Matrix: Rows and Columns of data

$$\begin{bmatrix} 9 & 2 \\ 4 & 7 \\ -8 & 0 \end{bmatrix}$$

Dimensions of a Matrix:

the matrix at the left has the following dimensions:

$\underbrace{3}_{\text{\# Rows}} \times \underbrace{2}_{\text{\# Columns}}$ "three by two"

Each number in a matrix is called an element

Matrices are named using a capital letter

Entering matrices on a calculator:

$$\begin{bmatrix} 9 & 2 \\ 4 & 7 \\ -8 & 0 \end{bmatrix}$$

1. press $\boxed{\text{2ND}}$ then $\boxed{X^{-1}}$
2. Arrow key to EDIT and press $\boxed{\text{ENTER}}$
3. Enter the dimensions you want (Rows x Columns)
4. Enter the elements row by row

A $\begin{bmatrix} 9 & 2 \\ 4 & 7 \\ -8 & 0 \end{bmatrix}$ B $\begin{bmatrix} -5 & 3 \\ 6 & 1 \\ 10 & -4 \end{bmatrix}$ C $\begin{bmatrix} 2 & -1 & 7 \\ -3 & 20 & 8 \end{bmatrix}$

Enter Matrices B and C too.

Matrix Operations

A $\begin{bmatrix} 9 & 2 \\ 4 & 7 \\ -8 & 0 \end{bmatrix}$ B $\begin{bmatrix} -5 & 3 \\ 6 & 1 \\ 10 & -4 \end{bmatrix}$ C $\begin{bmatrix} 2 & -1 & 7 \\ -3 & 20 & 8 \end{bmatrix}$

Which two matrices can be:

1. Added A + B
   B+ A
2. Subtracted A−B
   B − A
3. Multiplied
   AC, C·A, B·C, C·B
4. Divided NONE

To Add and Subtract two matrices they must have the exact same dimensions.

To multiply two matrices
the second matrix must have the same number of rows as the number of columns in the first matrix Their middle numbers must match:

A*C = 3 x 2 * 2 x 3 or C*B = 2 x 3 * 3 x 2

These "middle" numbers must match

the dimensions of the answer are the first and last numbers of the two matrices being multiplied.

A*C = 3 x 2 * 2 x 3 = 3 x 3 matrix

Find this qoutient.

Instead of dividing by a fraction you can multiply by the reciprocal.

$$\frac{\frac{8}{15}}{\frac{56}{55}}$$

$$\frac{8}{15} \cdot \frac{55}{56} \rightarrow \frac{8}{15} \cdot \left(\frac{56}{55}\right)^{-1}$$

Multiplying by the reciprocal is the same as multiplying by the inverse

You CAN'T do matrix division. However.....

You can multiply by the inverse:

Can't do this:

$$\begin{bmatrix} 4 & -7 \\ 1 & 5 \end{bmatrix} \div \begin{bmatrix} -9 & 2 \\ 0 & -8 \end{bmatrix}$$

Can do this:

$$\begin{bmatrix} 4 & -7 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} -9 & 2 \\ 0 & -8 \end{bmatrix}^{-1}$$

Solving a system of linear equations using matrices.

$$5x + 3y = 13$$
$$-4x + y = -7$$

Coefficient Matrix: A

2×2

$$\begin{bmatrix} 5 & 3 \\ -4 & 1 \end{bmatrix}$$

Constant Matrix: B
(Answer Matrix)

2×1

$$\begin{bmatrix} 13 \\ -7 \end{bmatrix}$$

Matrix Equation

$A\begin{bmatrix} X \\ Y \end{bmatrix} = B$

To solve for $\begin{bmatrix} X \\ Y \end{bmatrix}$ you would normally divide by matrix A.

But instead, we multiply by the inverse of matrix A.

$\begin{bmatrix} X \\ Y \end{bmatrix} = \underset{2\times2}{A^{-1}} \cdot \underset{2\times1}{B}$

Or

$\begin{bmatrix} X \\ Y \end{bmatrix} = \underset{2\times1}{B} \cdot \underset{2\times2}{A^{-1}}$

$A^{-1} \bullet B$ (2 x 2)(2 x 1)

This is the one that can be calculated because the middle numbers match. the answer will be a 2x1 matrix where x is the first number and y is the second.

$B \bullet A^{-1}$ (2 x 1)(2 x 2)

This can't be done because the middle numbers don't match up!

Solve the systems of equations in the bellwork using matrices.

1. $(7, -2)$

$y = x - 9 \rightarrow -x + y = -9$

$2x - 3y = 20$

$\begin{array}{cc} A & B \\ \begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix} & \begin{bmatrix} -9 \\ 20 \end{bmatrix} \end{array}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = A^{-1} \cdot B$

2. $(-8, 1)$

$3c - 7d = -31$

$4c + 7d = -25$

$\begin{array}{cc} A & B \\ \begin{bmatrix} 3 & -7 \\ 4 & 7 \end{bmatrix} & \begin{bmatrix} -31 \\ -25 \end{bmatrix} \end{array}$

3. $(4, -6)$

$5J - 8K = 68$

$3J - 7K = 54$

$\begin{array}{cc} A & B \\ \begin{bmatrix} 5 & -8 \\ 3 & -7 \end{bmatrix} & \begin{bmatrix} 68 \\ 54 \end{bmatrix} \end{array}$

4. $(3, 9)$

$5V - 3W = -12$

$8V + 4W = 60$

$\begin{array}{cc} A & B \\ \begin{bmatrix} 5 & -3 \\ 8 & 4 \end{bmatrix} & \begin{bmatrix} -12 \\ 60 \end{bmatrix} \end{array}$

How would you solve this system of equations using matrices?

