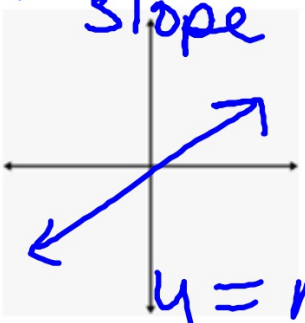
College Concepts 3.4: The Slope of a Line

**Objective 1: I can use two ordered pairs to find the slope of a line.**

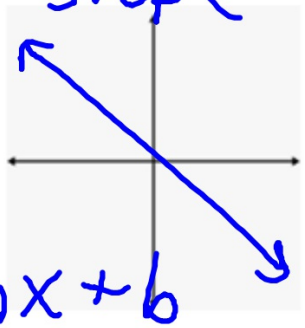
Recall from earlier math classes that the slope of a line is a measure of its steepness. A line with positive slope goes up & right. A line with negative slope goes down & right. A line with a slope of 0 is horizontal. A line with a slope that is undefined is vertical.

Let's draw some graphs of each line.

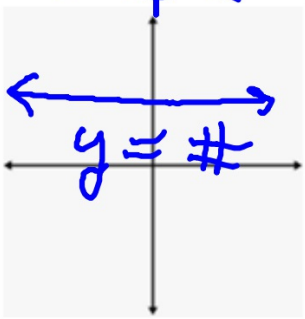
positive slope



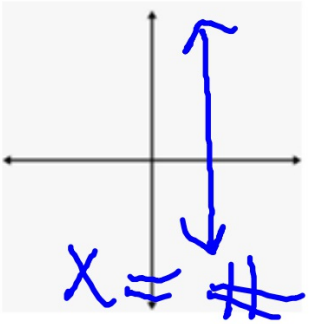
Negative slope



Zero Slope




Undefined slope




y = mx + b

If you know 2 points on the line, you can find its slope by using the following formula.

**Slope Formula**

If $(x_1, y_1)$ and $(x_2, y_2)$ are two points on a line, then the slope of the line is

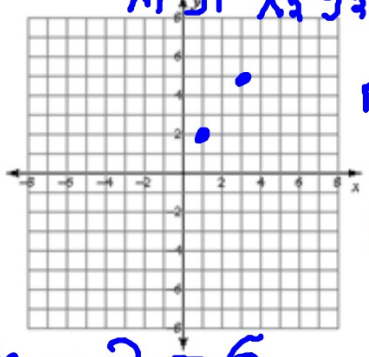
$$m = \frac{y_1 - y_2}{x_1 - x_2} \quad \frac{\text{rise}}{\text{run}}$$

**Hint:** Remember, it does not matter which point goes first, but you MUST stay in order when you subtract the two coordinates.

$$\neq \quad a - (-b)$$
$$a + b$$

Ex 1: Find the slope of the line between the points (1, 2) and (3, 5). Check by graphing.

$x_1\ y_1\ x_2\ y_2$



$m = \frac{rise}{run}$

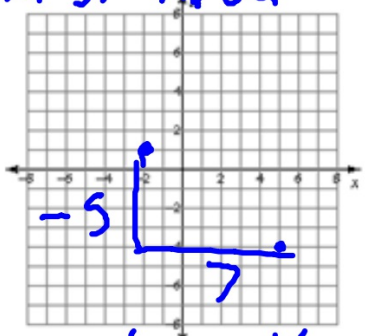
$m = \frac{3}{2}$

$m = \frac{2-5}{1-3} = \frac{-3}{-2}$

$m = \frac{3}{2}$

Ex 2: Find the slope of the line through (-2, 1) and (5, -4). Check by graphing.

$x_1\ y_1\ x_2\ y_2$

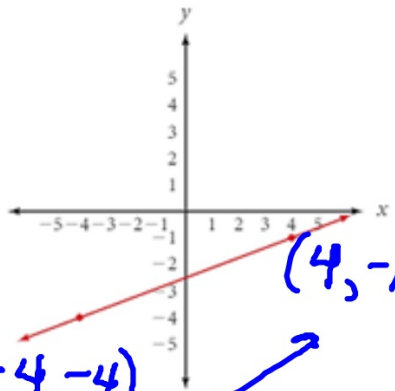



$m = \frac{1--4}{-2-5} = \frac{5}{-7}$

$m = \frac{-4-1}{5--2} = \frac{-5}{7}$

$-\frac{5}{7}$

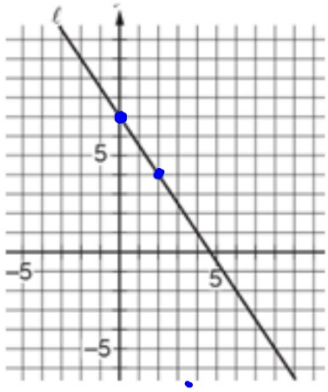
Ex 3: Find the slope of the line below.




