Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

No HA

Case 2: Degree of the Numerator = Degree of the Denominator

HA: y = ratio of the Leading Coefficients

Case 1: Degree of the Denominator > Degree of the Numerator

HA: y = 0

Determine by the equation the Horizontal Aysmptote for each rational function, if it has one.

1. $y = \dfrac{x^3 + 4x^2 - 9}{2x^2 + 6}$

NONE

2. $y = \dfrac{15x^2 - 2x + 10}{3x^2 + 5}$

$y = 5$

3. $y = \dfrac{20x + 13}{4x^2 + 9}$

$y = 0$

Use this function: $y = \dfrac{3x^2 + 15x}{x^2 - 2x - 8} = \dfrac{3x(x+5)}{(x-4)(x+2)}$

1. Find the HA, if any $y=3$
2. Find the VA, if any $X = 4, -2$
3. Sketch these asymptotes on a graph
4. Find the behavior of the graph on each side of each VA.
5. Sketch the graph.

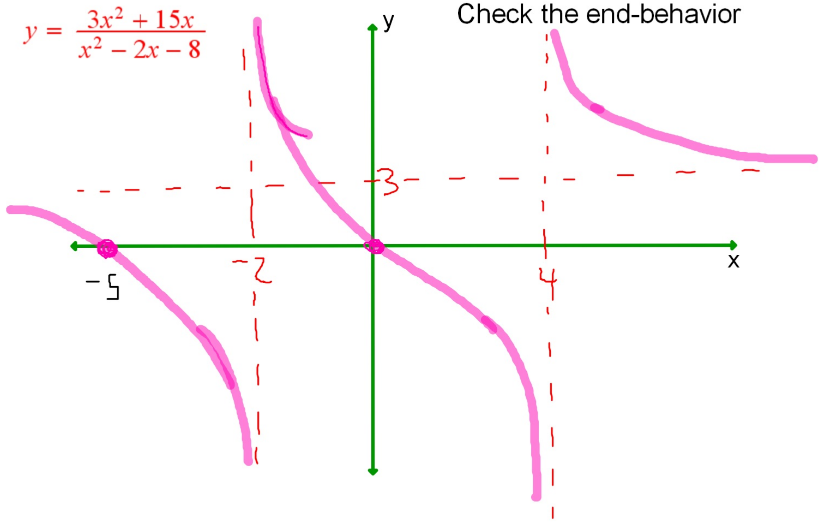
X-INT 0, −5

y-INT 0

X	3X	x+5	x−4	X+2	
−2.1	−	+	−	−	−
−1.9	−	+	−	+	+
3.9	+	+	−	+	−
4.1	+	+	+	+	+

$y = \dfrac{3x^2 + 15x}{x^2 - 2x - 8}$

Check the end-behavior



Y-intercepts of Rational Functions: replace x with zero.

$$y = \frac{3x^2 + 15x}{x^2 - 2x - 8} = \frac{0}{-8} = 0$$

What will the y-int of a rational function always turn out to be?

y-int = ratio of the constants.

X-intercepts of a Rational Function: Replace y with zero

$$y = \frac{x^2 - 9}{x^2 + 5x + 4} \longrightarrow 0 = \frac{x^2 - 9}{x^2 + 5x + 4}$$

A fraction equals zero only when....
the numerator is zero

$$0 = \frac{(x+3)(x-3)}{x^2 + 5x + 4}$$

X-intercepts of Rational Functions are zeros of the Numerator.

Use this function: $y = \frac{x - 7}{x^2 + 2x - 15} = \frac{x-7}{(x+5)(x-3)}$

1. Find the HA, if any $y = 0$
2. Find the VA, if any $x = -5, 3$
3. Sketch these asymptotes on a graph
4. Find the behavior of the graph on each side of each VA.
5. Sketch the graph.

X-int = 7

Y-int = $\frac{7}{15}$

$$y = \frac{x - 7}{x^2 + 2x - 15}$$

X	X−7	X+5	X−3	
−5.1	−	−	−	−
2.9	−	+	−	+