NC means No Calculator Portion

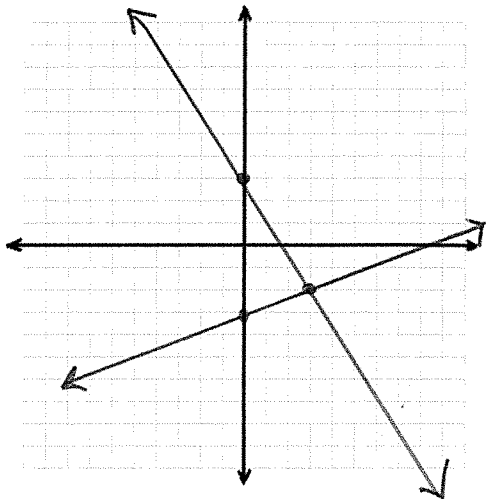
Name: KEY Hour: _____

Final Exam Study Guide

UNIT 1: SYSTEM OF EQUATIONS

1. **NC** Solve the following system of equations:

$$y = -\frac{5}{3}x + 3$$
$$y = \frac{1}{3}x - 3$$



   a. What method (elimination, substitution, or graphing) would be best to solve this system? Why?

   graphing, $y = mx + b$ form.

   b. Solve the system by graphing.

   to the right

   c. Check your solution.

   (3, -2) is soln.

2. **NC** Solve the following system of equations:

$$y = 6x - 11$$
$$-2x - 3y = -7$$

   a. Which method would be the best to use for this system? Why?

   substitution, y is by itself

   b. Solve the system by substitution.

$-2x - 3(6x - 11) = -7$

combine like terms → $-2x - 18x + 33 = -7$

$-20x + 33 = -7$

$-33 \quad -33$

$\frac{-20x}{-20} = \frac{-40}{-20}$

$\boxed{x = 2}$

$y = 6(2) - 11$

$= 12 - 11$

$\boxed{y = 1}$

3. **NC** Solve the following system of equations:

$$5x + y = 9$$
$$10x - 7y = -18$$

   a. Which method would be the best to use for this system? Why?

   elimination

   b. Solve by **elimination**.

$10x - 7y = -18 \rightarrow 10x - 7y = -18$

$-2(5x + y = 9) \rightarrow -10x + -2y = -18$

$-9y = -36$

$\boxed{y = +4}$

$5x + (+4) = 9$

$-4 \quad -4$

$5x = 5$

$\boxed{x = 1}$

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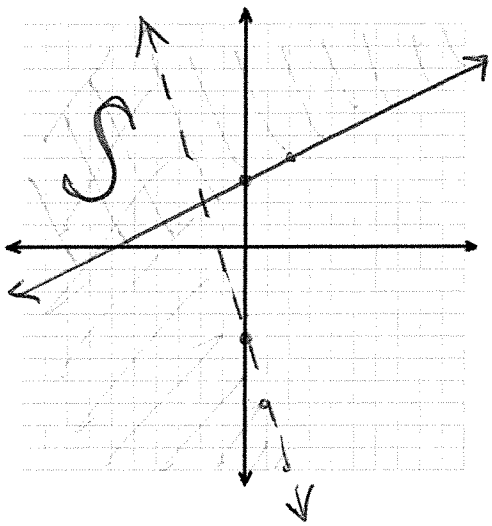
4. **NC** Graph the following system of inequalities. Then, circle the points below that are solutions to the system.

Shading
< below
> above

$y < -3x - 4$ (dashed)
$y \geq \frac{1}{2}x + 3$ (solid)



Circle the solutions to the system:		
(0, 0)	(-4, 3)	(-7, 1)
(5, 3)	(-2, -3)	(-3, -1)

5. **NC** Maria is hosting a party. She places an order at Pizza Hut for 5 pizzas and 4 breadsticks for a total of $51. Halfway through the party, she realizes that she will need one more pizza and 2 more order of breadsticks. This time she was given a total of $15. How much does a pizza cost? How much does a breadstick cost?

x = pizza
y = breadsticks

$5x+4y=51$
$x+2y=15$
$-2(x+2y=15)$

$5x+4y=51$
$-2x-4y=30$
$3x=81$
$x=27$

$x+2y=15$
$27+2y=15$
$2y=13$
$y=6.50$

6. **NC** Saleh has to buy apples and bananas. A bunch of bananas is $0.30 and an apple is $0.50. He spends a total of $7.00. He bought 3 times as many bananas as he did apples. How many apples did he buy? How many bananas did he buy?

x = banana
y = apple

$0.30x + 0.50y = \$7$
$x = 3y$
$0.30(3y) + 0.50y = 7$
$0.90y + 0.50y = 7$
$1.4y = 7$
$y = 5$
$x = 15$

7. Leila has $2 in her purse as coins. She has a total of 16 nickels and quarters. How many coins does she have of each type? Write a system of equations and solve algebraically.

