Geometry 6-2: Kites and Trapezoids Date: 1/27

Fill in the essential question for this section: How are diagonals and angle measures related in kites and trapezoids?

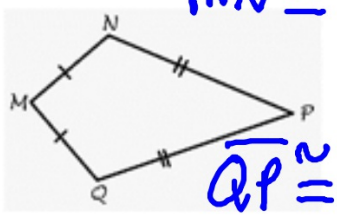
1) Fill in the Objective statement

**Objective:** I can use triangle congruence to understand kites and trapezoids.

DEFINTIONS

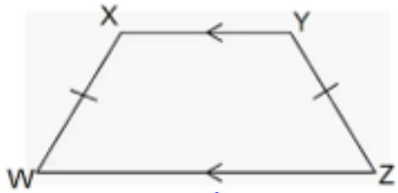
A **kite** has 2 pairs of consecutive congruent sides.

An **isosceles trapezoid** is a trapezoid with a congruent sides. $\overline{XW} \cong \overline{YZ}$




$\overline{MN} \cong \overline{MQ}$

$\overline{QP} \cong \overline{NP}$

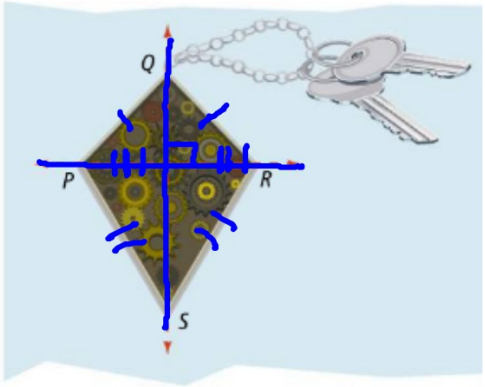



↑ parallel sides mark

2) Do "Critique and Explain", page 137 in your student companion.

## CRITIQUE & EXPLAIN

Manuel draws a diagram of kite ***PQRS*** with $\overleftrightarrow{QS}$ as the line of symmetry over a design of a kite-shaped key fob. He makes a list of conclusions based on the diagram.




- $\overline{PR} \perp \overline{QS}$
- $\overline{QP} \cong \overline{QR}$
- $\overline{SP} \cong \overline{SR}$
- $\overline{PR}$ bisects $\overline{QS}$.
- $\Delta PQR$ is an equilateral triangle.
- $\Delta PSR$ is an isosceles triangle.

A. Which of Manuel's conclusions do you agree with? Which do you disagree with? Explain.

B. Use Structure What other conclusions are supported by the diagram?

$\overline{QS}$ bisects $\overline{PR}$

## Habits of Mind

What theorems or definitions support your conclusions?

Isosceles $\triangle$s.

6.2

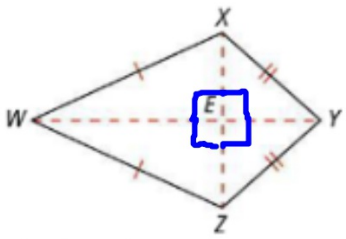
**THEOREM 6-3**

The diagonals of a kite are perpendicular.

meet at a 90° angle.

PROOF: SEE EXERCISE 12.

If...



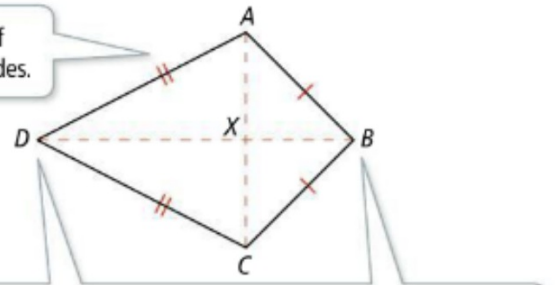

Then... $\overline{WY} \perp \overline{XZ}$

All the ∠s at E are 90°.

**EXAMPLE 1** **Investigate the Diagonals of a Kite**

How are the diagonals of a kite related?

A kite has two pairs of congruent adjacent sides.




Point $B$ is equidistant from the endpoints of $\overline{AC}$, as is $D$, so they lie on the perpendicular bisector of $\overline{AC}$.

