Horizontal Asymptotes:

Horizontal line (value of y) that the graph approaches the further left and right you move on the graph (bigger pos and neg values for x).

The graph flattens out on the ends and gets very close to this line.

END BEHAVIOR

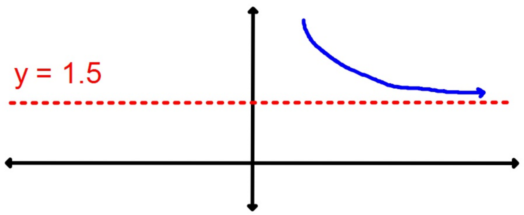
Since the HA is the END BEHAVIOR of the graph it only helps determine what the graph looks like at the far left and far right. Only one of two things will happen at each end.

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

For example as you move farther and farther to the right (larger and larger pos value for x)

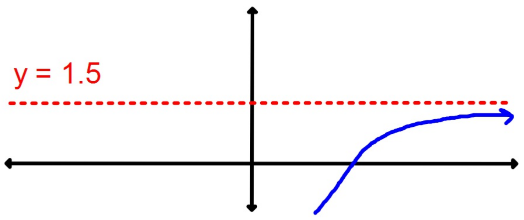
The graph might approach the HA from above

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$



OR It might approach the HA from below

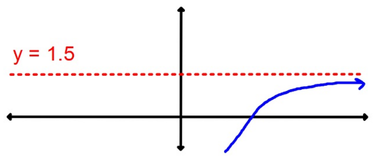
$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$



To find out if the graph approaches the HA off to the far right from above or below you can use the TABLE function on the graphing calculator and plug in large positive values for x:

X	Y
100	1.479038625
1000	1.497990263
10000	1.499799903
100000	1.499979999

The table shows that the value of y approaches 1.5 as larger and larger positive values for x are used (far to the right). But the values of y are just less than 1.5 so the graph is approaching the HA y=1.5 from below:



Vertical Asymptote:

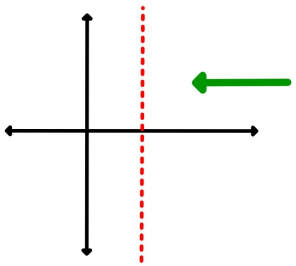
A break in the graph created by a zero of the denominator.

Behavior of a graph near a VA:

Near a VA the graph either increases or decreases without bound because as you plug in values for x that get closer and closer to the VA the denominator gets smaller and smaller thus making the fraction get bigger and bigger.

As the graph approaches a VA only one of two things will happen:

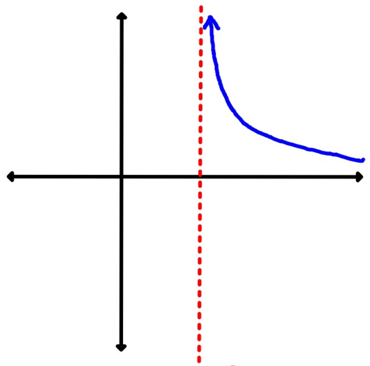
For example: As you approach x = 3 from the right the graph might:



x = 3

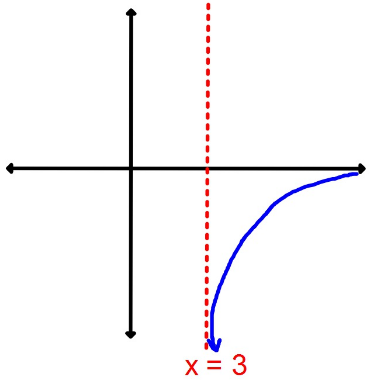
$$y = \frac{7x^2 - 14x}{x - 3}$$

Increase w/o bound



x = 3

OR Decrease w/o bound



Use the TABLE function on your graphing calc to find the behavior of the graph on both sides of the vertical asymptote x = 3 for the function below.

$$y = \frac{7x^2 - 14x}{x - 3}$$

To find the behavior of the graph on the left side try values for x that are very close to 3 but just a little less.

To find the behavior of the graph on the right side try values for x that are very close to 3 but just a little more.

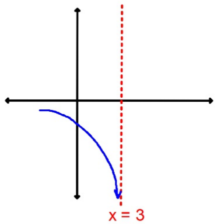
To find the behavior of the graph on the left side try values for x that are very close to 3 but just a little less.

X	Y
2.9	-182.7
2.99	-2072.07
2.999	-20972.007
2.9999	-209972

$$y = \frac{7x^2 - 14x}{x - 3}$$

The table shows that as x gets closer to 3 but a little less, from the left side, that the value of y gets bigger and bigger negative (going down).

The graphs decreases as you approach the VA x=3 from the left



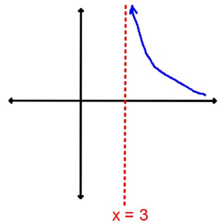
To find the behavior of the graph on the right side try values for x that are very close to 3 but just a little more.

X	Y
3.1	238.7
3.01	2128.07
3.001	21028.007
3.0001	210028.001

$$y = \frac{7x^2 - 14x}{x - 3}$$

The table shows that as x gets closer to 3 but a little more, from the right side, that the value of y gets bigger and bigger positive (going up).

