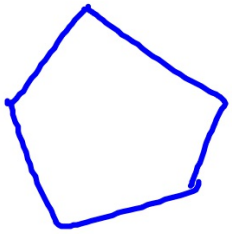
Geometry 6-1: The Polygon Angle-Sum Theorems Date: 1/23

Fill in the essential question for this section: How does the number of sides in a convex polygon, n, relate to the sum of the measures of its interior and exterior angles?

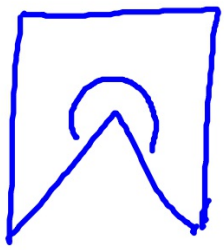
What is a convex polygon?

all interior angles are less than 180°.



What is a concave polygon?

1 or more angles is more than 180°.



Objective: I can find the sums of the measures of the interior angles and exterior angles of polygons.

Do "Explore and Reason" and "Habits of Mind" in your student companion, page 131.

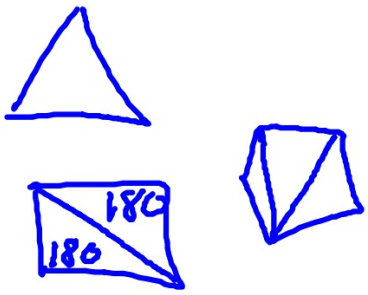
EXAMPLE 1 Explore Polygon Interior Angle Sums

How does the number of sides of a convex polygon, n, relate to the sum of measures of its interior angles?

0

# of sides	# of triangles
3	1
4	2
5	3
6	4
n	$n-2$

180
360
540
720



$$\text{angles} = 180(n-2)$$

Try It! 1. a. How many triangles are formed by drawing diagonals from a vertex in a convex octagon?

b. What is the interior angle sum for a convex octagon?

$n = 8$

6 triangles

$$180(8-2) = 180(6) = 1080^\circ$$

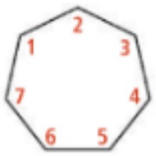
6.1

THEOREM 6-1 Polygon Interior Angle-Sum Theorem

The sum of the measures of the interior angles of a convex n-gon is $180° \cdot (n - 2)$.

PROOF: SEE EXERCISE 11.

If...



Then... $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 + m\angle 7 = 180° \cdot (7 - 2) = 900°$

n = # of sides

all angles

COROLLARY to Theorem 6-1

The measure of an interior angle of a regular n-gon is $\frac{180° \cdot (n - 2)}{n}$.

1 angle

6.1

equal sides & equal angles.

If...

$n = 5$

Then... $m\angle 1 = \frac{180° \cdot (5 - 2)}{5} = 108°$

$\frac{540}{5}$

EXAMPLE 2 Apply the Polygon Interior Angle-Sum Theorem

Jenna is building a corner cabinet to fit in a rectangular room. If she builds it with the angles shown, how can she determine whether the cabinet will fit?

90? 90 90 135 135

5 sides → 540°

540 − 90 − 90 − 135 − 135 = 90°

all angles: $180(n-2)$

one angle = $\frac{180(n-2)}{n}$

Try It! 2. a. What is the interior angle sum of a 17-gon?

b. Each angle of a regular n-gon measures 172.8°. How many sides does the n-gon have?

(a) $180(17-2) = 2700°$

(b) $172.8 = \frac{180(n-2)}{n} \cdot n$

$172.8n = 180n - 360$

$-180n \quad -180n$

$-7.2n = -360$

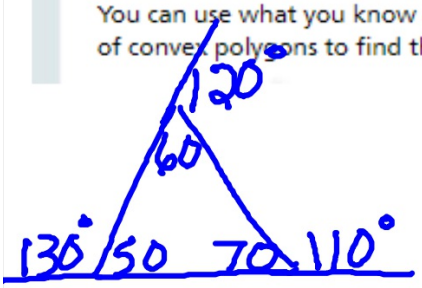
$n = 50$

Habits of Mind, page 132

EXAMPLE 3 Understand Exterior Angle Measures of a Polygon

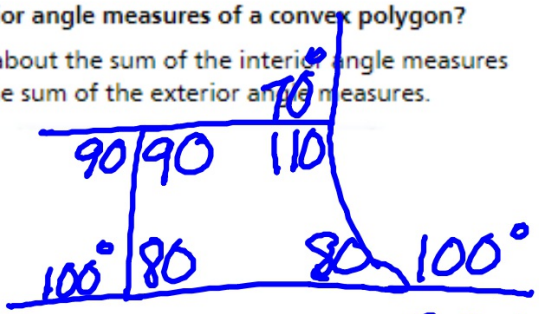
What is the sum of the exterior angle measures of a convex polygon?

