Simplifying Trigonometric Expressions:

A trigonometric identity is an equation that is true for all values of x that are in the domain of the functions.

Reciprocal identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Tangent and cotangent identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Simplifying a trig expression:

One method is to replace all trig functions with sines and cosines.

Simplify each trig expression:

1. sinx secx $\boxed{\text{Tanx}}$

$$\text{Sin}\,\frac{1}{\cos} = \frac{\text{Sin}}{\cos} =$$

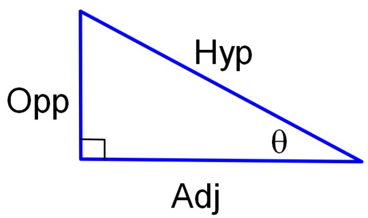
2. $\frac{\text{cosx secx}}{\text{tanx}} = \frac{\cos \cdot \frac{1}{\cos}}{\frac{\sin}{\cos}} = \frac{1}{\frac{\text{sinx}}{\cos}} = \frac{\cos}{\sin} = \text{cot X}$

Pythagorean Identity

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{\text{Opp}}{\text{Hyp}}\right)^2 + \left(\frac{\text{Adj}}{\text{Hyp}}\right)^2$$

$$\frac{\text{Opp}^2}{\text{Hyp}^2} + \frac{\text{Adj}^2}{\text{Hyp}^2} = \frac{\text{Opp}^2 + \text{Adj}^2}{\text{Hyp}^2} = \frac{\text{Hyp}^2}{\text{Hyp}^2}$$



Pythagorean Theorem:

$$\text{Leg}^2 + \text{Leg}^2 = \text{Hyp}^2$$

$$\text{Opp}^2 + \text{Adj}^2 = \text{Hyp}^2$$

Variations of $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$$\boxed{1 - \cos^2\theta = \sin^2\theta}$$

$$\boxed{1 - \sin^2\theta = \cos^2\theta}$$

Other Pythagorean Identities

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

Divide both sides by $\cos^2\theta$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

Other Pythagorean Identities

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

Divide both sides by $\sin^2\theta$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

Simplify each trig expression:

3. $\dfrac{\tan^2 x + 1}{1 + \cot^2 x}$

$$\frac{\sec^2\theta}{\csc^2\theta}$$

$$\frac{\frac{1}{\cos^2\theta}}{\cancel{\frac{1}{\sin^2\theta}}} \quad \frac{1}{\cos^2\theta} \cdot \frac{\sin^2\theta}{1} \quad \frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta$$

4. $\dfrac{1 - \cos^2 x}{\sin^2 x}$

$$\frac{\cancel{1 - \cos^2 x}}{\cancel{1 - \cos^2 x}} = 1$$

Simplify each Trigonometric Expression to a single trig function or a constant.

1. $\dfrac{\cos\theta \cdot \csc\theta}{\cot\theta} = 1$

$$\frac{\boxed{\cos \cdot \frac{1}{\sin}}}{\frac{\cos}{\sin}} = \frac{\frac{\cos}{\sin}}{\frac{\cos}{\sin}}$$

2. $\dfrac{1 + \tan x}{1 + \cot x} = \dfrac{1+\frac{\sin}{\cos}}{1+\frac{\cos}{\sin}}$

$$\frac{\frac{\cos + \sin}{\cos}}{\frac{\sin + \cos}{\sin}}$$

$$\frac{\cancel{\cos + \sin}}{\cos} \cdot \frac{\sin}{\cancel{\sin + \cos}}$$

$$\sin / \cos = \tan x$$

$\dfrac{\csc x - \sin x}{\csc x}$

$$\frac{\csc x - \sin x}{\csc x}$$

$$\frac{\frac{1}{\sin} - \sin}{\frac{1}{\cancel{\sin}}} \cdot \frac{\sin}{\cancel{\sin}} = 1 - \sin^2 = \boxed{\cos^2 x}$$

$$\frac{\tan x + 1}{\sec x} = \frac{\frac{\sin}{\cos} + 1}{\frac{1}{\cancel{\cos}}} \cdot \frac{\cos}{\cancel{\cos}} = \boxed{\sin + \cos}$$