The graph of Inverse Variation.

x	y
0.5	16
1	8
2	4
4	2
8	1
10	0.8
16	0.5

$$xy = 8 \quad or \quad y = \frac{8}{x}$$

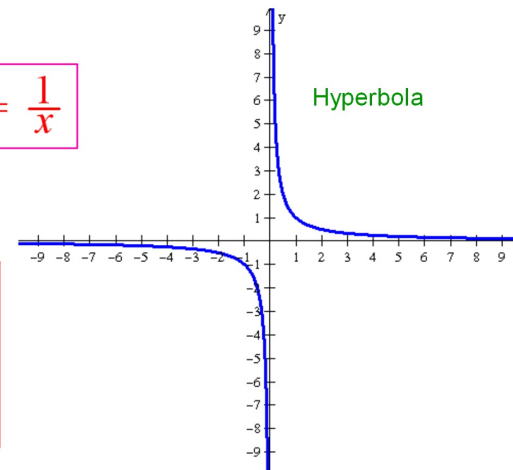A curve called a Hyperbola

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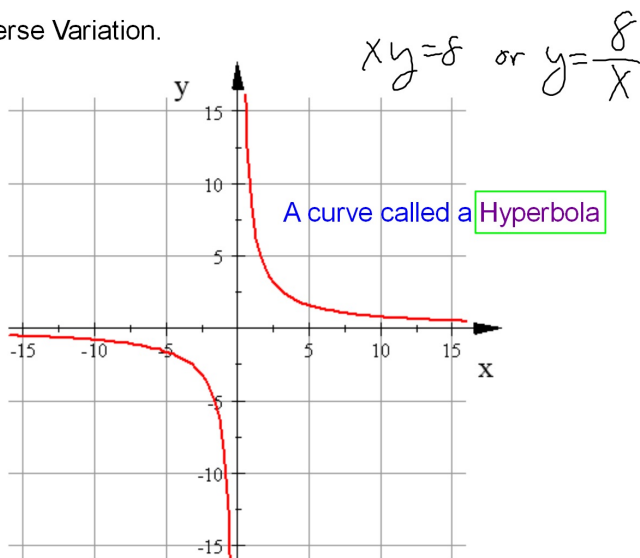
The Reciprocal Function: A member of the Rational Function Family

Parent Function: $y = \frac{1}{x}$

Hyperbola

The branches of the parent function are in which quadrants?

I   III

Graph in a standard window these three functions:

$$Y_1 = \frac{1}{x} \qquad Y_2 = \frac{5}{x} \qquad Y_3 = \frac{10}{x}$$

$5 \cdot \frac{1}{x}$

Note how they differ.

What happens to the graph as the value of the numerator increases?

Branches are stretched further from the origin.

Vertical Stretch factor

Graph in a standard window these three functions:

$$Y_1 = \frac{1}{x} \qquad Y_2 = \frac{-4}{x} \qquad Y_3 = \frac{-12}{x}$$

Note how they differ.

What happens to the location of the brances when the numerator is negative?
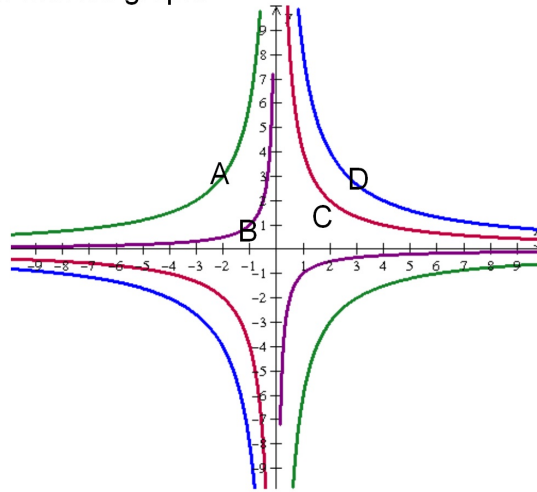
Branches are now in Quadrants II and IV

x-axis reflection

Panel 1 (top-left)

Match each equation with it's graph.

