Geometry   6-3: Properties of Parallelograms   Date: _1/30_
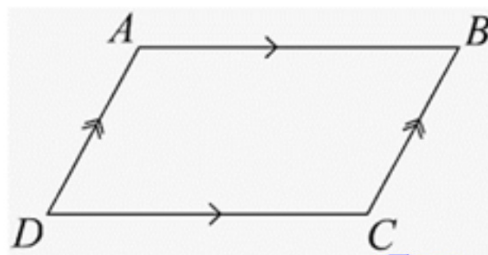
Fill in the essential question for this section: What are the relationships of the _sides_
the _angles_, and the _diagonals_ of a parallelogram?

Objective: I can use the properties of _the sides_, _the angles_, and _diagonals_ to investigate parallelograms.

DEFINITION

A **parallelogram** has 2 pairs of parallel sides.

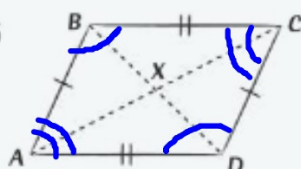$$\overline{AB} \parallel \overline{DC}, \overline{AD} \parallel \overline{BC}$$

CRITIQUE & EXPLAIN

Kennedy lists all the pairs of congruent triangles she finds in quadrilateral **ABCD**.

$\overline{AD} \parallel \overline{BC}$ $\overline{AB} \parallel \overline{CD}$
$\angle ABC \cong \angle CDA$
$\angle DAB \cong \angle BCD$

Congruent triangles:
$\triangle DAB \cong \triangle BCD$ by SAS
$\triangle ABC \cong \triangle CDA$ by SAS

A. Is Kennedy's justification for triangle congruence correct for each pair?

yes.

B. **Look for Relationships** Did Kennedy overlook any pairs of congruent triangles? If not, explain how you know. If so, name them and explain how you know they are congruent.

$\triangle ABX \cong \triangle DCX$ $\}$ ASA
$\triangle BXC \cong \triangle AXD$

Do "Critique and Explain" and "Habits of Mind" in your student companion, page 141.
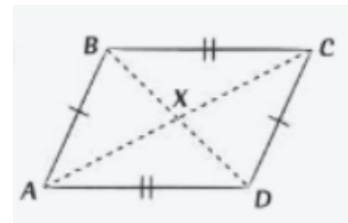
Answer these questions after you do the "Critique and Explain".

1) Is all of the given information marked in the diagram? Is all of the information in the diagram written as a statement?

$\overline{AD} \parallel \overline{BC}$ $\overline{AB} \parallel \overline{CD}$
$\angle ABC \cong \angle CDA$
$\angle DAB \cong \angle BCD$

2) How many small triangles do the diagonals form?

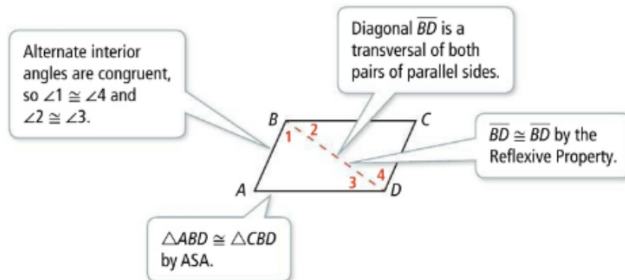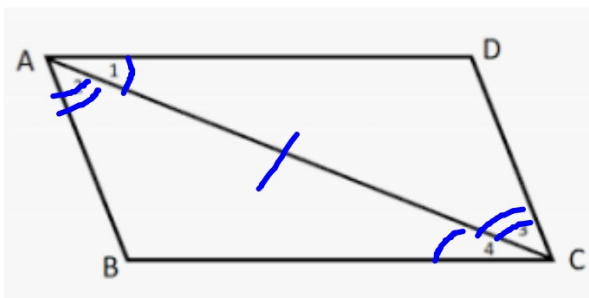3) How many triangles are formed by two adjacent small triangles?

Habits of Mind (pg 141)

Why might it be useful to start analyzing a parallelogram by decomposing it into triangles?

