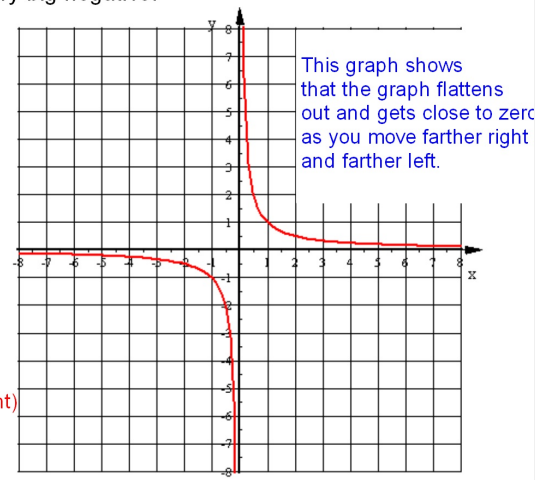
Horizontal Asymptote → End Behavior

The horizontal line that the graph approaches as it flattens out at the far left end and far right end. In other words, it's the value that y approaches as x gets very big positive and very big negative.

$y = 1/x$

x	y
10	0.1
100	0.01
1000	0.001
10000	0.0001
1000000	0.000001

This table shows that y approaches zero as x increases (move to the right)

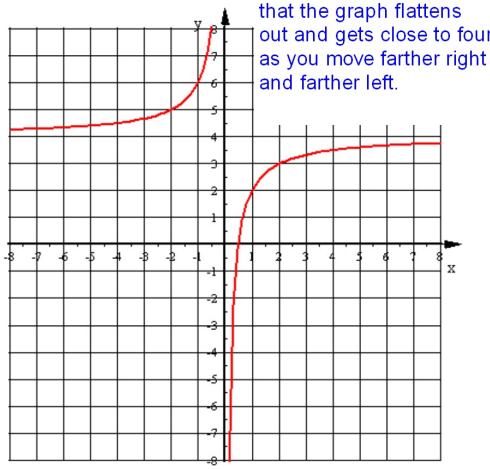


This graph shows that the graph flattens out and gets close to zero as you move farther right and farther left.

$y = -2/x+4$

x	y
10	3.8
100	3.98
1000	3.998
10000	3.9998
1000000	3.999998

This table shows that y approaches 4 as x increases (move to the right)

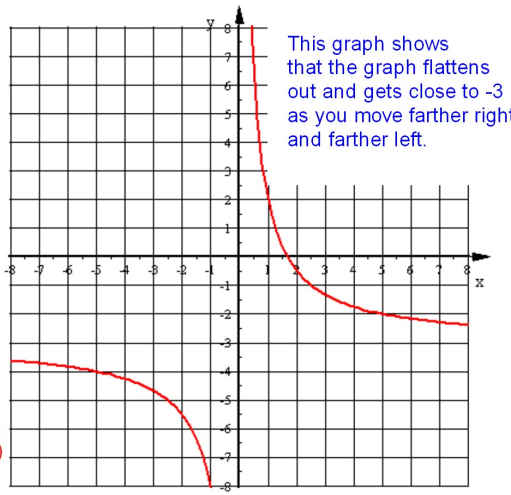


This graph shows that the graph flattens out and gets close to four as you move farther right and farther left.

$y = 5/x-3$

x	y
10	-2.5
100	-2.95
1000	-2.995
10000	-2.9995
1000000	-2.999995

This table shows that y approaches -3 as x increases (move to the right)



This graph shows that the graph flattens out and gets close to -3 as you move farther right and farther left.

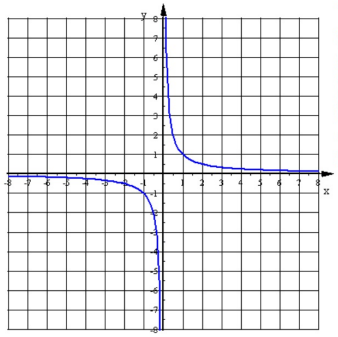
Vertical Asymptote

A vertical line the graph approaches. It's created by zeros of the denominator. As the graph approaches a vertical asymptote it either increases without bound or it decreases without bound.