The diagonals of a kite are perpendicular to each other. Exactly one diagonal bisects the other.

# Put in your s.c. next to TI 1:

**One of the diagonals gets** bisected, **or cut** in half.

**Try It!**

1. a. What is the measure of $\angle AXB$? $90^\circ$

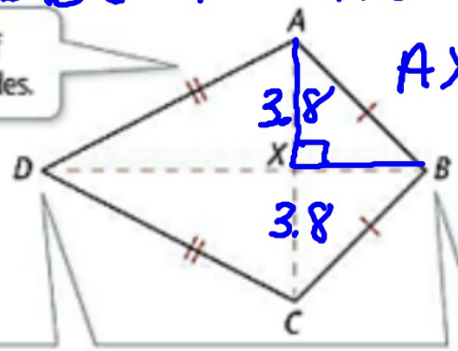
b. If $AX = 3.8$, what is $AC$? $2(3.8) = 7.6$

c. If $BD = 10$, ~~does $BX = 5$~~? Explain.

No, b/c AC was bisected, not BD.

**Draw this kite in your S.C.**

A kite has two pairs of congruent adjacent sides.

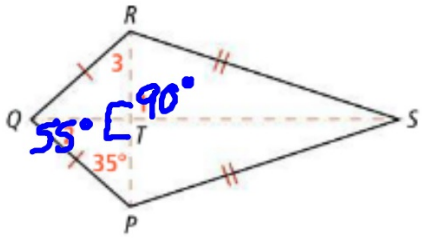



$AX = XC$

Point $B$ is equidistant from the endpoints of $\overline{AC}$, as is $D$, so they lie on the perpendicular bisector of $\overline{AC}$.

## EXAMPLE 2 Use the Diagonals of a Kite

Quadrilateral ***PQRS*** is a kite with diagonals $\overline{QS}$ and $\overline{PR}$.




**Draw this in you SC or take the examples sheet.**

**COMMON ERROR**
You may incorrectly assume angles are congruent just from their appearance. Always check that you can prove congruence first.

**A. What is $m\angle 1$?**

The diagonals of a kite are perpendicular, so $m\angle 1 = 90^\circ$.

**B. What is $m\angle 2$?**

The sum of the angles of $\triangle PQT$ is $180^\circ$.

$$m\angle 2 + 35^\circ + 90^\circ = 180^\circ$$

$$m\angle 2 = 55^\circ$$

**C. What is $m\angle 3$?**

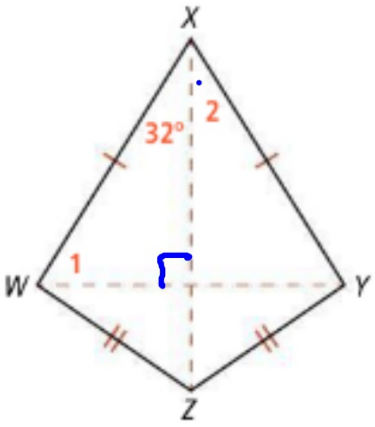
Since $\triangle PQR$ is an isosceles triangle, $\angle 3 \cong \angle QPT$.

So, $m\angle 3 = 35^\circ$.

**Try It!** 2. Quadrilateral *WXYZ* is a kite.

a. What is $m\angle 1$?

b. What is $m\angle 2$? 32°




180
−90
−32
58°

## Habits of Mind

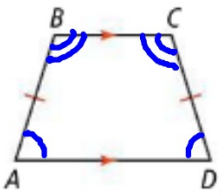
What ideas about triangles have you learned that could be useful in investigating kites?

## THEOREM 6-4

In an isosceles trapezoid, each pair of base angles is congruent.

PROOF: SEE EXERCISE 13.

If...




Then... $\angle BAD \cong \angle CDA$, $\angle ABC \cong \angle DCB$

$\angle A \cong \angle D$

$\angle B \cong \angle C$

$\angle A + \angle B = 180°$

$\angle C + \angle D = 180°$

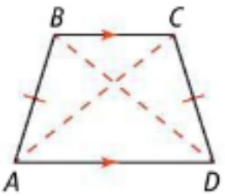
same-side interior $\angle$s

## THEOREM 6-5

The diagonals of an isosceles trapezoid are congruent.