$m = \frac{-4 - -1}{-4 - 4}$

$m = \frac{-3}{-8} = \frac{3}{8}$

Ex 4: Find the slope of the line below.




$\frac{3}{-2}$

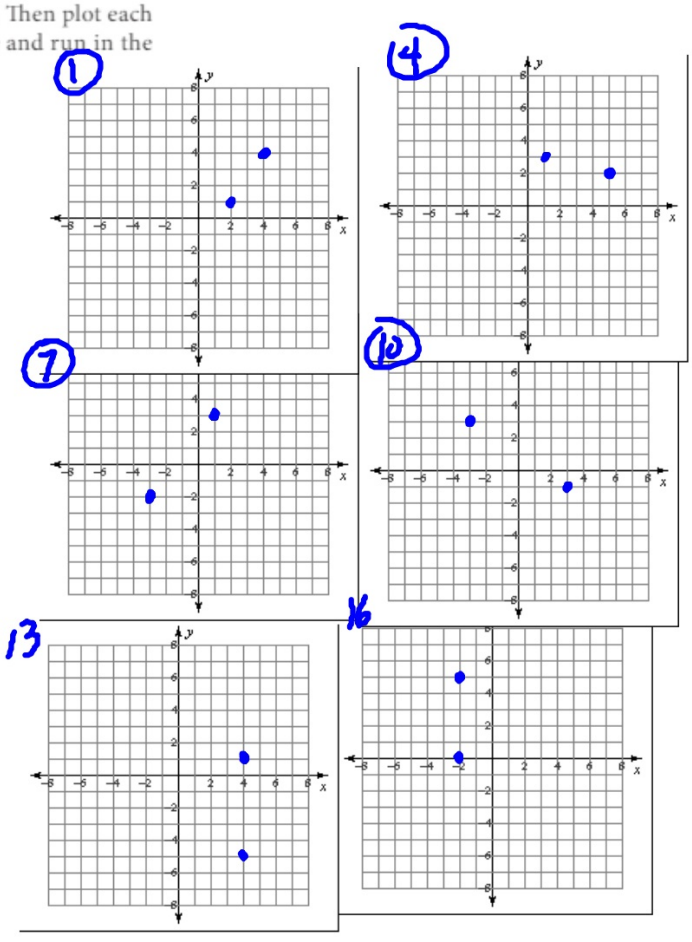
$m = \frac{\text{rise}}{\text{run}} \quad -\frac{3}{2}$

**HW page 227 #1, 4, 7, 10, 13, 16, 19-22**

Find the slope of the line through the following pairs of points. Then plot each pair of points, draw a line through them, and indicate the rise and run in the graph in the same manner shown in Examples 1 and 2.

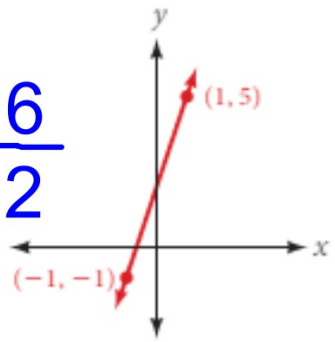
1. $(2, 1), (4, 4)$ $m = \frac{3}{2}$
4. $(1, 3), (5, 2)$ $m = \frac{-1}{4}$
7. $(-3, -2), (1, 3)$ $m = \frac{5}{4}$
10. $(-3, 3), (3, -1)$ $m = \frac{-4}{6}$
13. $(4, -5), (4, 1)$ $m = \frac{6}{0} = \text{und}$
16. $(-2, 0), (-2, 5)$

$m = \frac{5}{0} = \text{und}$



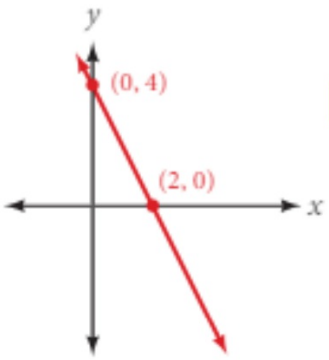
Find the slope for each line.