B/C we know a lot about triangles.

Example 1: How do the lengths of opposite sides of a parallelogram compare to each other?

Given: ABCD is a parallelogram.

A ∠1) D

B 4 C

alternate
interior
∠s
are
≅

Alternate interior angles are congruent, so ∠1 ≅ ∠4 and ∠2 ≅ ∠3.

Diagonal $\overline{BD}$ is a transversal of both pairs of parallel sides.

B 1 2 C

A 3 4 D

$\overline{BD} \cong \overline{BD}$ by the Reflexive Property.

△ABD ≅ △CBD by ASA.

By CPCTC, $\overline{AD} \cong \overline{CB}$ and $\overline{AB} \cong \overline{CD}$, so the lengths of the opposite sides are congruent to each other.
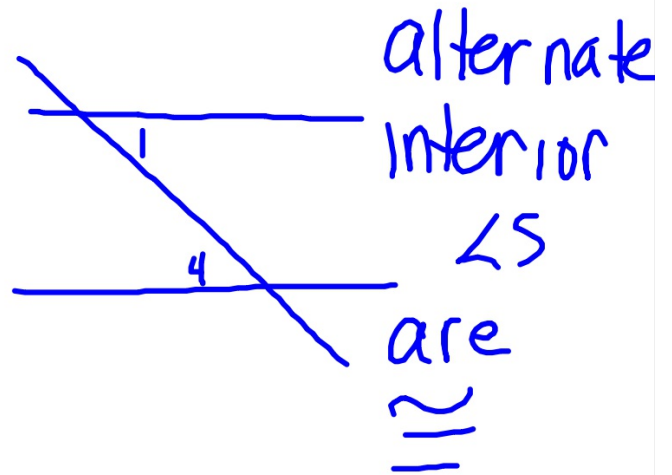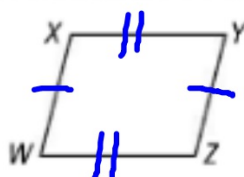
SC: Opposite sides of a parallelogram are _congruent_.

THEOREM 6-7

,3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

PROOF: SEE EXERCISE 13.

If...

$\overline{WX} \parallel \overline{ZY}$
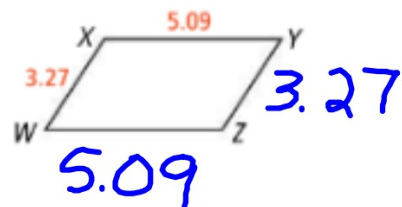$\overline{WZ} \parallel \overline{XY}$

Then... $\overline{WX} \cong \overline{YZ}$
$\overline{WZ} \cong \overline{XY}$
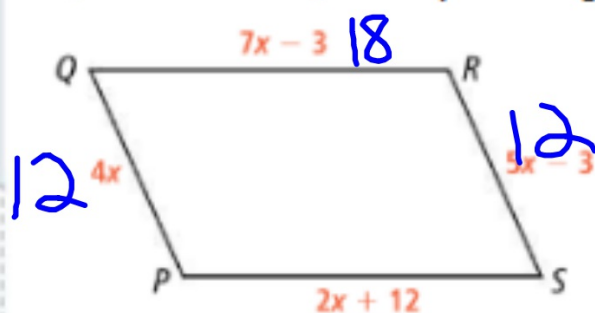
Do Try It 1, page 142 in your student companion.

☑ **Try It!**   1. Given parallelogram WXYZ, what is YZ?

X —— 5.09 —— Y

3.27

3.27

W —— Z

5.09

Quadrilateral *PQRS* is a parallelogram.

7x − 3 18

Q   R

12 4x

12

5x − 3

P

2x + 12

18

S

$$7x - 3 = 2x + 12$$

OR

$$4x = 5x - 3$$

A) What is the value of x?

$$x = 3$$

B) What is the length of each side of PQRS?

Do Try It 2 and "Habits of Mind", page 142 in your student companion.