You can use what you know about the sum of the interior angle measures of convex polygons to find the sum of the exterior angle measures.



$130 + 120 + 110$

$= 360°$



$100 + 100 + 90 + 70$

$200 + 160$

$= 360°$

Try It! 3. What is the sum of exterior angle measures of a convex 17-gon?

$360° - n$ does not matter.

6.1

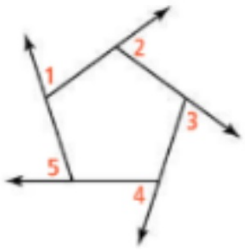
THEOREM 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a convex polygon, one at each vertex, is 360°.

# of sides (n) does not matter.

PROOF: SEE EXERCISE 15.

If...

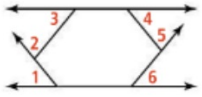


Then... $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

EXAMPLE 4 Find an Exterior Angle Measure

Suppose $\angle 1 \cong \angle 3$, $m\angle 1 = 3x°$, and $m\angle 2 = 2x°$. What is the measure of each exterior angle?

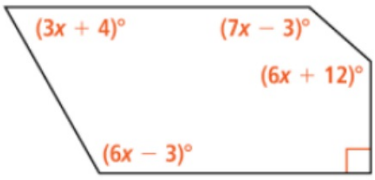
Try It! 4. Suppose $\angle 1 \cong \angle 3 \cong \angle 4 \cong \angle 6$, $\angle 2 \cong \angle 5$, and $m\angle 3 = m\angle 2 + 30°$. What is $m\angle 4$?



Do Try It 4 and "Habits of Mind", page 133 in your student companion.

Example 5

What are the measures of the interior angles of the pentagon shown?



Try It! 5. The measure of each interior angle of a regular 100-gon is $(3x + 26.4)°$. What is the value of x?

Do Try It 5 and "Habits of Mind", page 133 in your student companion.

Write these on your paper notes.

Check for Understanding

1) What is the sum of the angle measures of a linear pair?

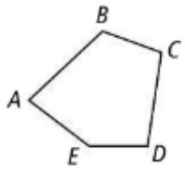
2) What is the sum of an interior angle and an exterior angle at any vertex of a polygon?

3) Does the number of sides matter when finding the sum of the exterior angles?

CONCEPT SUMMARY Polygon Angle Sums

THEOREM 6-1 Polygon Interior Angle-Sum Theorem

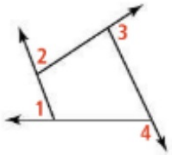
The sum of the measures of the interior angles of a convex polygon is $180° \cdot (n - 2)$, where n is the number of sides of the polygon.



$$m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = 180°(5 - 2)$$
$$= 540°$$

THEOREM 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a convex polygon, one at each vertex, is 360°.



$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$$

In your Book

1) page 249: Concept Summary

2) page 249 #1-10 (page 134 in your student companion)

Tomorrow's assignment is page 249 #13, 14, 16-25, 28, 29

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1) page 249: Concept Summary

2) page 249 #1-10 (page 134 in your student companion)

Tomorrow's assignment is page 249 #13, 14, 16-25, 28, 29

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does the number of sides in convex polygons relate to the sums of the measures of the exterior and interior angles?

2. Error Analysis In the calculation shown, what is Danielle's error?

$n = 25$
Sum of exterior angles is
$180° \cdot (25 - 2) = 4{,}140°$. ✗

3. Make Sense and Persevere What is the minimum amount of information needed to find the sum of the interior angles of a regular polygon?

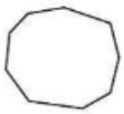
4. Reason A convex polygon can be decomposed into 47 triangles. How many sides does the polygon have? Explain.

Do You KNOW HOW?

Use polygon A for Exercises 5 and 6.