PROOF: SEE EXERCISE 18.

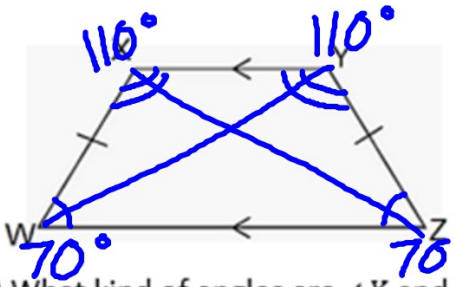
If...




Then... $\overline{AC} \cong \overline{DB}$

P139

**Example 3:** Explore properties of an isosceles trapezoid.




a) Because an isosceles trapezoid has 2 congruent sides, there must be congruent angles.

b) Which angles do you think are congruent in the figure?

These angles are called the base angles.

c) What kind of angles are $\angle X$ and $\angle W$? Is there another pair of similar type angles?

same side interior $\angle$s. add to 180° ∴ $\angle Y$ & $\angle Z$

d) Draw in the diagonals of the figure. Name them: $\overline{XZ}$ and $\overline{WY}$.

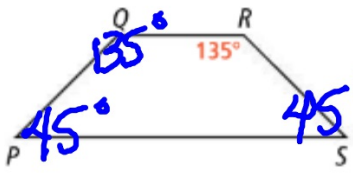
What do you think is true about these diagonals? Can we verify that?

they are congruent.

e) If $m\angle W = 70°$, find the measures of the remaining angles.

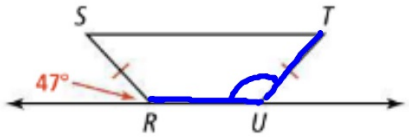
p139

**Try It!** 3. a. Given isosceles trapezoid $PQRS$, what are $m\angle P$, $m\angle Q$, and $m\angle S$?




180
-135

b. Given $\overline{ST} \parallel \overline{RU}$, what is the measure of $\angle TUR$?

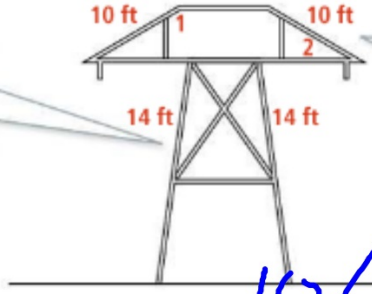



180
- 47
133°

$\angle TUR = 133°$

## EXAMPLE 4 Solve Problems Involving Isosceles Trapezoids

**All horizontal beams of the high-voltage transmission tower are parallel to the ground.**




The top section is an isosceles trapezoid.

The center section is an isosceles trapezoid.


**MAKE SENSE AND PERSEVERE**
What other strategy might you use to solve this problem?

**A. If $m\angle 1 = 138°$, what is $m\angle 2$?**

The sum of the interior angle measures of a quadrilateral is 360°.

$$m\angle 1 + m\angle 1 + m\angle 2 + m\angle 2 = 360°$$
$$138° + 138° + 2(m\angle 2) = 360°$$
$$276° + 2(m\angle 2) = 360°$$
$$2(m\angle 2) = 84°$$
$$m\angle 2 = 42°$$

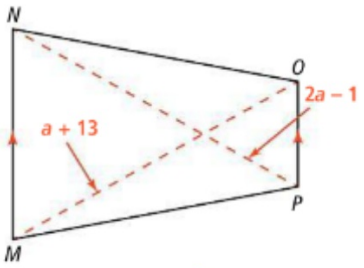
The base angles are congruent.

$$180 - 138 = 42$$

The measure of $\angle 2$ is 42°.

p 139

**Try It!** 4. Given isosceles trapezoid *MNOP* where the given expressions represent the measures of the diagonals, what is the value of *a*?




$NP = 2a - 1$

$MO = a + 13$

27

27

Diagonals are equal.