Try It! 2. The 600-meter fence around City Park forms a parallelogram. The fence along Chaco Road is twice as long as the fence along Grover Lane. What is the length of the fence along Jones Road?
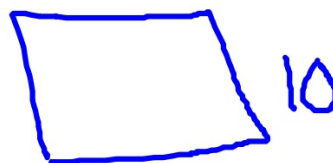
$2x$

Chaco Rd.

Jones Rd.

City Park

Grover Ln.

x $\quad$ x

Village Dr.

$2x$

100 ft (circled)
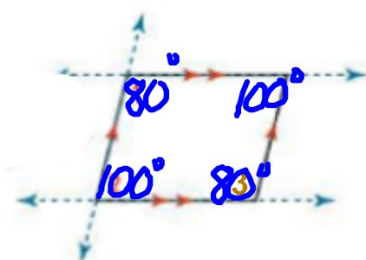
$6x = 600$

$x = 100$

Habits of Mind pg 142

If you know the perimeter of a parallelogram, what information would you need to know in order to determine the lengths of the sides?

1 side

10

Example 3: Explore the angle measures in a parallelogram.

A) How are consecutive angles in a parallelogram related?

$$\angle 1 + \angle 2 = 180°$$

$$\angle 2 + \angle 3 = 180°$$

} consecutive interior $\angle s$

B) How are opposite angles in a parallelogram related?

$$\angle 1 \cong \angle 3$$

STUDY TIP
You can visualize a parallelogram as parallel lines intersected by transversals that are also parallel. This may help you determine how the angles are related.

THEOREM 6-8

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If... B ⟋⟍ C $\overline{AB} \parallel \overline{DC}$
A ⟋⟍ D $\overline{AD} \parallel \overline{BC}$

Then... $m\angle A + m\angle B = 180°$
$m\angle B + m\angle C = 180°$
$m\angle C + m\angle D = 180°$
$m\angle D + m\angle A = 180°$

⎫
⎬ Consecutive
⎭ interior angles

PROOF: SEE EXERCISE 15.

THEOREM 6-9

If a quadrilateral is parallelogram, then opposite angles are congruent.

If... B ⟋⟍ C $\overline{AB} \parallel \overline{DC}$
A ⟋⟍ D $\overline{AD} \parallel \overline{BC}$

Then... $\angle A \cong \angle C$
$\angle B \cong \angle D$

PROOF: SEE EXERCISE 23.

Do Try It 3, page 143 in your student companion.

☑ **Try It!** 3. a. Given parallelogram *ABCD*, what are
   $m\angle A$ and $m\angle C$?

   b. What is $m\angle B$?
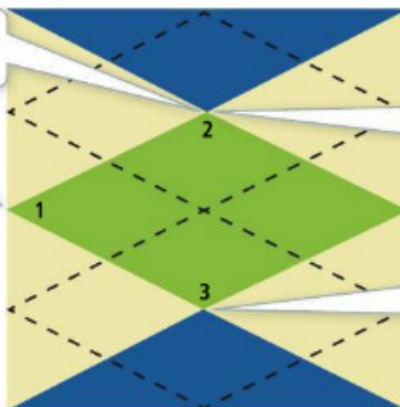
180

714

114°   66°

66°   114°   D

A
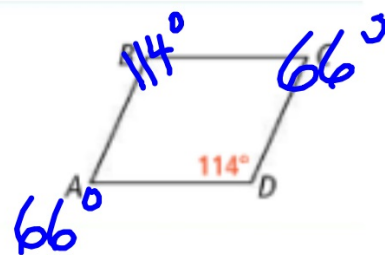
👆 **EXAMPLE 4** **Use Angles of a Parallelogram**

The green shape in the fabric design is a parallelogram. The measure of $\angle 2$ is twice the measure of $\angle 1$. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?

As consecutive angles,
$m\angle 1 + m\angle 2 = 180°$

$m\angle 2 = 2m\angle 1$
is given.

As opposite
angles, $\angle 2 \cong \angle 3$.

