Sec 6-7 Outcomes, Permutations, and Combinations.

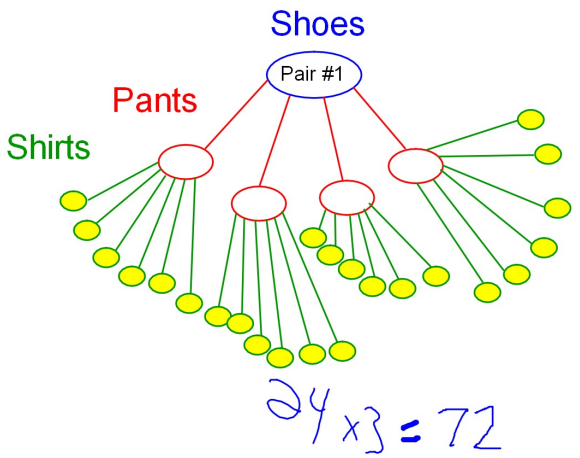
Each morning you must decide what to wear. An outfit consists of a pair of shoes, a pair of pants, and a shirt.

You have the following to choose from:

- 3 pairs of shoes
- 4 pairs of pants
- 6 shirts

How many different outfits are possible? = 72

Tree Diagram:



Multiplication Counting Principle:

multiplying the number of choices for each step

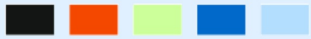
$$\underset{\text{Shoes}}{3} \cdot \underset{\text{Pants}}{4} \cdot \underset{\text{Shirts}}{6} = \underset{\text{\# of outfits}}{72}$$

Example: You are buying a new car.

There are **2** body styles:

sedan or hatchback

There are **5** colors available:

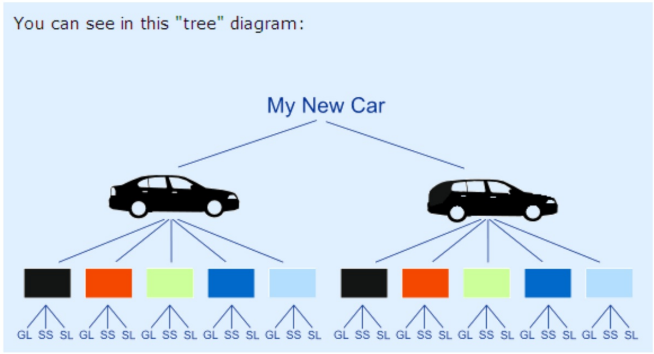


There are **3** models:

- GL (standard model),
- SS (sports model with bigger engine)
- SL (luxury model with leather seats)

How many total choices?

You can see in this "tree" diagram:



You can count the choices, or just do the simple calculation:

Total Choices = 2 × 5 × 3 = 30

Sarah is going to get a pizza for dinner. She stops by her favorite pizza place and buys one of their specials: 1 topping, any size, any crust, just $4.99.

Sizes(3): Small, Medium, Large
Toppings(5): Pepperoni, Onion, Bacon, Mushrooms, or Ham.
Crusts(4): Butter, Garlic, Cajun, Sesame.

How many different pizzas does Sarah have to choose from?

$3 \cdot 5 \cdot 4 = 60$

There are 5 people running a race. How many different ways can 1st through 5th place be awarded?

$$\underset{\text{1st Place}}{5} \cdot \underset{\text{2nd Place}}{4} \cdot \underset{\text{3rd Place}}{3} \cdot \underset{\text{4th Place}}{2} \cdot \underset{\text{5th Place}}{1}$$

$= 120$

Permutation: An arrangement of items when the order or arrangement IS important.

Factorial: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

5! is said: "five factorial"

Factorial is usually used if you are arranging ALL of the available items.

Each day the school puts out 7 flags in front of the school.

How many different arrangements of the 7 flags are possible?

$7 \cdot 6 \cdot 5 \cdot 4 \cdots$

$7! = 5040$

What if prizes are only awarded to the top three finishers of the race?

In other words, how many ways can 1st, 2nd, and 3rd places be awarded to 5 people running in the race?

Multiplication Counting Principle:

$\underset{\text{1st Place}}{5} \cdot \underset{\text{2nd Place}}{4} \cdot \underset{\text{3rd Place}}{3} = 60$

Permutation: Arrangment of 5 things 3 at a time

Permutation Formula:

Ways to arrange n items r at a time.

$${}_nP_r = \frac{n!}{(n-r)!}$$

n= total # items

r = # arranging at a time

${}_5P_3 = 60$

60 different ways to award 1st thru 3rd place to five people

You are playing Scrabble. You choose 7 tiles from the pile.

1. How many ways can you arrange all 7 in front of you?

   $7!$ or ${}_7P_7 = 5040$

2. If you can only play 3 at a time, find the number of ways you can arrange 3 of the 7 tiles on the board.

$7 \cdot 6 \cdot 5 = 210$

${}_7P_3 = 210$

There are 24 students in a class and there are 24 desks. How many different seating charts are possible?

Does order/arrangement matter? Yes

$24!$ or ${}_{24}P_{24}$

6.20×10^{23}

There are 24 students in a class and there are 30 seats in the room. How many different seating charts are possible?

Does order/arrangement matter? Yes

${}_{30}P_{24}$

Ali must finish making his schedule for the fall semester and has three spots open, 1st, 2nd, and 3rd hours.

He can choose from the 8 remaining available classes to complete his schedule. Each class is offered all three hours. How different schedules can be created?

Does order/arrangement matter? Yes

${}_{8}P_{3} = 336$

You want to order a two topping pizza. If there are only 4 toppings to choose from (Onions, Ham, Pepperoni, and Mushrooms) how many different 2 topping pizzas are possible?

O, H, P, M

Does order/arrangement matter? No

OH HP PM
OP HM
OM

6 different two topping pizzas

You order a shake at a shop. There are 7 ingrediants to choose from. You buy a shake that contains 4 ingredients. How many different shakes are possible?

See next page for answer

Does order/arrangement matter? No

Combination:

Selecting a number of items when ORDER DOESN'T matter.

Combination Formula:

Ways to choose n items r at a time.

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

n= total # items

r = # selecting at a time

$_4C_2 = 6$

6 different 2 topping pizzas

$_7C_4 = 35$

35 different shakes

There are 24 students in the class.

How many ways could President, Vice-President, and Secretary be assigned?

$_{24}P_3 = 12,144$

How many ways could a committee of 3 students be created to organize an after school club?

$_{24}C_3 = 2024$

A license plate has 3 letters and 3 digits.

The letters can be anything but O or I and any digit from 0 to 9 can be used.

How many different license plates are possible:

1. Letters and numbers can repeat. $24 \cdot 24 \cdot 24 \cdot 10 \cdot 10 \cdot 10 = 13,824,000$

2. Letters and numbers can't repeat. $24 \cdot 23 \cdot 22 \cdot 10 \cdot 9 \cdot 8 = 8,743,680$