

Section 11-4: Arithmetic Series

Series: When the terms of a sequence are added.

Find the sum of this series:

$$1+2+3+\dots+10+11+12 = 78$$

This is called a FINITE SERIES


has a last term.

$$3+5+7+9+11+\dots$$

What do these three dots mean?

What is another word that means
to continue on forever?

What is a good name for this kind of series?

INFINITE SERIES


There is no last term

What is the sum of an INFINITE SERIES?

It doesn't have one!

An Arithmetic Series is the sum of
the terms in an Arithmetic Sequence.

All FINITE Arithmetic Series have a sum.

Find the sum of the first 100 natural numbers:

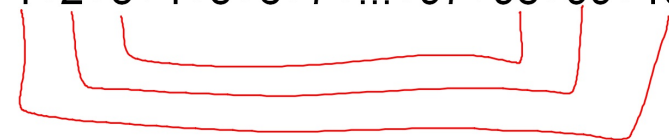
$$1+2+3+4+5+6+7+\dots+97+98+99+100 =$$

The famous mathematician Karl Friedrich Gauss was 10 years old, when his teacher assigned a problem to the class: find the sum of the first 100 natural numbers, thinking this would keep the class busy for a while.

Gauss, being quite bright, came up with the answer within minutes.

Find the sum of the first 100 natural numbers:

$$1+2+3+4+5+6+7+\dots+97+98+99+100 =$$



$(101 + 10) + 10) \dots$
Each pair of number adds to 101
there will be 50 pairs
 $101(50) = 5050$

The sum of a Finite Arithmetic Series:

S_n represents the sum of the first n terms of a finite Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

n = Number of terms

a_1 = First term

a_n = Last Term

What is the Explicit Formula for this Arithmetic Sequence?

4, 10, 16, 22, 28, ...

$$a_n = 4 + (n-1)6$$

How many terms are there in this Finite Arithmetic Series?

4, 10, 16, 22, 28, ..., 106

$$\begin{aligned} 106 &= 4 + (n-1)6 \\ 102 &= (n-1)6 \\ 17 &= n-1 \\ 18 &= n \end{aligned}$$

Find the sum this Finite Arithmetic Series

4+10+16+22+28+ ...+ 106

$$S_{18} = \frac{18}{2}(4 + 106)$$

290

Find the sum of this Finite Arithmetic Series

$$21+29+37+45+ \dots +117$$

$$n=13$$

$$a_n = 21 + (n-1)8$$
$$117 = 21 + (n-1)8$$
$$n=13$$

$$S_{13} = \frac{13}{2} (21 + 117)$$
$$= 897$$


Find the sum of this Finite Arithmetic Sequence

$$74 + 65 + 56 + 47 + \dots + -52$$

$$n=15$$

$$S_{15} = \frac{15}{2} (74 + -52)$$
$$= 165$$

Section 11-4

Σ  Capital Greek letter Sigma
"sum of"

Summation Notation

$$\sum_{n=1}^6 8n - 3$$

"the sum of $8n-3$ from 1 to 6"

Upper Limit,
greatest value of n

Lower Limit,
least value of n

Explicit formula
for the sequence

$$\sum_{n=1}^6 (8n - 3)$$

$5 + 13 + 21 + \dots$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_6 = \frac{6}{2}(5 + 45)$$

$$= 150$$