Find $m\angle 1$.

$$m\angle 1 + m\angle 2 = 180°$$
$$m\angle 1 + 2m\angle 1 = 180°$$
$$m\angle 1 = 60°$$

Find $m\angle 2$.

$$m\angle 2 = 2m\angle 1$$
$$m\angle 2 = 2(60°)$$
$$m\angle 2 = 120°$$

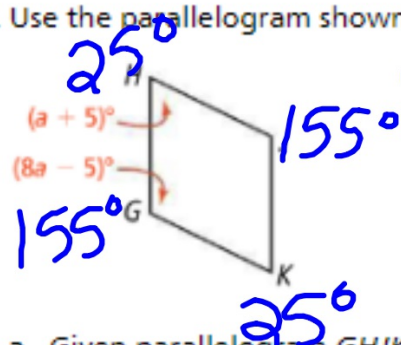Find $m\angle 3$.

$$m\angle 3 = m\angle 2$$
$$m\angle 3 = 120°$$

The measures of $\angle 1$, $\angle 2$, and $\angle 3$ are 60°, 120°, and 120°, respectively.

Do Try It 4 and Habits of Mind, page 144 in your student companion.

Try It!   4. Use the parallelogram shown.

25°

$(a + 5)°$

$(8a - 5)°$

155°G

155°

$a + 5 + 8a - 5 = 180°$

$9a = 180$

a. Given parallelogram GHJK, what is the value of a? $a = 20$

25°

b. What are m∠G, m∠H, m∠J, and m∠K?

Habits of Mind pg 144

Under what conditions can a pair of consecutive angles in a parallelogram be congruent? Explain.
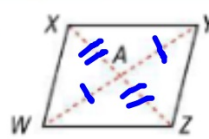
EXIT TICKET

HW pg 268 #4, 7, 8, 11, 12, 14, 16, 18, 19, 21, 26, 28 (#1-12 is page 146 in SC)
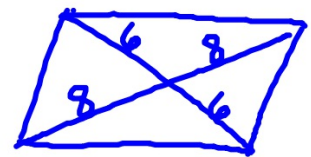
THEOREM 6-10

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

6.3

cuts each
other in
half.

If... $\overline{WX} \parallel \overline{ZY}$
$\overline{WZ} \parallel \overline{XY}$

Then... $\overline{AW} \cong \overline{AY}$
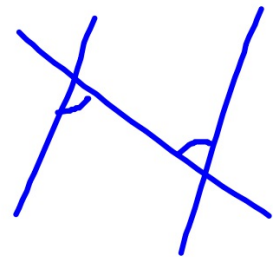$\overline{AX} \cong \overline{AZ}$

PROOF SEE EXAMPLE 5.
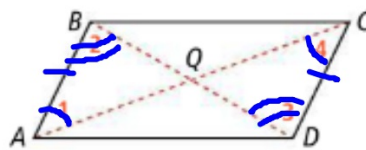
Example 5: $\overline{AC}$ and $\overline{BD}$ are the diagonals of parallelogram, ABCD.

Given: ABCD is a parallelogram.

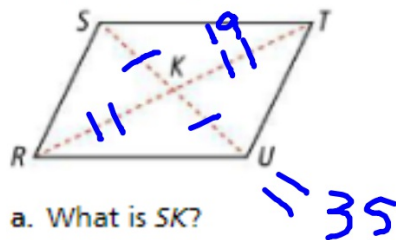**Prove:** $\overline{AQ} \cong \overline{CQ}, \overline{BQ} \cong \overline{DQ}$

Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. $\overline{AB} \cong \overline{DC}$	2. Thm 6-9
3. $\overline{AB}||\overline{DC}$	3. Def'n of parallelogram
4. $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$	4. Alternate interior $\angle$s.
5. $\triangle ABQ \cong \triangle CDQ$	5. ASA
6. $\overline{AQ} \cong \overline{CQ}$ and $\overline{BQ} \cong \overline{DQ}$	6. CPCTC

Do Try It 5, page 145 in your student companion.

☑ **Try It!** 5. Use parallelogram *RSTU* with *SU* = 35 and *KT* = 19.