5. What is the sum of the measures of the interior angles?

6. What is the sum of the measures of the exterior angles?

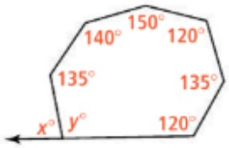


Polygon A

Use polygon B for Exercises 7 and 8.

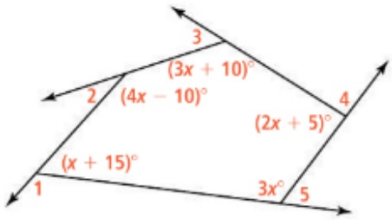
7. What is the value of y?

8. What is the value of x?



Polygon B

9. What are the measures of the exterior angles of the polygon shown?



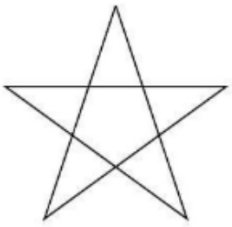
10. The sum of the interior angles of a regular n-gon is 6,120°. What is the measure of each interior angle?

13. **Reason** Explain why a regular polygon cannot have an interior angle that is 40°.

14. **Error Analysis** Jayesh makes the calculation shown to find the measure of each interior angle of a regular nonagon. What is his error?

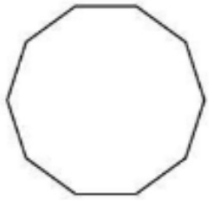
$n = 9$
$n - 2 = 7$
Interior angles:
$180 \cdot 9 = 1620°$
$1620 \div 7 = 231°$ ✗

16. **Higher Order Thinking** The star shown is constructed by extending each side of a regular pentagon. Explain why the surrounding triangles are isosceles and congruent.

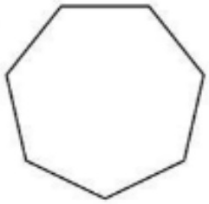


For Exercises 17 and 18, find the sum of the interior angles and the measure of each angle for the given regular polygon. SEE EXAMPLES 1 AND 2

17.



18.

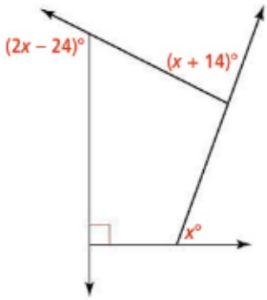


19. How many sides does a regular polygon have if the measure of each interior angle is 160°?
SEE EXAMPLES 1 AND 2

20. What is the measure of each exterior angle of a regular polygon with 72 sides? SEE EXAMPLE 3

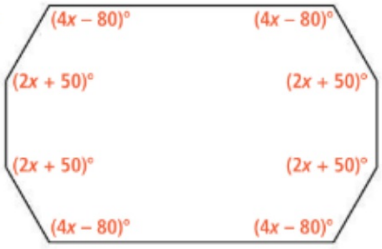
21. How many sides does a regular polygon with an exterior angle measure of 60° have?
SEE EXAMPLE 3

22. What is the value of x? What is the measure of each exterior angle? SEE EXAMPLE 4



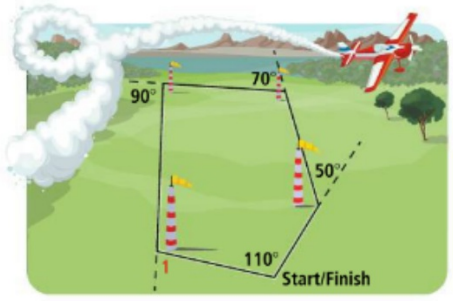
For Exercises 23 and 24, find the value of x and the measure of each interior angle. SEE EXAMPLE 5

23.



24.

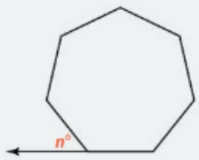
25. **Model With Mathematics** An airplane is navigating a polygon-shaped course. Each turn is labeled with the measure of the external angle at the striped post. What is $m\angle 1$?



28. Match the number of sides of a regular polygon with the measure of each interior angle.

I. 4	A. 120°
II. 6	B. 157.5°
III. 16	C. 160°
VI. 18	D. 90°

29. **SAT/ACT** Suppose the figure below is a regular polygon. What is the value of n? Round to the nearest whole number.



Ⓐ 45 Ⓑ 51 Ⓒ 129 Ⓓ 135