The graphs increases as you approach the VA x=3 from the right

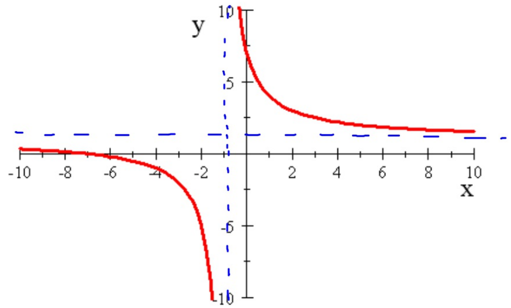


When the denominator of a rational function is zero there is a break in the graph - because this value of x can never be used.

These breaks in the graph are one of two types.

Graph the rational function f(x) in a standard window.

$$f(x) = \frac{x+7}{x+1}$$



There is a break in the graph at x = -1

This kind of break in the graph is called a

Vertical Asymptote

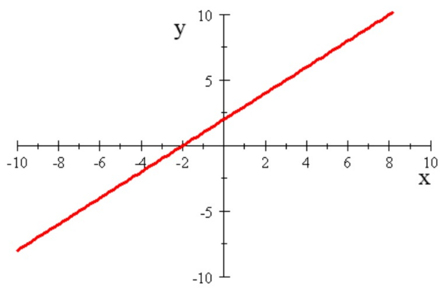
Graph the rational function f(x) in a standard window.

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

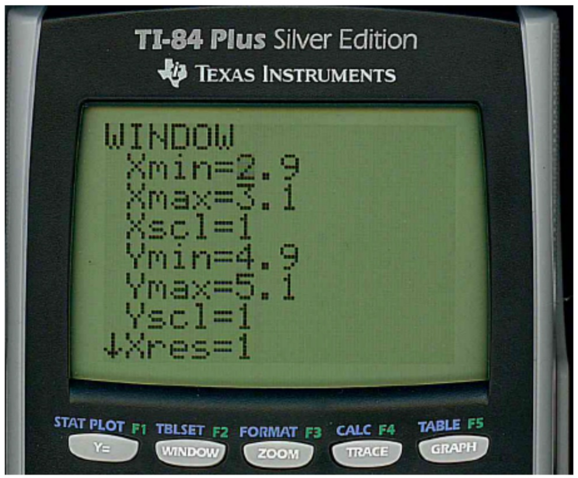
Do you see a vertical asymptote?

Why do you think that there isn't a vertical asymptote at x = 3?

because x-3 appears in both the numerator and denominator.

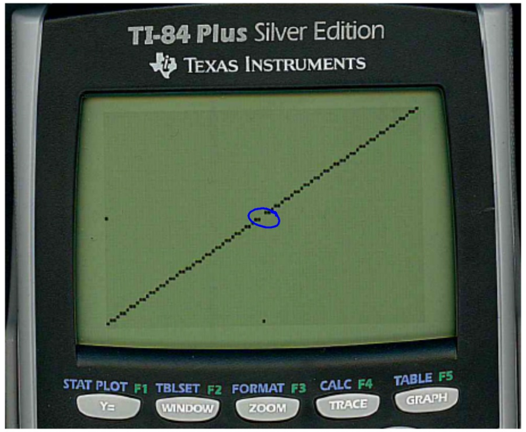


Look at the graph in the following window:



What do you see?

This kind of break in the graph is called a **Hole**



Why did this graph have a Vertical Asymptote at x = -1

$$f(x) = \frac{x+7}{x+1}$$

because x+1 occurs in the denominator ONLY

and

this graph have a hole at x = 3?

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

because x-3 occurs in the denominator AND numerator

Breaks in the graph are caused by zeros of the denominator and are called:

Points of Discontinuity

Holes or **Vertical Asymptotes**

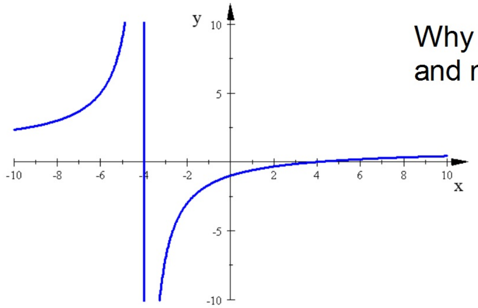
Holes: Occur at values of x that are zeros of both the denominator AND numerator

Vertical Asymptotes: Occur at values of x that are zeros of the denominator ONLY.

An exception to this rule:

$$y = \frac{x^2-16}{x^2+8x+16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$

$$y = \frac{x^2 - 16}{x^2 + 8x + 16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$



Why is there a VA at x=-4 and not a hole?

Even though the factor x+4 occurs in the both the numerator and denominator when you cancel there is still x+4 left in the denominator. If the extra x+4 didn't exist there would be a hole at -4.

Properties **Vertical Asymptotes**

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a hole in the graph or a vertical asymptote at $x = a$.

Find any points of discontinuity and classify them as Vertical Asymptotes or Holes.

1. $y = \frac{2x(x+1)(x-6)}{(x-1)(x-6)}$

PTS of DIS: 1, 6

VA: $x = 1$

Holes: $x = 6$

2. $y = \frac{(x-2)}{x^2+4}$

VA: NONE

Holes: There are no zeros of the denominator

3. $y = \frac{x^2 - x - 12}{x^2 - 16} = \frac{(x-4)(x+3)}{(x+4)(x-4)}$

PTS of DISC: ± 4

VA: $x = -4$

Holes: $x = 4$

4. $y = \frac{x^2 + 6x + 9}{x^2 + 5x + 6} = \frac{(x+3)(x+3)}{(x+3)(x+2)}$

PTS of DISC: $-3, -2$

VA: $x = -2$

Holes: $x = -3$

You can now finish the following problems from
Hwk #3: 2-4, 13, 17, 18