S ——————— T
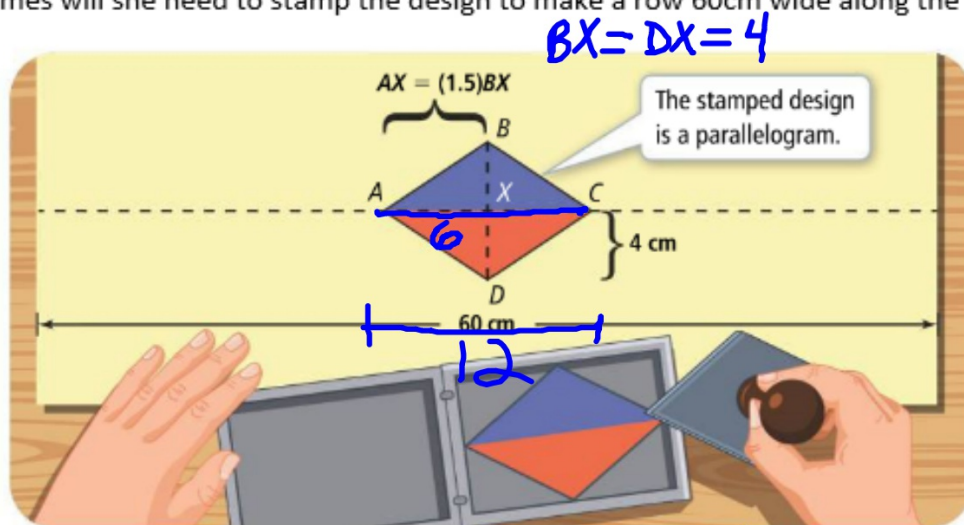K — 19
— 11
R —————— U
= 35

a. What is *SK*?

b. What is *RT*?

a) $\frac{35}{2} = 17.5$

b) $RT = 2 \cdot 19 = 38$

Example 6: Corey Stamps the pattern shown on the front of a poster she is making. How many times will she need to stamp the design to make a row 60cm wide along the dashed line?
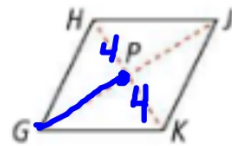
$BX = DX = 4$

$AX = (1.5)BX$

B

The stamped design is a parallelogram.

A   X   C

6

4 cm

D

60 cm

12

$AX = 1.5(4) = 6$

$AC = 6 + 6 = 12$

$\dfrac{60cm}{12cm} = 5$ stamps

Try It! 6. Given parallelogram $GHJK$, if $PK = 4$ and $HK = \frac{2}{3}(GJ)$, what is GP?

$8 = \frac{2}{3}(GJ)$ $y\frac{1}{2}(GJ)$
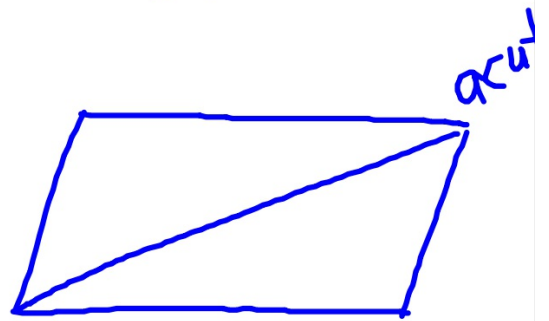
$\frac{3}{2} \cdot \frac{8}{1} = \frac{3}{2} \cdot \frac{2}{3}(GJ)$

$12 = GJ$

$GP = 6$

$HK = 4 + 4 = 8$

acute

acute

Habits of Mind

How can you tell which diagonal of a parallelogram has the greater length?   The one that connects the 2 smaller angles.
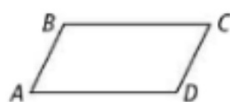
CONCEPT SUMMARY Properties of Parallelograms

Angles of Parallelograms	Sides and Diagonals of Parallelograms

WORDS

Consecutive angles of a parallelogram are supplementary. Opposite angles of a parallelogram are congruent.

Opposite sides of a parallelogram are congruent. Diagonals of a parallelogram bisect each other.

SYMBOLS

If...