$2a - 1 = a + 13$

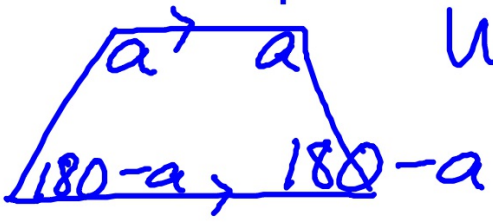
$-a \quad +1 \quad -a \quad +1$

$a = 14$

## Habits of Mind

p139

What is the minimum information you need to find the measures of all 4 interior angles in an isosceles trapezoid?



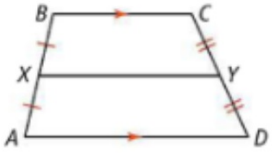

We need 1 angle.

**THEOREM 6-6** Trapezoid Midsegment Theorem

In a trapezoid, the midsegment is parallel to the bases, and the length of the midsegment is half the sum of the lengths of the bases.

PROOF: SEE LESSON 9-2.

If...



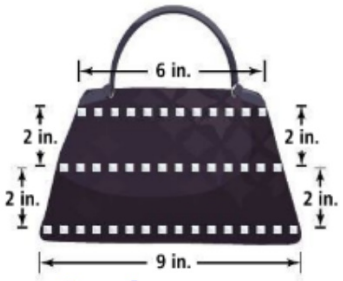

Then... $\overline{XY} \parallel \overline{AD}$, $\overline{XY} \parallel \overline{BC}$, and $XY = \frac{1}{2}(AD + BC)$

6.2

Midsegment a line that connects the midpoints of 2 segments.

## EXAMPLE 5 Apply the Trapezoid Midsegment Theorem

**Paxton makes trapezoidal handbags for her friends. She stiches decorative trim along the top, middle, and bottom on both sides of the handbags. How much trim does she need for three handbags? Explain.**




**Formulate** ◀ The top and bottom sides of the handbag are the bases of a trapezoid. The left and right sides are the legs. Since the middle segment divides both legs in half, it is the midsegment of the trapezoid. The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs.

Let $x$ represent the length of the midsegment in inches.

**Compute** ◀ **Step 1** Find the value of $x$.

$x = \frac{1}{2}(6 + 9)$ — Apply the Trapezoid Midsegment Theorem with the base lengths 6 and 9.

$x = 7.5$

The length of the midsegment is 7.5 in.

Now find the amount of trim that she needs.

First, find the amount for one side.

$6 + 9 + 7.5 = 22.5$

Then, multiply by 2 for the number of sides per handbag and by 3 for the number of handbags.

$22.5 \cdot 2 \cdot 3 = 135$

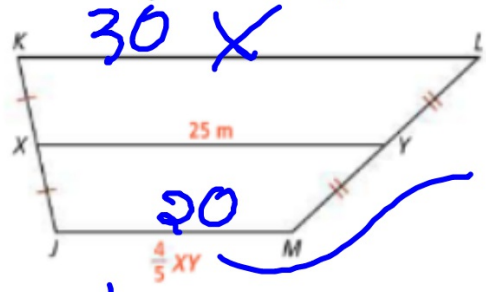
**Interpret** ◀ Paxton needs 135 inches of trim.


$x = \frac{1}{2}(6+9)$

$x = \frac{1}{2}(15) = 7.5$ in

45"
x 3
135"

**Try It!** 5. Given trapezoid *JKLM*, what is *KL*?




30 X

$\frac{4}{5}(25) = \frac{100}{5} = 20$

$XY = \frac{1}{2}(KL + JM)$

$25 = \frac{1}{2}(X + 20)$

$25 = \frac{1}{2}x + 10$

$-10 \quad -10$

$2(15) = (\frac{1}{2}x)2$

$30 = x$

## Habits of Mind

What information is represented in the diagram but not given in words?

XY is a midsegment.