$y = \dfrac{-1}{x}$ B

$y = \dfrac{4}{x}$ C

$y = \dfrac{-6}{x}$ A

$y = \dfrac{8}{x}$ D

Panel 2 (top-right)

After completing this section you will be able to:

1. Write an equation of a reciprocal function given:
   a. A written description of the transformations
   b. The H.A. and V.A.
   c. The graph

2. Given the equation of a reciprocal function you will be able to:
   a. Describe the transformations
   b. State the H.A. and V.A.
   c. Sketch the graph

Panel 3 (bottom-left)

Describe what happens to the graph as you move farther and farther right

flattens out and getting close to the x-axis (Horiz line Y=0)

and

as you move farther and farther left.

flattens out & gets close to the x axis (y=0)

$y = \dfrac{1}{x}$

Panel 4 (bottom-right)
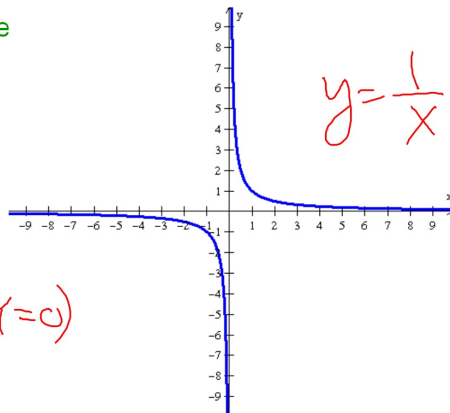
Using the table function on the calculator test some values of x to explain what happens to the value of the function (y) as

$X \longrightarrow \infty$
bigger and bigger positive values for x

$y \longrightarrow 0^+$

and

$X \longrightarrow {}^-\infty$
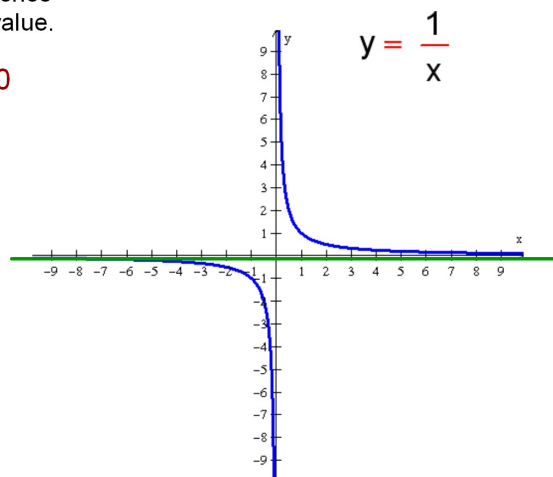bigger and bigger negative values for x

$y \longrightarrow 0^-$

$y_1 = \dfrac{1}{x}$

The Parent Reciprocal Function

Horizontal Asymptote:
a line that the graph approaches
as x increases in absolute value.

Horiz Asymptote: $y = 0$

$$y = \frac{1}{x}$$

Use a graph and/or the table function to find the
Horizontal Asymptote of each reciprocal function.

1. $y = \dfrac{2}{x}$

   HA: $y = 0$

2. $y = \dfrac{-4}{x}$

   HA: $y = 0$

3. $y = \dfrac{6}{x} - 3$

   HA: $y = -3$

4. $y = \dfrac{1}{x} + 2$

   HA: $y = 2$

$$y = \frac{a}{x} + k$$

What is the Horizontal Asymptote?

$y = k$

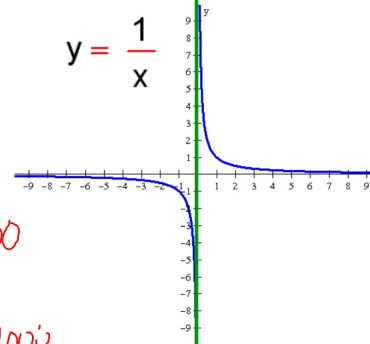Explain what happens to the graph as x gets closer and closer to zero

