Section 5-5: Direct Variation

Direct Variation is a special Linear Function.

- It has a constant ratio $\frac{Y}{X} = k$

k = the Variation Constant

- Direct Variation Equation:

$\frac{y}{x} = k$ or $y = kx$

Graph of direct variation

- The graph must be a line that passes through the origin.

Does each table of values represent a Direct Variation relationship?

1. NO

X	Y	$\frac{Y}{X}$
6	28.5	4.75
11	52.25	4.75
19	89	4.68
26	119.6	
42	201.6	

These are NOT equal

2. NO

X	Y	$\frac{Y}{X}$
4	5.4	1.35
14	18.9	1.35
22	21.6	0.9
27	36.45	
34	45.9	

These are NOT equal

Does each table represent a Direct Variation relationship?

1. NO

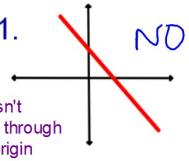
X	Y	$\frac{Y}{X}$
–6	–21	pos
–4	–14	pos
10	–35	neg
15	52.5	
18	63	

Sometimes you can see that a relationship is NOT Direct Variation by considering if the ratio will be Pos or Neg or how big or small the ratio is.

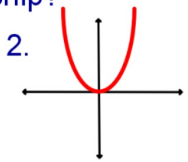
2. NO

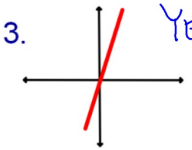
X	Y	$\frac{Y}{X}$
–5	–38	> 1
–2	–15.2	> 1
4	30.4	> 1
12	91.2	> 1
45.6	18	< 1

Does each graph represent a Direct Variation relationship?

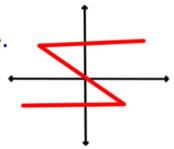
1.  NO

Doesn't pass through the origin

2.  NO this is not a linear function

3.  YES

This is both a LINE and it passes through the ORIGIN

4.  NO

This in not even a function, it doesn't pass the Vertical Line Test.

Direct Variation Equations:

$\frac{y}{x} = k$ or $y = kx$

If you can rewrite these equations to look like one of the above then they must be Direct Variation

Is each equation direct variation?
If yes, find the variation constant.

1. $4x + 2y = 10$

$-4x \quad -4x$

$\frac{2y}{2} = \frac{10}{2} - \frac{4x}{2}$

$y = 5 - 2x$

This isn't the same as y=kx so 4x+2y=10 is NOT Direct Variation

2. $6 + 7y = 5 - 3x + 1$

$6 + 7y = 6 - 3x$

$-6 \quad -6$

$\frac{7y}{7} = \frac{-3x}{7} \rightarrow y = -\frac{3}{7}x$

Yes, this equation is Direct Variation.

K= $-\frac{3}{7}$

Given the table shows a direct variation relationship, find the value of ?.

To solve Direct Variation situations you can use either equation or you can use a Proportion

X	Y
4	9
10	22.5
24	54
35	?

$\frac{y}{x}$

$2.25 = k$

Find the variation constant then write a Direct Variation equation and use it to solve for ?

$\frac{y}{x} \quad \frac{9}{4} = \frac{?}{35} \quad \frac{y}{x}$

$y = 2.25x$

$y = 2.25(35)$

$y = 78.75$

$? = 78.75$

1. Use this table to write a Direct Variation equation.

X	Y
2	12.2
9	54.9
15	91.5
18	109.8
23	140.3

$\frac{y}{x}$

$6.1 = k$

$\frac{y}{x} = 6.1$ or $y = 6.1x$

2. Find the value of x when y=50

Use one of the Direct Variation equations: $50 = 6.1x \rightarrow x = 8.20$

3. Find the value of y when x=20

You could also use a proportion to solve these problems.

$\frac{12.2}{2} = \frac{y}{20}$

$y = 122$

Remember the phrase: "Y varies directly with X"

The amount of water (Y) in the tub varies directly with the amount of time (X) the water has running. After 5 minutes there are 12 gallons in the tub.

1. Model this situation with a Direct Variation equation.

First find k: $k = \frac{Y}{X} = \frac{12 \text{ gal}}{5 \text{ min}} = 2.4 \text{ gal/min}$ $\rightarrow$ $y = 2.4x$ or $\frac{Y}{X} = 2.4$

2. Find the amount of time it takes to fill a 32 gallon tub. (32 $\rightarrow$ y)

Use one of the Direct Variation equations:

$\frac{32}{2.4} = \frac{2.4x}{2.4}$

$x = 13.33 \text{ min}$

OR use a Proportion:

$\frac{12 \text{ gal}}{5 \text{ min}} = \frac{32 \text{ gal}}{x}$ $\quad x = 13.33 \text{ min}$

The number of air conditioners (Y) built varies directly with the number of workers (X) in the factory.

When there are 120 workers on the job 270 air conditioners are manufactured.

1. What is the variation constant?

$k = \frac{Y}{X} = \frac{270 \text{ A.C.}}{120 \text{ work}}$

$k = 2.25 \text{ AC/work}$

2. Write a direct variation equation to model this situation.

$Y = 2.25X$ OR $\frac{Y}{X} = 2.25$

3. Find the number of air conditioners that can be produced if there are 140 workers present (140 $\rightarrow$ X)

Use one of the Direct Variation equations:

$y = 2.25(140)$

$= 315 \text{ A.C.}$

OR use a Proportion:

$\frac{270 \text{ AC}}{120 \text{ W}} = \frac{?}{140 \text{ W}}$ $\quad ? = 315 \text{ A.C.}$