Concept Summary Assess

## CONCEPT SUMMARY Kites and Trapezoids

**WORDS**

### Kites

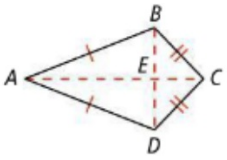
A kite is a quadrilateral with two pairs of adjacent sides congruent and no pairs of opposite sides congruent. Exactly one diagonal is a perpendicular bisector of the other.

### Trapezoids

A trapezoid is a quadrilateral with exactly one pair of parallel sides. The length of the midsegment is the average of the lengths of the two bases. A trapezoid with congruent legs is an isosceles trapezoid that has congruent base angles and congruent diagonals.

**DIAGRAMS**

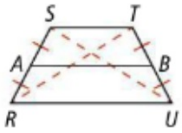
Quadrilateral $ABCD$ is a kite.




$\overline{AC} \perp \overline{BD}$

$BE = ED$

Quadrilateral $RSTU$ is an isosceles trapezoid.




$\overline{SU} \cong \overline{RT}$

$AB = \frac{1}{2}(ST + RU)$

$\overline{AB} \parallel \overline{ST} \parallel \overline{RU}$

$m\angle S = m\angle T$

$m\angle R = m\angle U$

**In your Book**

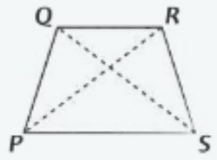
Read Concept Summary and #1-11 page 259 (page 140 in your student companion).

Tomorrow's assignment is page 260 #14, 16, 17, 19, 20, 22 (1760 yd = 1 mile), 24, 25, 26A

## Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are diagonals and angle measures related in kites and trapezoids?

2. Error Analysis What is Reagan's error?




By Theorem 6-5, $\overline{PR} \cong \overline{QS}$ ✗

3. Vocabulary If $\overline{XY}$ is the midsegment of a trapezoid, what must be true about point $X$ and point $Y$?

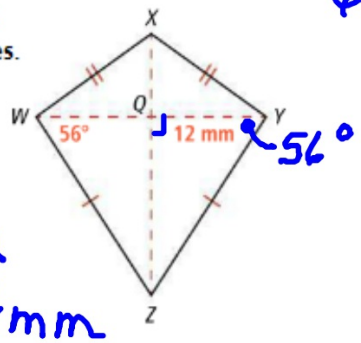
4. Construct Arguments Emaan says every kite is composed of 4 right triangles. Is he correct? Explain.

yes b/c diagonals are perpendicular.

## Do You KNOW HOW?

For Exercises 5–7, use kite ***WXYZ*** to find the measures.

5. $m\angle XQY$ 90°
6. $m\angle YZQ$ 34°
7. $WY$



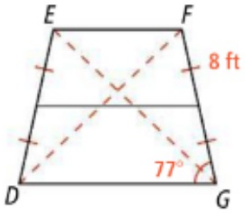

$WY = 2 \cdot 12$

$WY = 24\text{mm}$

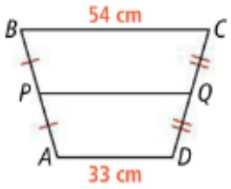
6) $m\angle YZQ = 180 - 90 - 56$

For Exercises 8–10, use trapezoid ***DEFG*** with ***EG*** = 21 ft and $m\angle DGF = 77°$ to find each measure.

8. $ED$
9. $DF$
10. $m\angle DEF$

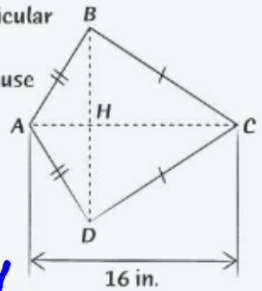



11. What is the length of $\overline{PQ}$?



**14.** Error Analysis What is Emery's error?

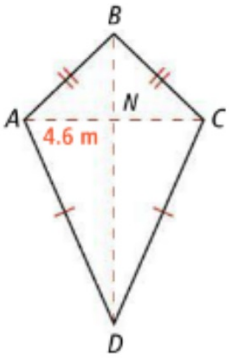
$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$, so $HC = 8$ in. because $AC = 16$ in.




False, $\overline{BD}$ is bisected, not $\overline{AC}$.

