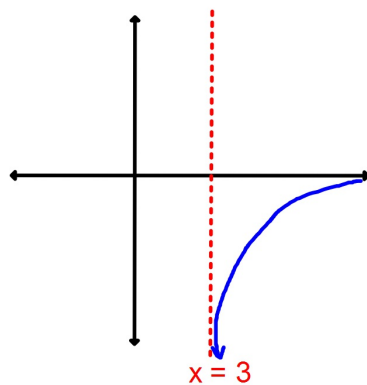


$$y = \frac{7x^2 - 14x}{x - 3}$$

Increase w/o bound



OR Decrease w/o bound



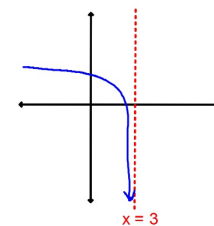
To find the behavior of the graph on the left side try values for x that are very close to 3 but just a little less.

X	Y
2.9	-182.7
2.99	-2072.07
2.999	-20972.007
2.9999	-209972

$$y = \frac{7x^2 - 14x}{x - 3}$$

The table shows that as x gets closer to 3 but a little less, from the left side, that the value of y gets bigger and bigger negative (going down).

The graphs decreases as you approach the VA $x=3$ from the left



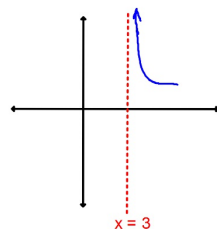
To find the behavior of the graph on the **right side** try values for x that are very **close to 3 but just a little more**.

X	Y
3.1	238.7
3.01	2128.07
3.001	21028.007
3.0001	210028.001

The table shows that as x gets closer to 3 but a little more, from the right side, that the value of y gets bigger and bigger positive (going up).

The graphs increases as you approach the VA $x=3$ from the right

$$y = \frac{7x^2 - 14x}{x - 3}$$



Use this function: $y = \frac{8x^2 - 7x}{2x^3 + 74x^2 - 240x}$

This function has a VA at $x = -40$

Use the TABLE function on the graphing calc to find out:

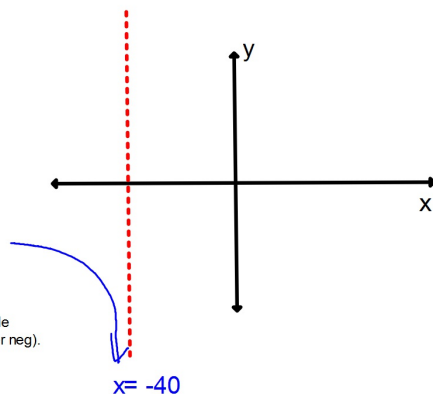
1. Does the graph increase or decrease without bound as it approaches the VA from the left?
2. Does the graph increase or decrease without bound as it approaches the VA from the right?

1. Does the graph increase or decrease without bound as it approaches the VA from the left?

$$y = \frac{8x^2 - 7x}{2x^3 + 74x^2 - 240x}$$

X	Y
-40.1	-38.028
-40.01	-380.237
-40.001	-3802.330
-40.0001	-38023.260

As the value of x gets closer to -40 but stays to the left side (a little more neg) the value of y keeps decreasing (bigger neg). This means that as the graph approaches the VA $x = -40$ from the left the graph goes down.

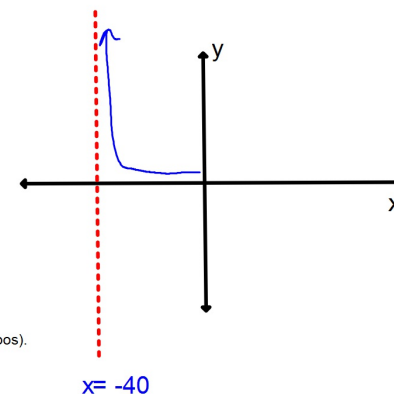


2. Does the graph increase or decrease without bound as it approaches the VA from the right?

$$y = \frac{8x^2 - 7x}{2x^3 + 74x^2 - 240x}$$

X	Y
-39.9	38.019
-39.99	380.228
-39.999	3802.321
-39.9999	38023.251

As the value of x gets closer to -40 but stays to the right side (a little less negative) the value of y keeps increasing (bigger pos). This means that as the graph approaches the VA $x = -40$ from the right the graph goes up.



Horizontal Asymptotes:

The horizontal line that a graph approaches as you move farther and farther to the left and right. (END BEHAVIOR)

The value of y that the function approaches as x gets larger and larger (pos and neg).

Horizontal Asymptote Exploration:

1. $y = \frac{x-6}{x+5}$ Horizontal Asymptote: $y=1$
2. $y = \frac{3x+1}{x+4}$ Horizontal Asymptote: $y=3$
3. $y = \frac{8x^2+x-6}{2x^2-21}$ Horizontal Asymptote: $y=4$

4. $y = \frac{4x+9x}{x^2-3}$ Horizontal Asymptote: $y=0$
5. $y = \frac{x^2-13}{x^2+7}$ Horizontal Asymptote: $y=0$
6. $y = \frac{x-5}{2x^3+3}$ Horizontal Asymptote: $y=0$

7. $y = \frac{5x^2-4}{x+3}$ Horizontal Asymptote: NO HA
8. $y = \frac{-2x^3+5x-8}{x^2+3x-1}$ Horizontal Asymptote: NO HA

Without using a table or a graph how could you tell from the equation what the horizontal asymptote is or if it even has one?

Predict the Horizontal Asymptote for each of the rational functions below.

a. $y = \frac{10x+7}{5x-3}$

HA: $y = \frac{10}{5} = 2$

b. $y = \frac{6x^2-5}{2x+3}$

HA: NONE

c. $y = \frac{12x-11}{3x^2-1}$

HA: $y=0$

Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

No HA

Case 2: Degree of the Numerator = Degree of the Denominator

HA: $y =$ ratio of the Leading Coefficients

Case 1: Degree of the Denominator > Degree of the Numerator

HA: $y = 0$