19.



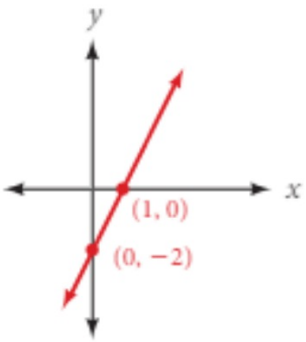

$m = \frac{6}{2}$

20.



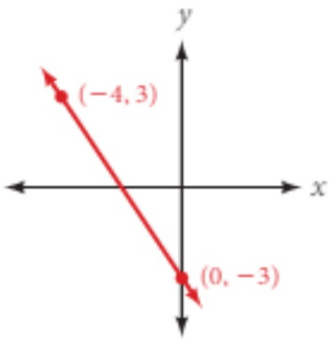

$m = \frac{-4}{2}$

21.




$m = \frac{2}{1}$

22.

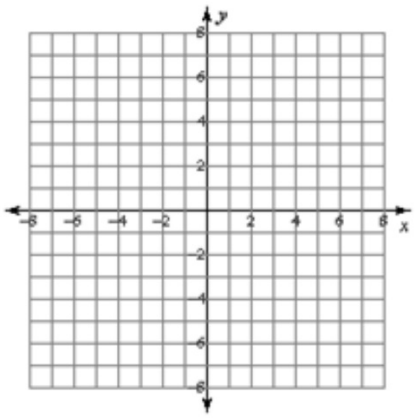



$m = \frac{-6}{4}$

**Objective 2: I can graph a line if given a point on the line and the slope.**

If we are given some information about a line, besides the equation, we can often use that information tc graph the line.

Ex 5: Graph the line with slope $-\frac{3}{2}$ passing through the point (1, 1).

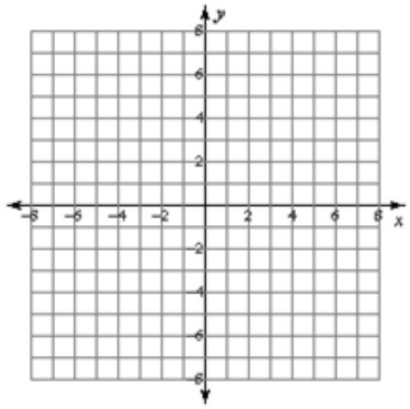



Step 1: Plot the given point

Step 2: Use m = rise to get more points
run

Step 3: Connect the points.

Ex 6: Graph the line with slope 4 that passes through the point (-3, -2).




1) Plot (-3, -2)

2) Use m = 4 to move

up 4 and right 1

$$\frac{4}{1}$$

Earlier in this chapter, we learned to graph vertical and horizontal lines. Let's look at their slope.

Ex 7: Find the slope of the line passing through the points (3, -1) and (3, 4).

Undefined slope
This is a vertical line.
Notice the x's are the same #!

Ex 8: Find the slope of the line passing through the points (-4, -1) and (3, -1).

This is a slope of zero.
This is a horizontal line.
Notice the y's are the same #!

## SPECIAL SLOPES

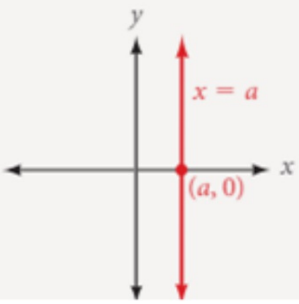
**Vertical Line**

Equation: x = #

Slope: undefined

x-intercept: (#, 0)

y-intercept: none



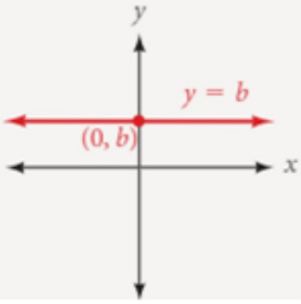

**Horizontal Line**

Equation: y = #

Slope: 0 (zero)

x-intercept: none

y-intercept: (0, #)



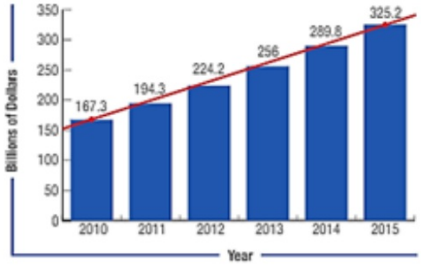

** y = mx will cross the x and y axes!!!

**Objective 3: I can interpret slope as a rate of change**

If two quantities have a relationship that can be described by a linear equation, then we can interpret the _____ of the line as the _____ between these 2 quantities.