$\overline{AD} \parallel \overline{CB}$
$\overline{AB} \parallel \overline{DC}$

Then... $m\angle A + m\angle B = 180°$
$m\angle B + m\angle C = 180°$
$m\angle C + m\angle D = 180°$
$m\angle D + m\angle A = 180°$
$m\angle A = m\angle C$
$m\angle B = m\angle D$

If...
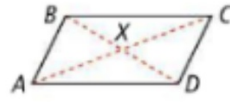
$\overline{AD} \parallel \overline{CB}$
$\overline{AB} \parallel \overline{DC}$

Then... $AB = CD$
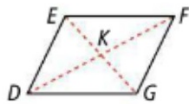$AD = BC$
$AX = CX$
$BX = DX$

Do You UNDERSTAND?

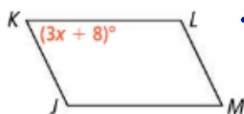1. **? ESSENTIAL QUESTION** What are the relationships of the sides, the angles, and the diagonals of a parallelogram?

2. **Error Analysis** What is Carla's error?

$\overline{PR} \cong \overline{QS}$
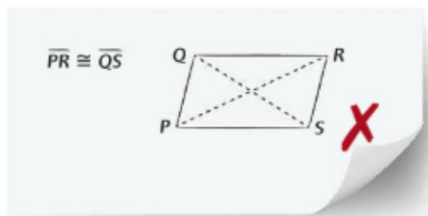
3. **Make Sense and Persevere** If you knew the length of $\overline{DF}$ in parallelogram *DEFG*, how would you find the length of $\overline{DK}$? Explain.

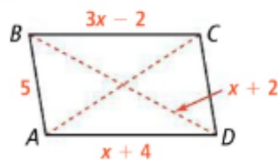4. **Reason** Given parallelogram *JKLM*, what could the expression $180 - (3x + 8)$ represent? Explain.

∠L or ∠J – consecutive ∠s are supplementary

Do You KNOW HOW?

For Exercises 5 and 6, use parallelogram **ABCD** to find each length.

5. *BC*

6. *BD*

Diagram of parallelogram ABCD with diagonals. Top side BC = $3x - 2$, left side A = 5, right side near D = $x + 2$, bottom side = $x + 4$.

For Exercises 7 and 8, use parallelogram **WXYZ** to find each angle measure.

7. m∠WXY **99°**

8. m∠XYZ **81°**

Diagram of parallelogram WXYZ with angle at Z = 99°.

(handwritten) 99°

(handwritten) 81°

(handwritten)
$$180$$
$$-\ 99$$
$$81°$$

For Exercises 9 and 10, use parallelogram **EFGH** to find each length.

9. *EJ*

10. *FH*

Diagram of parallelogram EFGH with diagonals intersecting at J. Segment marked 4 near F, segment marked 5 near G.
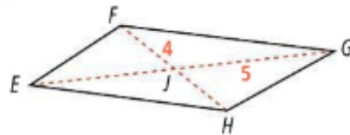
For Exercises 11 and 12, use parallelogram **MNPQ** to find each angle measure.

11. m∠NPQ **56° 124**

12. m∠PQM **124° 56**

Diagram of parallelogram MNPQ with angle N = $(9t - 2)°$ and angle M = $(3t + 14)°$.
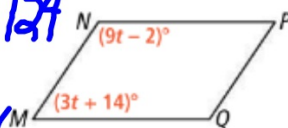
(handwritten)
$$9t - 2 + 3t + 14 = 180$$
$$12t + 12 = 180$$
$$t = 14$$

14. Error Analysis In the statements shown, explain the student's error. What shape is the quadrilateral?

$\overline{JK} \cong \overline{KL}$ and $\overline{LM} \cong \overline{MJ}$.
$\angle MJK \cong \angle KLM$
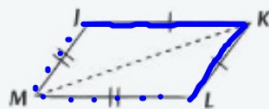
Therefore, $\triangle MJK \cong \triangle KLM$ by SAS. The triangular halves of JKLM are congruent, so JKLM must be a parallelogram. ✗
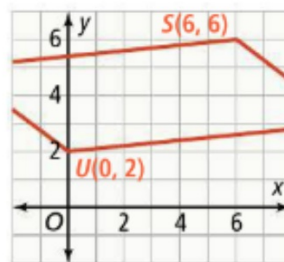
The opposite Sides need to be congruent. Kite