$y = 1/x$

x	y
0	ERROR
0.01	100
0.001	1000
0.0001	10000
0.00001	100000

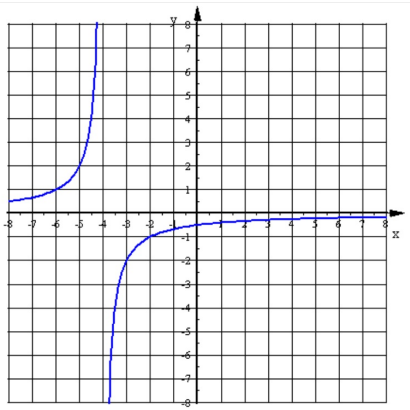
This table shows that as x approaches zero the y values increase dramatically but x cannot equal zero.



This shows that as the graph approaches zero it rises rapidly on the right side and declines rapidly on the left side.

$y = -2/(x+4)$

x	y
-4	ERROR
-3.99	-200
-3.999	-2000
-3.9999	-20000
-3.99999	-200000

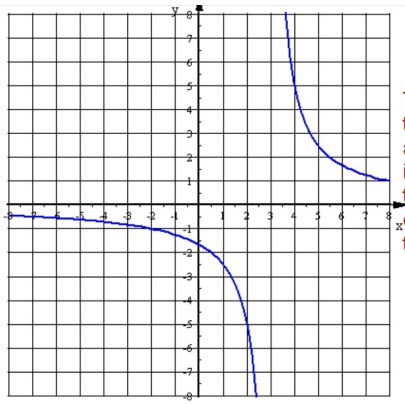


This shows that as the graph approaches -4 it rises rapidly on the left side and declines rapidly on the right side.

This table shows that as x approaches -4 the y values increase dramatically but x cannot equal -4.

$y = 5/(x-3)$

x	y
3	ERROR
2.99	-500
2.999	-5000
2.9999	-50000
2.99999	-500000



This shows that as the graph approaches 3 it rises rapidly on the right side and declines rapidly on the left side.

This table shows that as x approaches 3 the y values increase dramatically but x cannot equal 3.

Transformations of the Parent Function $y = \frac{1}{x}$

$$y = \frac{a}{x-h} + k$$

$a > 0$
Branches in Quadrants I and III

$a < 0$
Branches in Quadrants II and IV
(x-axis reflection)

h :
Horizontal Translation
VA: x = h

k :
Vertical Translation
HA: y = k

$0 < a < 1$
Vertical Shrink
Branches closer to the origin

$a > 1$
Vertical Stretch
Branches further from origin

Sec 9-2 The Reciprocal Function

After completing this section you will be able to:

1. Write an equation of a reciprocal function given:
   a. A written description of the transformations
   b. The H.A. and V.A.
   c. The graph

2. Given the equation of a reciprocal function you will be able to:
   a. Describe the transformations
   b. State the H.A. and V.A.
   c. Sketch the graph

1. Write the equation of this function which is a transformation of $y = \frac{2}{x}$

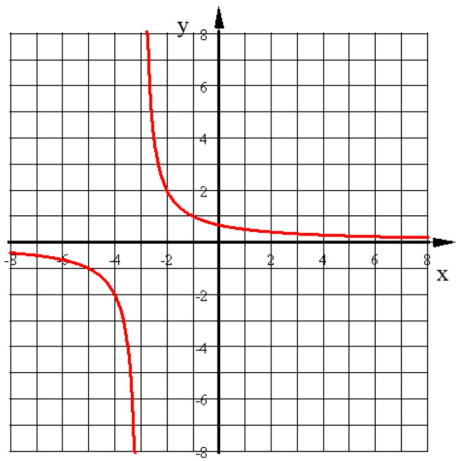
HA: $y=0$

VA: $x=-3$

x-axis Reflection?

