$$y = \frac{4}{x} + 3$$

Why is the horizontal Asymptote y = 3?

What happens to the value of $\frac{4}{x}$ when x becomes very large?

When x increases 4/x decreases so much that it essentially becomes zero. Therefore, the original equation becomes y=3.

Horizontal Asymptote → End Behavior

It's the value that y approaches as x gets very big positive and very big negative.

$$y = \frac{2}{x - 1}$$

Why is the Vertical Asymptote x = 1?

The denominator is undefined when x = 1 so the graph can't exist there. This causes a break in the graph.

What happens to the value of $\frac{2}{x - 1}$ when x gets close to 1?

The closer that x gets to 1 the smaller the denominator becomes and thus the larger the value of y becomes (either large positive or large negative)

Transformations of the Parent Function $y = \frac{1}{x}$

$$y = \frac{a}{x - h} + k$$

$a > 0$
Branches in Quadrants I and III

$a < 0$
Branches in Quadrants II and IV (x-axis reflection)

$h :$
Horizontal Translation
VA: x = h

$k :$
Vertical Translation
HA: y = k

$0 < a < 1$
Vertical Shrink
Branches closer to the origin

$a > 1$
Vertical Stretch
Branches further from origin

Sec 9-2 The Reciprocal Function

After completing this section you will be able to:

1. Write an equation of a reciprocal function given:
   a. A written description of the transformations
   b. The H.A. and V.A.
   c. The graph

2. Given the equation of a reciprocal function you will be able to:
   a. Describe the transformations
   b. State the H.A. and V.A.
   c. Sketch the graph

Write the equation of each transformation of the parent reciprocal function.

1. The branches are in quadrants II and IV & it has been shifted 7 units left and 2 units up.

2. The branches are in quadrants I and III, it is half as tall as it used to be, and the asymptotes are:

VA: $x = -5$ HA: $y = -9$

3. Describe the transformations represented in this reciprocal function:

$$y = \frac{9}{x - 4} + 11$$

4. State the equations of the HA and VA:

$$y = \frac{7}{x + 15} - 21$$

Section 9-3

Definition **Rational Function**

A **rational function** $f(x)$ is a function that can be written as

$$f(x) = \frac{P(x)}{Q(x)},$$ Ratio of two polynomials

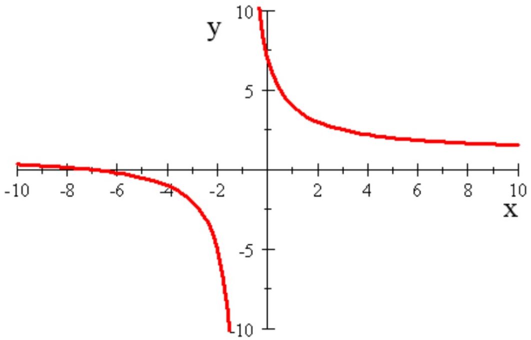
where $P(x)$ and $Q(x)$ are polynomial functions. The domain of $f(x)$ is all real numbers except those for which $Q(x) = 0$. The denominator can't = 0

When the denominator is zero there is a break in the graph - because this value of x can never be used.

These breaks in the graph are one of two types:

Graph the rational function f(x) in a standard window.

$$f(x) = \frac{x+7}{x+1}$$



This kind of break in the graph is called a

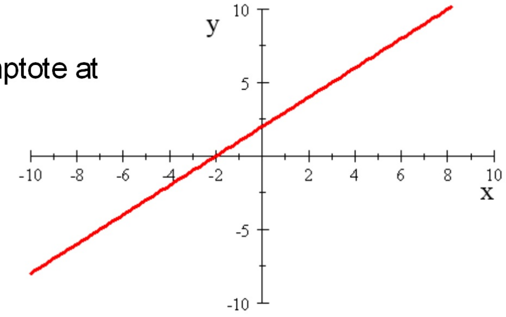
Vertical Asymptote

Graph the rational function f(x) in a standard window.

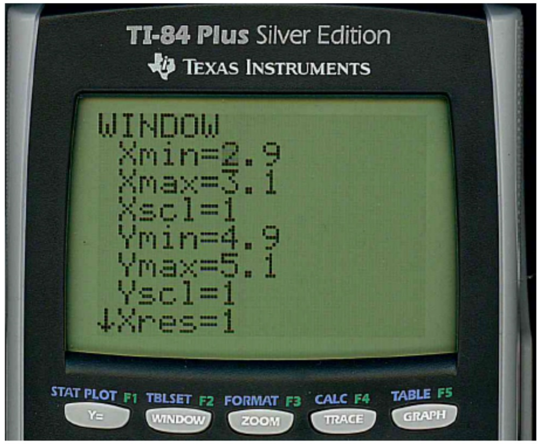
$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

Do you see a vertical asymptote?

Why do you think that there isn't a vertical asymptote at x = 3?



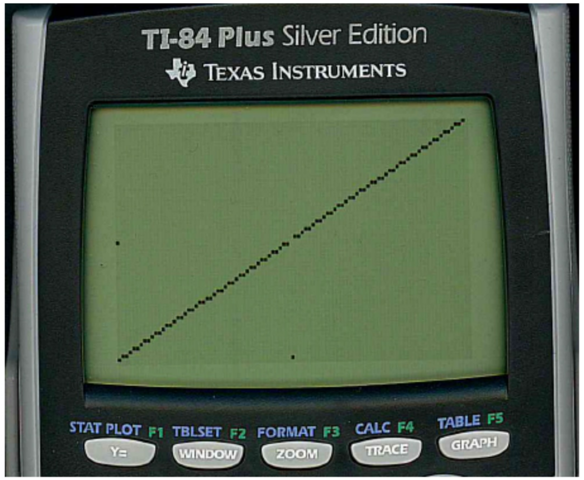
Look at the graph in the following window:



What do you see?

What appears to be a line.

This kind of break in the graph is called a **Hole**



Why did this graph have a Vertical Asymptote at x = -1

$$f(x) = \frac{x+7}{x+1}$$

and

this graph have a hole at x = 3?

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

Breaks in the graph are caused by zeros of the denominator and are called:

Points of Discontinuity

Holes or **Vertical Asymptotes**

Holes: Occur at values of x that are zeros of both the denominator and numerator

Vertical Asymptotes: Occur at values of x that are zeros of the denominator ONLY.

Find any points of dicontinuity and classify them as Vertical Asymptotes or Holes.

1. $y = \frac{2x(x+1)(x-6)}{(x-1)(x-6)}$

6 is a hole

1 is a vertical asymptote

2. $y = \frac{(x-2)}{x^2+4}$

There are no points of discontinuity because there are no real zeros of the denominator.