**16.** Given kite $ABCD$, in which $AN = 4.6$ m, what is $AC$?

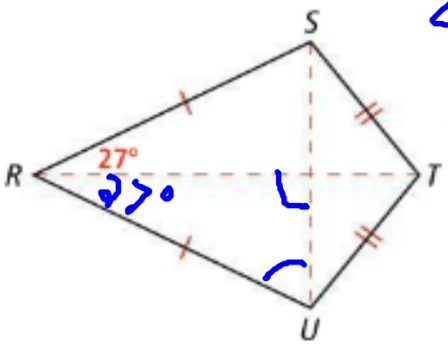
SEE EXAMPLE 1




$AC = 2(4.6)$

$AC = 9.2\text{m}$

**17.** Given kite $RSTU$, what is $m\angle RUS$? SEE EXAMPLE 2




$\angle RUS = 180 - 90 - 27$

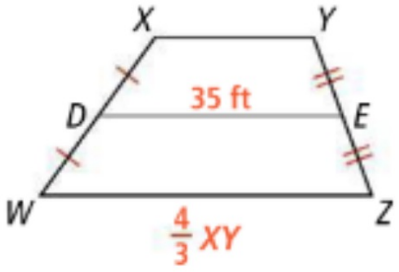
$\angle RUS = 63°$

**19.** Given trapezoid $MNPQ$, what is $m\angle MNP$?

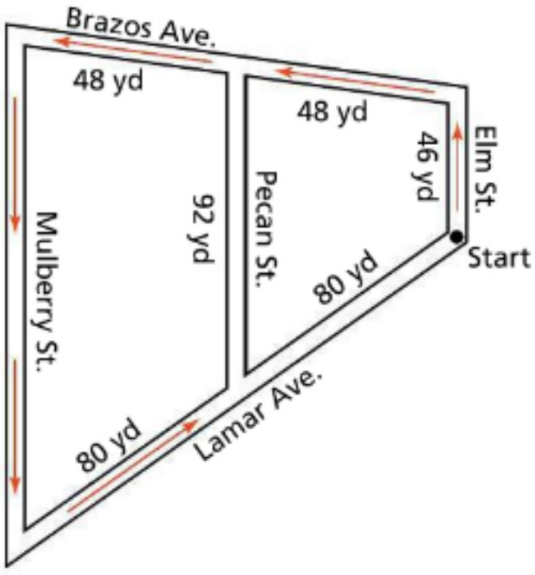
SEE EXAMPLE 4


**20.** Given trapezoid $WXYZ$, what is $XY$?

SEE EXAMPLE 5



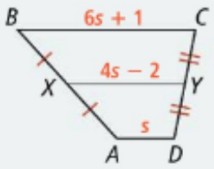
**22.** Reason Coach Murphy uses the map to plan a 2-mile run for the track team. How many times will the team run the route shown?




1760 yd = 1 mile

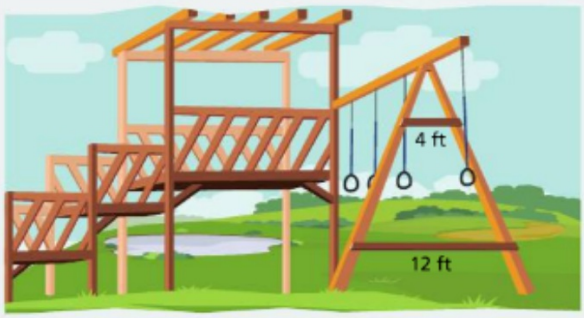
**24.** The ____?____ of a kite are always ____?____

**25.** SAT/ACT Given trapezoid $ABCD$, what is the length of $\overline{XY}$?




Ⓐ $3\frac{3}{5}$ Ⓑ $4\frac{2}{3}$ Ⓒ 5 Ⓓ 11 Ⓔ 18

**26.** Performance Task Cindy is a member of a volunteer group that built the play structure shown.




**Part A** Cindy wants to add three more trapezoid boards evenly spaced between the bottom and top boards of the triangular frame. Based on the average lengths of the top and bottom boards shown, what will be the average lengths of each of the three additional boards? Explain.