What are the points of discontinuity, if any? Are they vertical asymptotes or holes?

$$y = \frac{x-6}{36-x^2} = \frac{x-6}{(6+x)(6-x)} = \frac{x-6}{-1(x-6)(x+6)}$$

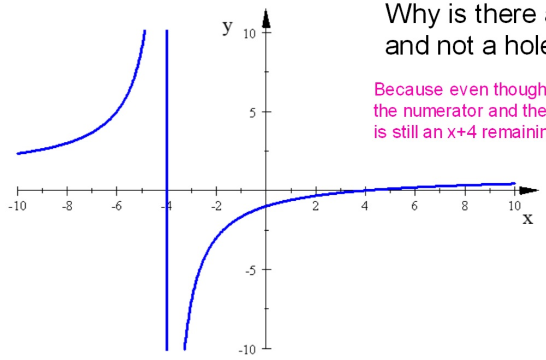
$6 \to$ Hole

$-6 \to$ VA

An exception to this rule:

$$y = \frac{x^2-16}{x^2+8x+16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$

$$y = \frac{x^2-16}{x^2+8x+16} = \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+4)}$$



Why is there a VA at x=-4 and not a hole?

Because even though x+4 cancels in both the numerator and the denominator there is still an x+4 remaining in the denominator.

Properties **Vertical Asymptotes**

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a hole in the graph or a vertical asymptote at $x = a$.

$$y = \frac{x+6}{x-2}$$

VA at x = 2

The closer the graph gets to a vertical asymptote the faster it increases (or decreases)

(this means y will become very big positive or very big negative)

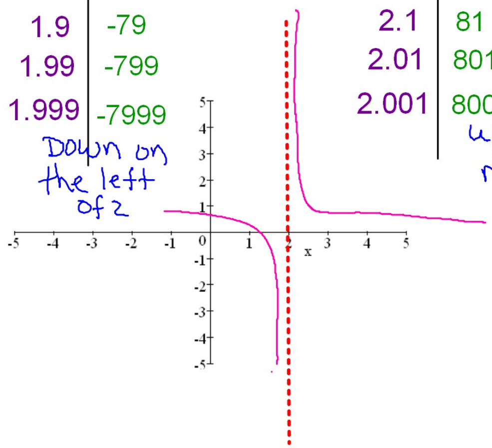
the closer x gets to 2 the closer the denominator gets to zero. Thus, this makes y become very big Positive or very big Negative.

x	y
1.9	-79
1.99	-799
1.999	-7999

Down on the left of 2

x	y
2.1	81
2.01	801
2.001	8001

up on the right of 2



The closer a graph gets to a Vertical Asymptote the larger positive or larger negative y gets.

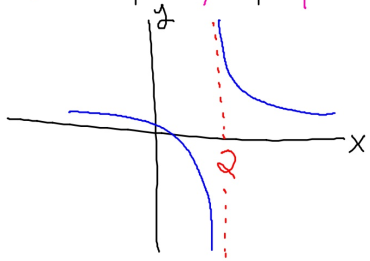
Sign Analysis

$$y = \frac{x+6}{x-2}$$

A number just to the left of 2 →

A number just to the right of 2 →

x	x+6	x-2	quotient	
1.9	+	−	−	Down on the left of 2
2.1	+	+	+	Up on the right of 2



$$y = \frac{x+6}{x-2}$$

A number just to the left of 2 →

A number just to the right of 2 →

x	$\frac{x+6}{x-2}$		
1.9	$\frac{+}{-}$	−	Down on the left of 2
2.1	$\frac{+}{+}$	+	Up on the right of 2

Use Sign Analysis to determine what happens on each side of each vertical asymptote.

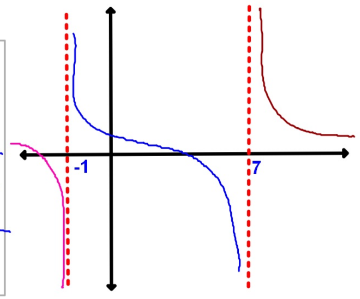
$$y = \frac{x-3}{(x+1)(x-7)}$$

VA: x = -1

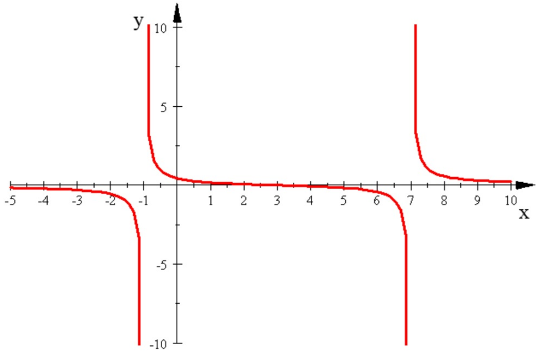
x	x-3	x+1	x-7	
-1.1	−	−	−	−
-.9	−	+	−	+

VA: x = 7

x	x-3	x+1	x-7	
6.9	+	+	−	−
7.1	+	+	+	+



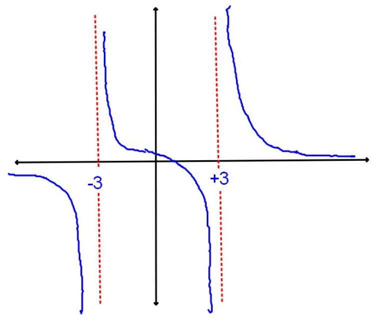
$$y = \frac{x-3}{(x+1)(x-7)}$$



What happens on each side of each VA?

$$y = \frac{x+1}{x^2-9} = \frac{x+1}{(x+3)(x-3)}$$

X	X+1	X+3	X−3	QUOTIENT
−3.1	−	−	−	− Down
−2.9	−	+	−	+ Up
2.9	+	+	−	− Down
3.1	+	+	+	+ Up



What happens on each side of each VA?

$$y = \frac{x-5}{x^2+2x+1} = \frac{x-5}{(x+1)(x+1)}$$

X	X−5	X+1	X+1	
−.9	−	+	+	− down
−1.1	−	−	−	− down