NO

EQ: $y = \frac{2}{x+3}$



2. Write the equation of this function which is a transformation of $y = \frac{3}{x}$

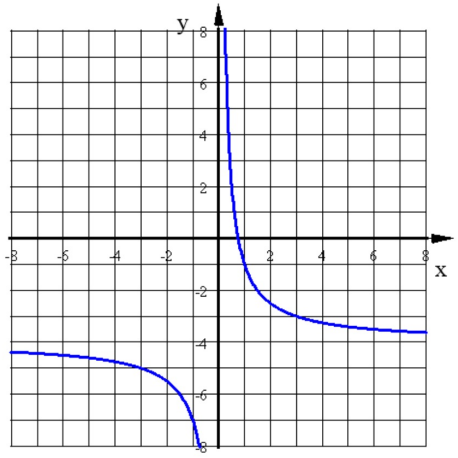
HA: $y=-4$

VA: $x=0$

x-axis Reflection?

NO

EQ: $y = \frac{3}{x} - 4$



3. Write the equation of this function which is a transformation of $y = \frac{1}{x}$

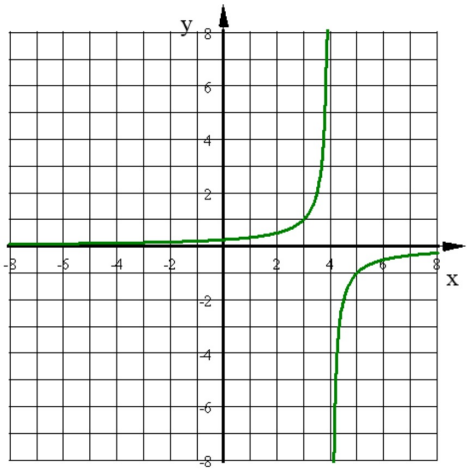
HA: $y=0$

VA: $x=4$

x-axis Reflection?

yes

EQ: $y = \frac{-1}{x-4}$



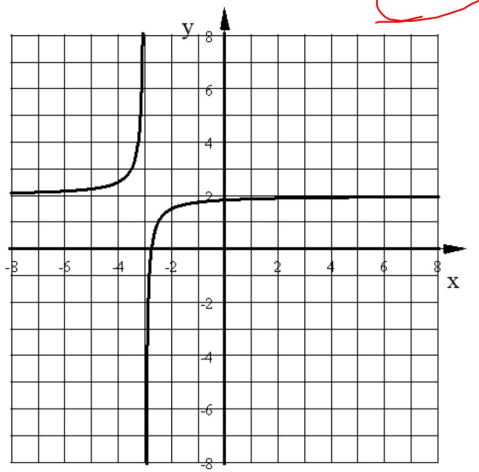
4. Write the equation of this function which is a transformation of $y = \frac{0.5}{x}$

HA:

VA:

x-axis Reflection?

EQ: $y = \frac{-0.5}{x+3} + 2$



$$y = \frac{8}{x-11} + 13$$

HA: $y = 13$

VA: $x = 11$

Location of branches:

I III

Describe the transformations shown by each reciprocal function.

$$y = \frac{4}{x+7} - 3$$

Vert strech factor = 4
7 units left
3 units down

HA: y = -3

VA: x= -7

$$y = \frac{-0.25}{x-1} + 9$$

Vert shrink factor = 0.25
x-axis reflection
1 unit right
9 units up

HA: y = 9

VA: x = 1

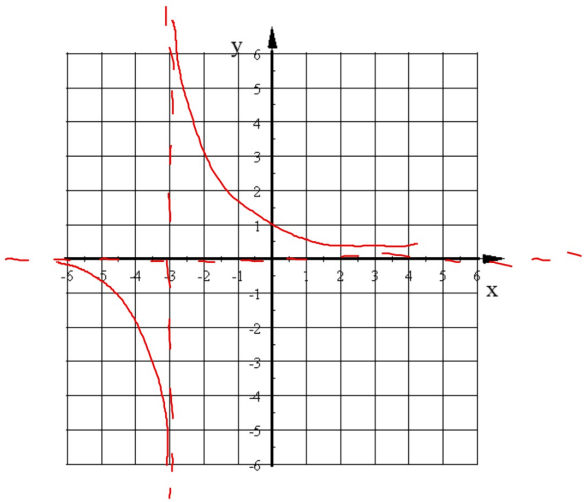
1. Graph this reciprocal function. Show the asymptotes as dashed lines.