$y = 4x + 9 \longrightarrow -4x + y = 9$

$y = 21 \qquad y = 21$

$\begin{array}{cc} A & B \\ \begin{bmatrix} -4 & 1 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 9 \\ 21 \end{bmatrix} \end{array}$

$[A]^{-1}[B] \rightarrow (0, 9)$

Solve this system of equations using matrices.

$$3(8x + 2y = 14)$$
$$2(12x + 3y = 18)$$

You must use algebra to find out which answer applies

$$24x + 6y = 42$$
$$24x + 6y = 36$$
$$0 = 6$$

ERR: Singular Matrix

A matrix that doesn't have an inverse

this means that there is either no solution or many solutions

No Solution!

Solve this system of equations using matrices:

___C ___D = ___

___C ___D = ___

fill in the blanks with any real numbers and solve using matrices.

Hwk #12

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Due tomorrow

problems 26, 27, 33, 34, 42

Don't have a graphing calculator to solve a system of equations with matrices?

- Borrow one
- Use the internet → Check my blog!

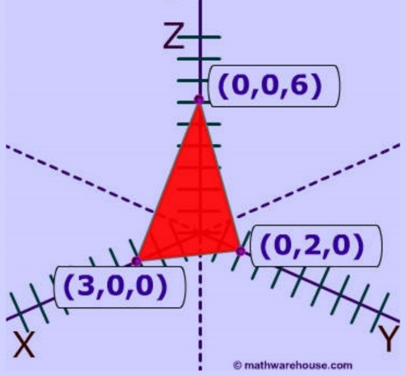
Equation in three variables ⟶ 3 Dimensions

$$2x + 3y + z = 6$$

x-int = 3
y-int = 2
z-int = 6

What is a linear equation with 3 variables?

$2x + 3y + Z = 6$



It is a plane !
The picture on the left is the graph of the plane $2x + 3y + z = 6$.

The red triangle is the portion of the plane when x, y, and z values are all positive. This plane actually continues off in the negative direction. All that is pictured is the part of the plane that is intersected by the positive axes (the negative axes have dashed lines).

What is a <u>system</u> of 3 variables equations?

Just like a system of linear equations with 2 variables is more than 1 line, a system of 3 variable equations is just more than plane.

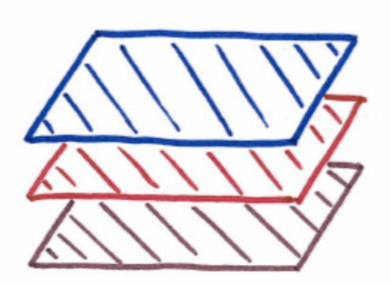
How many solutions could a system of equations with three variables have?

the same outcomes as a system of equations with two variables!

------------------- No Solutions, 1 Solution or Infinite Solutions -------------------

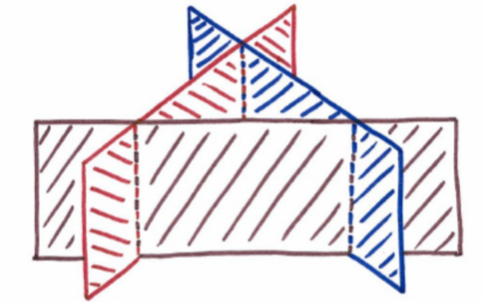
How many solutions?

No
Solution



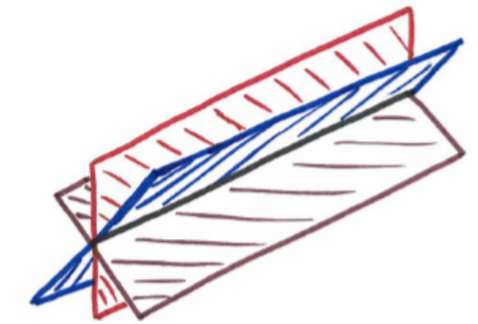
How many solutions?

No
Solution



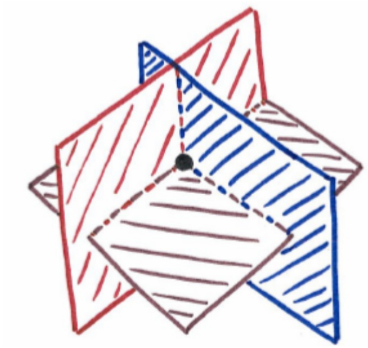
How many solutions?

Many
Solutions



How many solutions?

One
Solution



How would you solve this system of equations in three variables?

Use matrices!

$$x + 3y - 5z = -24$$
$$7x + 4z = 48$$
$$-3x - 4y + z = -3$$

$$A \; 3\text{x}3 \quad \begin{bmatrix} 1 & 3 & -5 \\ 7 & 0 & 4 \\ -3 & -4 & 1 \end{bmatrix} \qquad B \; 3\text{x}1 \quad \begin{bmatrix} -24 \\ 48 \\ -3 \end{bmatrix} \longrightarrow [A]^{-1}[B]$$

$$= \boxed{(4, -1, 5)}$$

$$\begin{array}{ccc} x & y & z \end{array}$$