Theorem **Remainder Theorem**

If a polynomial $P(x)$ of degree $n \geq 1$ is divided by $(x - a)$, where a is a constant, then the remainder is $P(a)$.

Synthetic Division

Uses the zero of the divisor.
By reversing the sign of the divisor you can ADD throughout the process instead of subtracting.

Works only when the leading coefficient of the divisor is 1.

Meaning either $\div(x + a)$ or $\div(x - a)$

$$\frac{x^3 - 2x^2 - 31x + 20}{x + 5}$$

Zero of the Divisor

Coefficients of the dividend in Standard Form

-5	1	-2	-31	20	
		-5	35	-20	Multiply and ADD
Bring down the first #	1	-7	4	0	

$x^2 - 7x + 4 \quad r = 0$

Find each quotient using Synthetic Division

1. $$\frac{4x^3 - 6x^2 - 7x - 33}{x - 3}$$

2. $$\frac{2x^4 + 18x^3 + 34x^2 + 43x + 10}{x + 7}$$

1. $\dfrac{4x^3 - 6x^2 - 7x - 33}{x - 3}$

$$
\begin{array}{r|rrrr}
3 & 4x^3 & -6x^2 & -7x & -33 \\
 & & +12 & +18 & 33 \\
\hline
 & 4 & 6 & 11 & 0
\end{array}
$$

Answer $\underline{4x^2 + 6x + 11}$

2. $\dfrac{2x^4 + 18x^3 + 34x^2 + 43x + 10}{x + 7}$

$$
\begin{array}{r|rrrrr}
-7 & 2 & 18 & 34 & 43 & 10 \\
 & & -14 & -28 & -42 & -7 \\
\hline
\end{array}
$$

$2x^3 + 4x^2 + 6x + 1 \quad R = 3$

Find this quotient using Synthetic Division.

$$\frac{4x^3 - x + 9}{x - 3}$$

$$
\begin{array}{r|rrrr}
3 & 4 & 0 & -1 & 9 \\
 & & 12 & 36 & 105 \\
\hline
 & 4 & 12 & 35 & 114
\end{array}
$$

$4x^2 + 12x + 35 \quad r = 114$

Is X+7 a factor of $x^3 - 2x^2 + 10x - 21$?

You could do any kind of division and find out the remainder. If the remainder is zero then x + 7 is a factor.

OR

You could use the Remainder Theorem:

$P(-7) = (-7)^3 - 2(-7)^2 + 10(-7) - 21 = -532$

Since the remainder is -532 this tells us that x + 7 is NOT a factor.

Given $f(x) = 3x^4 - 5x^3 + 8x^2 - 7x + 10$

Find $f(2) = 36$

How could you use Synthetic Division to find f(2)?

$$\begin{array}{r|rrrrr} 2 & 3 & -5 & 8 & -7 & 10 \\ & & 6 & 2 & 20 & 26 \\ \hline & 3 & 1 & 10 & 13 & 36 \end{array}$$

Given $x - 5$ is a factor of $2x^3 - 11x^2 - 16x + 105$
Use synthetic division to help find the other two factors.

$$\begin{array}{r|rrrr} 5 & 2 & -11 & -16 & 105 \\ & & 10 & -5 & -105 \\ \hline & 2 & -1 & -21 & \end{array}$$

$$\underbrace{2 \quad -1 \quad -21}$$

$2x^2 - x - 21$

-42

-7 6

-1

	$2x$	-7
x	$2x^2$	$-7x$
$+3$	$+6x$	-21

$\boxed{(2x-7)(x+3)}$

Hwk #30 Due tomorrow.

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Problems 4, 9, 14, 15, 24, 37, 41