From the
left side

$$y = \frac{1}{x}$$

From the
right side

$$\frac{1}{-0.1} = -10$$

$$\frac{1}{0.1} = 10$$

$$\frac{1}{-0.01} = -100$$

$$\frac{1}{0.01} = 100$$

$$\frac{1}{-0.001} = -1000$$

$$\frac{1}{0.001} = 1000$$

The closer the denominator gets to zero
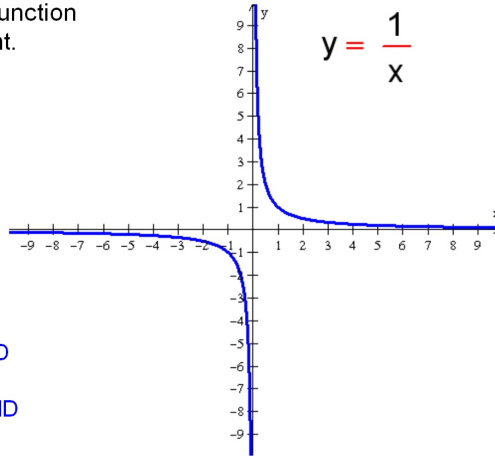the larger the value of y becomes (+ or -)

large + y means graph goes up
large - y means graph goes down

Vertical Asymptote:
a line that the graph approaches but can never reach because the function becomes undefined a that point.

Vert Asymptote: x = 0

As a graph approaches
a Vertical Asymptote it
INCREASES WITHOUT BOUND
OR
DECREASES WITHOUT BOUND

The Parent Reciprocal Function

$$y = \frac{1}{x}$$

The vertical asymptote of the parent reciprocal function is the line x = 0

What must be done to the equation of the parent reciprocal function in order to change the Vertical Asymptote?

Shift the graph left or right.
(change the zero of the denominator)

$$\frac{a}{x \pm h}$$

$$y = \frac{1}{x}$$
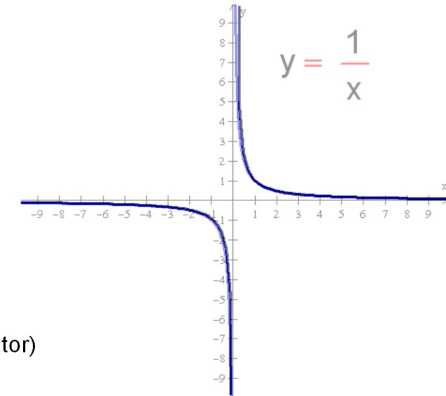
Find the equation of the Vertical Asymtptote of each reciprocal function.

1. $y = \frac{20}{x}$

VA: X= ◯

2. $y = \frac{-11}{x + 6}$

VA: X= -6

3. $y = \frac{1}{x - 5}$

VA: X= 5

What is the Vertical Asymptote?

$$y = \frac{a}{x - h}$$

VA: X=h

Transformation of the Parent Function: $y = x^2$

$$y = a(x - h)^2 + k$$

$a > 0$
Opens Up

$a < 0$
Opens Down
(x-axis Reflection)

$h :$
Horizontal Translation
x-coord of the Vertex

$k :$
Vertical Translation
y-coord of the Vertex

$0 < a < 1$
Vertical Shrink

$a > 1$
Vertical Stretch

Transformations of the Parent Function $y = \frac{1}{x}$

$$y = \frac{a}{x - h} + k$$

$a > 0$
Branches in Quadrants I and III

$a < 0$
Branches in Quadrants II and IV
(x-axis reflection)

$h :$
Horizontal Translation
VA: x = h

$k :$
Vertical Translation
HA: y = k

$0 < a < 1$
Vertical Shrink
Branches closer to the origin

$a > 1$
Vertical Stretch
Branches further from origin

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1. Write the equation of this function which is a transformation of $y = \frac{2}{x}$
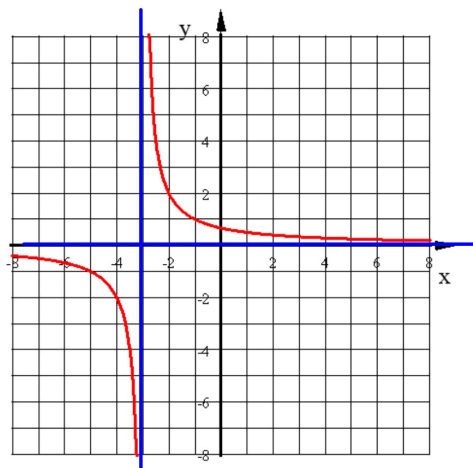
HA: $y = 0$