**RATE OF CHANGE IS JUST A FANCY PHRASE FOR _____!**

Ex 9: The graph below shows the rise in US retail internet commerce over a 5-year period, and is approximately linear. Use the graph to find the slope, and then interpret the slope as a rate of change.



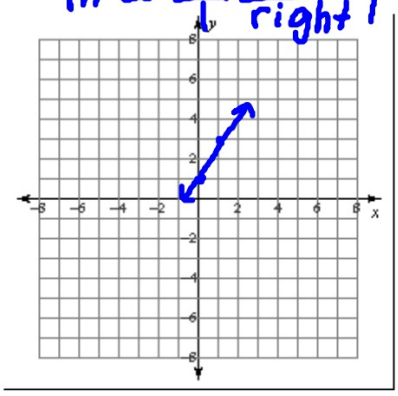

*This means we spend $_____ billion more EACH year!

**HW page 227 #23, 27, 31, 35, 37, 39, 40, 43-46, 63, 65**

In each of the following problems, graph the line having the given slope and passing through the given point.

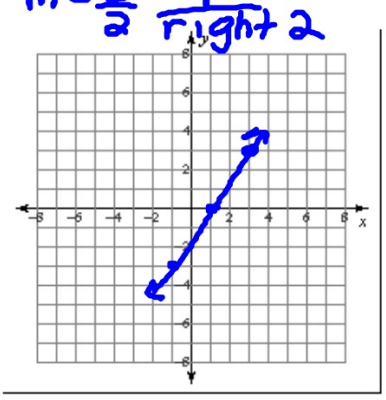
**23.** $m = 2, (0, 1)$

$m = \frac{2}{1}$ up 2 / right 1



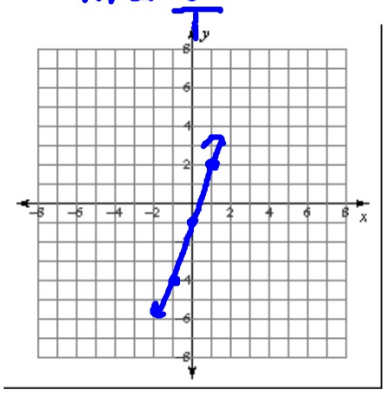
**27.** $m = \frac{3}{2}, (-1, -3)$

$m = \frac{3}{2}$ up 3 / right 2



**31.** $m = 3, (-1, -4)$

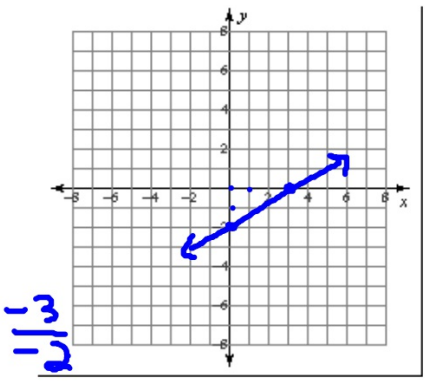
$m = \frac{3}{1}$



1) Plot the given point.

2) Use $m = \frac{\text{rise}}{\text{run}}$ for more points.

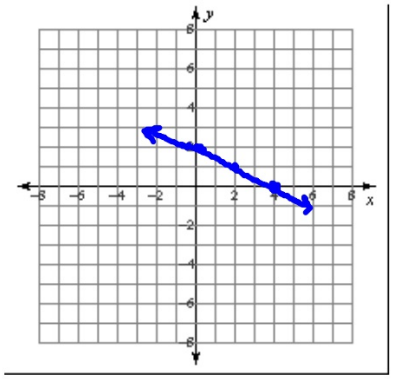
**35.** Graph the line that has an $x$-intercept of 3 and a $y$-intercept of $-2$. What is the slope of this line?




Plot the two intercepts and use rise/run to find the slope.