$$y = \frac{2}{x+3}$$

HA: $y = 0$

VA: $x = -3$

I & III



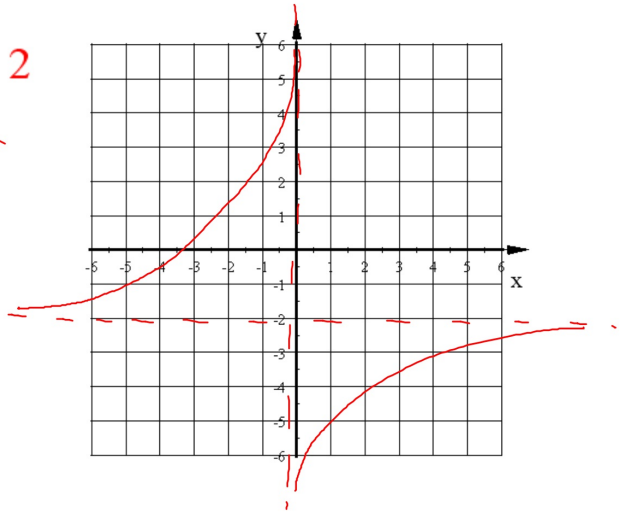
2. Graph this reciprocal function. Show the asymptotes as dashed lines.

$$y = \frac{-5}{x} - 2$$

HA: $y = -2$

VA: $x = 0$

II IV

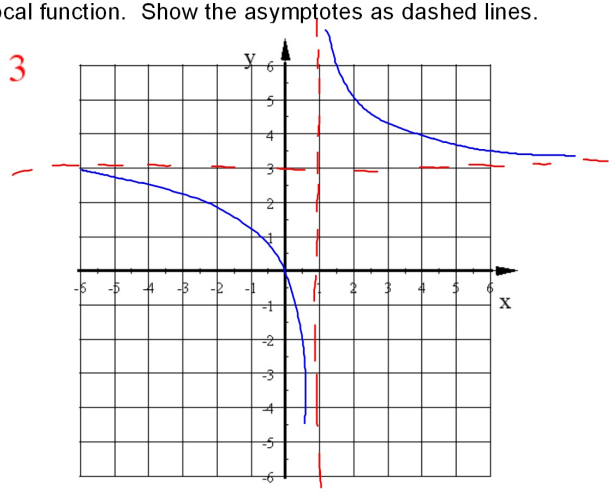


3. Graph this reciprocal function. Show the asymptotes as dashed lines.

$y = \frac{1}{x-1} + 3$

HA: $y=3$

VA: $x=1$



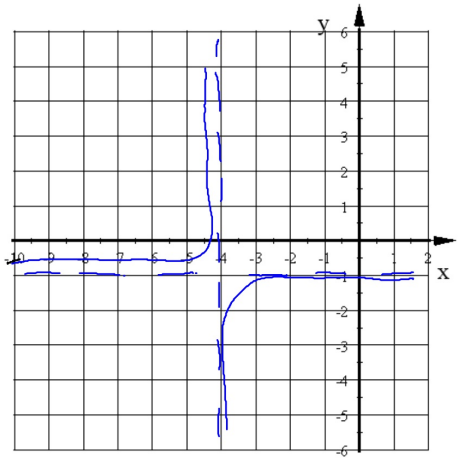
4. Graph this reciprocal function. Show the asymptotes as dashed lines.

$y = \frac{-0.25}{x+4} - 1$

HA: $y=-1$

VA: $x=-4$

II IV



Write each equation in the form $y = \frac{k}{x}$

1. $y = \frac{1}{5x} = \frac{1}{5} \cdot \frac{1}{x}$

$= .2 \frac{1}{x}$

$\frac{.2}{x}$

2. $\frac{4xy}{4x} = \frac{10}{4x}$

$y = \frac{2.5}{x}$

Write the equation of each transformation of the reciprocal function:

1. 5 units left, twice as tall, branches are in quadrants I and III

$\frac{2}{x+5}$

2. 8 units up, half as tall, branches are in quadrants II and IV

$y = \frac{.5}{x} + 8$

3. 3 units right, 2 units down, branches are in quadrants II and IV

$\frac{-1}{x-3} - 2$

Write the equation of the translation of the parent reciprocal function that has the given asymptotes:

1. x = 8 and y = -7

$$\frac{1}{x-8} - 7$$

2. x = -12 and y = 0

$$y = \frac{1}{x+12}$$