VA: $x = -3$

x-axis Reflection? NO

EQ: $y = \frac{2}{x + 3}$

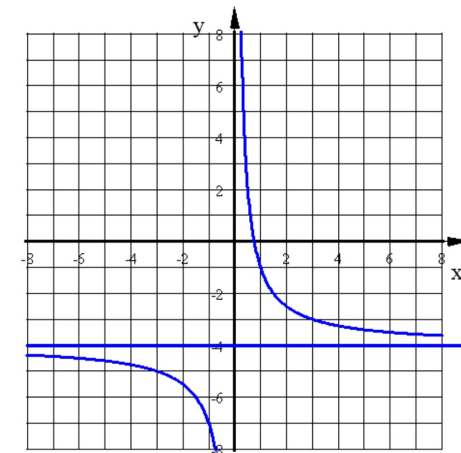2. Write the equation of this function which is a transformation of $y = \frac{3}{x}$

HA: $y = -4$

VA: $x = 0$

x-axis Reflection?

EQ: $y = \frac{3}{x} - 4$

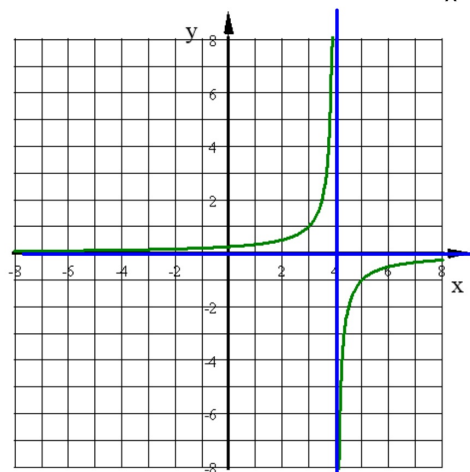3. Write the equation of this function which is a transformation of $y = \frac{1}{x}$

HA: $y = 0$

VA: $x = 4$
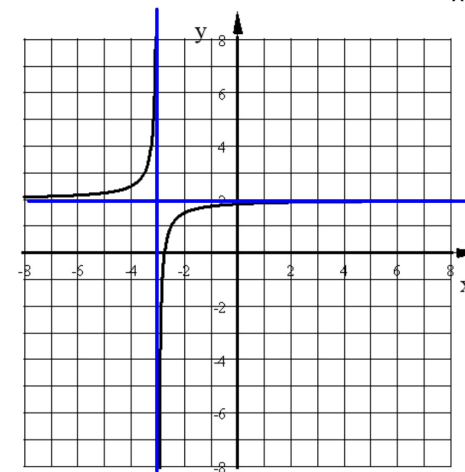
x-axis Reflection?

EQ: $y = \frac{-1}{x-4}$

4. Write the equation of this function which is a transformation of $y = \frac{0.5}{x}$

HA: $y = 2$

VA: $x = -3$

x-axis Reflection?

EQ: $y = \frac{-5}{x+3} + 2$

$$y = \frac{8}{x-11} + 13$$

HA $y = 13$

VA $x = 11$

Describe the transformations shown by each reciprocal function.

$$y = \frac{4}{x+7} - 3 \qquad y = \frac{-0.25}{x-1} + 9$$

Vert strech factor = 4
7 units left
3 units down

HA: y = -3
VA: x= -7

Vert shrink factor = 0.25
x-axis reflection
1 unit right
9 units up

HA: y = 9

VA: x = 1

1. Graph this reciprocal function. Show the asymptotes as dashed lines.

$$y = \frac{2}{x+3}$$

HA: $y = 0$

VA: $x = -3$