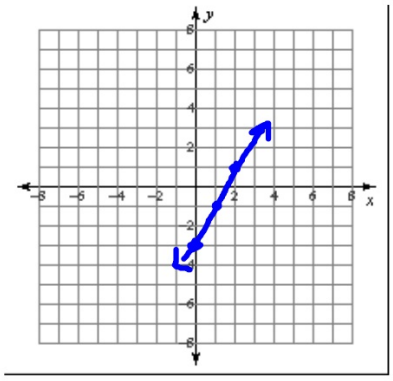
$$m = \frac{\text{up } 2}{\text{right } 3} = \frac{2}{3}$$

**37.** Graph the line with $x$-intercept 4 and $y$-intercept 2. What is the slope of this line?




$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{down } 2}{\text{right } 4} = \frac{-2}{4} \text{ or } \frac{-1}{2}$$

**39.** Graph the line $y = 2x - 3$, then name the slope and $y$-intercept by looking at the graph.

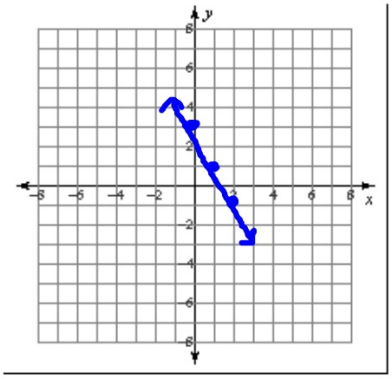



$b = -3$

$m = 2 = \frac{2}{1}$

Use the method from earlier:
1) Plot b on the y-axis
2) Use m to get more points

**40.** Graph the line $y = -2x + 3$, then name the slope and $y$-intercept by looking at the graph.




$b = 3$

$m = -2 = -\frac{2}{1}$

For each equation in each table, give the slope of the graph.

**43.**

| Equation | Slope |
|---|---|
| $x = 3$ | und |
| $y = 3$ | 0 |
| $y = 3x$ | 3 |

$y = mx$

**44.**

| Equation | Slope |
|---|---|
| $y = \frac{3}{2}$ | 0 |
| $x = \frac{3}{2}$ | und |
| $y = \frac{3}{2}x$ | $\frac{3}{2}$ |

**45.**

| Equation | Slope |
|---|---|
| $y = -\frac{2}{3}$ | 0 |
| $x = -\frac{2}{3}$ | und |
| $y = -\frac{2}{3}x$ | $-\frac{2}{3}$ |

**46.**

| Equation | Slope |
|---|---|
| $x = -2$ | und |
| $y = -2$ | 0 |
| $y = -2x$ | $-2$ |

**63. Garbage Production** The table and completed line graph give the annual production of garbage in the United States for some specific years.

**a.** Find the slope of each of the four line segments, $A$, $B$, $C$, and $D$.

**b.** Interpret the slope of line segment $A$ as a rate of change. garbage increased 3.3 million tons each year from 1970–1980

| Year | Garbage (millions of tons) |
|---|---|
| 1960 | 88 |
| 1970 | 121 |
| 1980 | 152 |
| 1990 | 205 |
| 2000 | 224 |

A { 1960, 1970 } B

C { 1980, 1990 } D


A: $m = \frac{121-88}{1970-1960} = \frac{33}{10} = 3.3$

B: $m = \frac{152-121}{1980-1970} = \frac{31}{10} = 3.1$

C: $m = \frac{205-152}{1990-1980} = \frac{53}{10} = 5.3$

D: $m = \frac{224-205}{2000-1990} = \frac{19}{10} = 1.9$

**65. Non-Camera Phone Sales** The table and line graph here each show the projected non-camera phone sales each year from 2006 to 2010.

**a.** Find the slope of each of the three line segments, $A$, $B$, and $C$.

**b.** Interpret the slope of segment $B$ as a rate of change.

non-camera phone sales decreased by 75 million from 2007–2008

| Year | Sales (in millions) |
|---|---|
| 2006 | 300 |
| 2007 | 250 |
| 2008 | 175 |
| 2009 | 150 |
| 2010 | 125 |

A < , > B

C <


$A: m = \frac{250-300}{2007-2006} = -50$

$B: m = \frac{175-250}{2008-2007} = -75$

$C: m = \frac{125-150}{2010-2009} = -25$

**Objective 4: I can determine if two lines are parallel or perpendicular.**

Recall from geometry, that _____lines are lines that _____. For 2 lines NOT to meet, they must go in the same direction, or have the _____.