16. Use Appropriate Tools In a parallelogram, opposite sides are congruent, and opposite angles are congruent. If all sides in a parallelogram are congruent, are all angles congruent also? Draw a picture to explain your answer. No
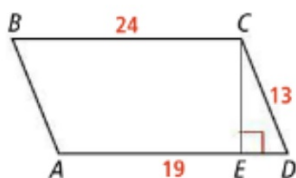
17. Mathematical Connections Only one pair of opposite vertices of parallelogram RSTU is shown on the coordinate plane. Is there enough information to find the point where the diagonals of RSTU intersect? Explain and find the point of intersection, if possible.
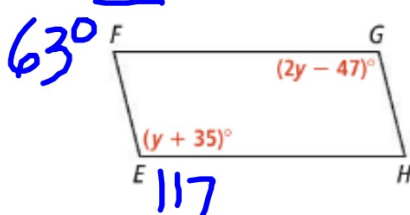
18. What are the values of *AB* and *DE* in parallelogram *ABCD*? SEE EXAMPLES 1 AND 2

B —— 24 —— C

13

A —— 19 —— E D

$AB = CD = \boxed{13}$

$DE = 24 - 19 = \boxed{5}$

19. Quadrilateral *EFGH* is a parallelogram. What is *m∠F*? SEE EXAMPLES 3 AND 4

63° F   G

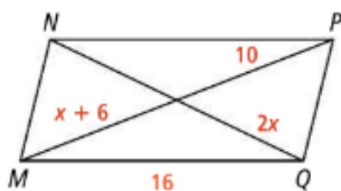$(2y - 47)°$

$(y + 35)°$

E 117   H

$2y - 47 = y + 35$

$y = 82$

$\angle F = 180 - \angle E$

20. Quadrilateral *MNPQ* is a parallelogram. What is *NQ*? SEE EXAMPLES 5 AND 6

N   P

10

x + 6   2x

M   16   Q

21. The figure below can be divided into two parallelograms. What is the angle measure of the point at the bottom?
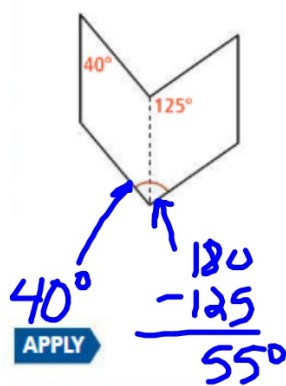
(handwritten) The entire angle is 40+55 or 95°

40°

125°

(handwritten) 40°

(handwritten) 180 −125 ──── 55°

APPLY

24. Model With Mathematics All four arms of a mechanical jack are the same length, and they form a parallelogram. Turning the crank pulls the arms together, raising the top of the jack. How high is the top of the jack when the crank is 5 inches off the ground? Explain.

5 in.

26. Higher Order Thinking Reagan designs a pattern consisting of large squares of the same size, small squares of the same size, and some parallelograms. She wants to replicate the pattern using tiles for her bathroom. Are the vertical and horizontal parallelograms congruent? Explain.

Yes b/c the sides of each are the sides of the squares.

27. Find the values of a, b, and c in the parallelogram.

a
5.2
b
10.4
10.4
c
12.8
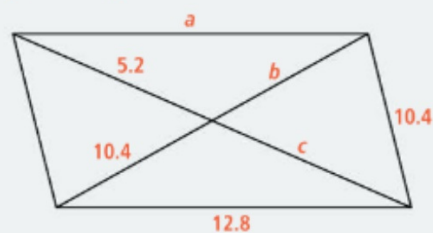
28. SAT/ACT In parallelogram *ABCD*, which angle is congruent to ∠*ABC*?

Ⓐ ∠*ABD*   Ⓒ ∠*BCD*

Ⓑ ∠*CDA*   Ⓓ ∠*DAB*