Lines that are _____ meet at a _____. Because they meet, they must have different slopes, but their slopes will be _____.

| | |
|---|---|
| Parallel lines have _____slopes. | Perpendicular lines have _____slopes. |

Ex 10: A line has a slope of 3.

a) What would the slope be for a parallel line?

b) What would the slope be for a perpendicular line?

Ex 11: Determine if the line passing through the given pairs of points are parallel, perpendicular, or neither.

(-4, -4) and (2, 5), (-2, 0) and (0, -3).

## HW pg 228 #55-62, 67-70

Determine if the lines passing through the given pairs of points are parallel, perpendicular or neither.

**55.** $(-3, 2)$ and $(6, 8)$, $(0, -1)$ and $(3, 1)$

**56.** $(-4, -4)$ and $(2, 5)$, $(-2, 0)$ and $(0, -3)$

**57.** $(-2, 6)$ and $(2, 4)$, $(-3, -5)$ and $(1, 3)$

**58.** $(0, -1)$ and $(4, 7)$, $(-4, -5)$ and $(-1, 1)$

**59.** $(-4, 0)$ and $(1, 5)$, $(0, -4)$ and $(2, -6)$

**60.** $(0, -2)$ and $(2, 6)$, $(-4, 1)$ and $(4, -1)$

**61.** Find $y$ if the line through $(4, 2)$ and $(6, y)$ has a slope of 2.

**62.** Find $y$ if the line through $(1, y)$ and $(7, 3)$ has a slope of 6.

## Learning Objectives Assessment

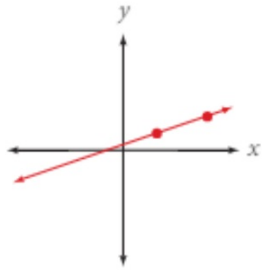
The following problems can be used to help assess if you have successfully met the learning objectives for this section.

**67.** Find the slope of the line passing through the points $(-3, 2)$ and $(1, -4)$.

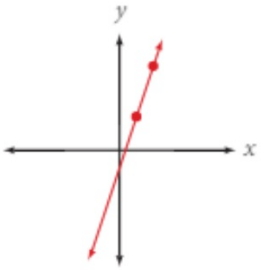
**a.** $-\frac{3}{2}$ **b.** $1$ **c.** $\frac{1}{2}$ **d.** $-\frac{2}{3}$

**68.** Sketch the graph of the line with slope 3 and passing through the point $(2, 1)$.

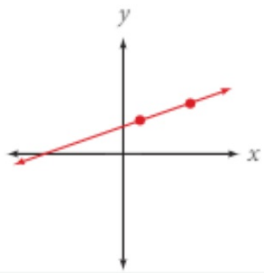
**a.**



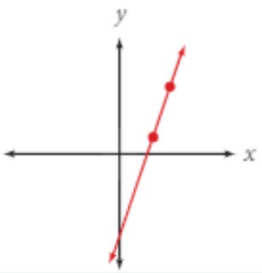

**b.**




**c.**




**d.**



**69.** The table below shows the federal minimum wage from two past years. Find the slope of the line passing through these two points and then interpret the slope as a rate of change.

| Year | Minimum Wage |
|---|---|
| 1979 | $2.90 |
| 2009 | $7.25 |

**a.** The federal minimum wage increased at a rate of $6.90 per year from 1979 to 2009.

**b.** The federal Minimum wage increased at a rate of $4.35 per year from 1979 to 2009.

**c.** The federal minimum wage increased at a rate of 0.145 years per dollar from 1979 to 2009.

**d.** The federal minimum wage increased at a rate of 14.5 cents per year from 1979 to 2009.

**70.** Find the slope of a line that would be perpendicular to the line passing through $(-1, -3)$ and $(2, 3)$.

**a.** $-\frac{1}{2}$ **b.** $2$ **c.** $-2$ **d.** $\frac{1}{2}$

## Getting Ready for the Next Section

Solve each equation for $y$.

**71.** $-2x + y = 4$ **72.** $-4x + y = -2$ **73.** $2x + y = 3$

**74.** $3x + 2y = 6$ **75.** $4x - 5y = 20$ **76.** $-2x - 5y = 10$