8. For each system below, solve by **two methods**: algebraically and by graphing. Sketch a rough picture of the graph, labeling the solutions clearly.

c. $y = x^2 - 8x + 12$
$y = 4x - 8$

$4x - 8 = x^2 - 8x + 12$
$-4x \quad -4x$
$-8 = x^2 - 12x + 12$
$+8 \quad +8$
$0 = x^2 - 12x + 20$
$0 = (x-10)(x-2)$
$x = 10, 2$

20: x, x; -10, -2

$y = 4(10) - 8$
$y = 40 - 8$
$y = 32$
(10, 32)

$y = 4(2) - 8$
$y = 8 - 8$
$y = 0$
(2, 0)

d. $y = x^2 - 9x + 18$
$y = x - 3$

$x - 3 = x^2 - 9x + 18$
$-x \quad +3 \quad -x \quad +3$
$0 = x^2 - 10x + 21$
$0 = (x-7)(x-3)$
$x = 7, 3$

21: x, x; -7, -3; -10

$y = 7 - 3$
$y = 4$
(7, 4)

$y = 3 - 3$
$y = 0$
(3, 0)

* where the quadratic & linear eqv. intersect are the answers when graphing

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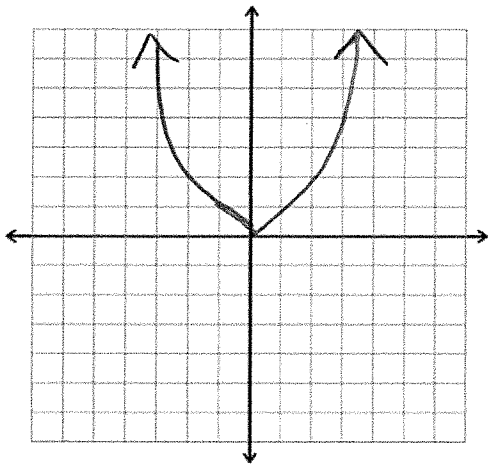
UNIT 2: FUNCTION FAMILIES

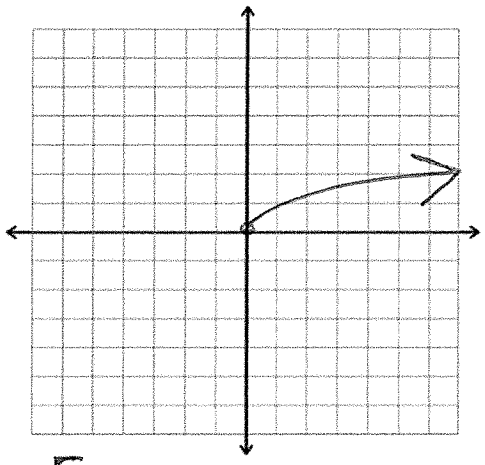
1. Given the family name, find the parent equation and graph it. Determine the domain, range, and end behavior.

Family: Quadratic
Equation: $y = x^2$



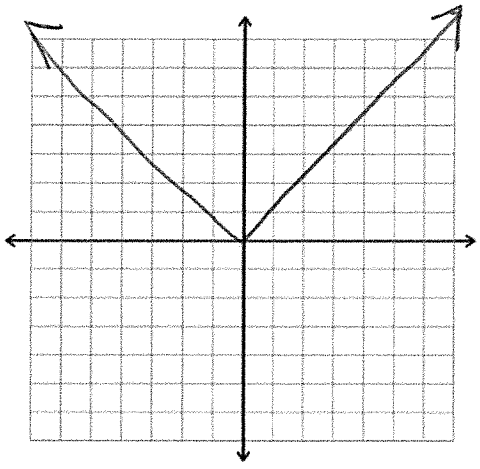
Domain: $(-\infty, +\infty)$
Range: $[0, +\infty)$
~~Positive: $(0, +\infty)$~~
~~Negative: $(-\infty, 0)$~~
Increasing: $(0, +\infty)$
Decreasing: $(-\infty, 0)$

Family: ~~Radical~~ square root
Equation: $f(x) = \sqrt{x}$



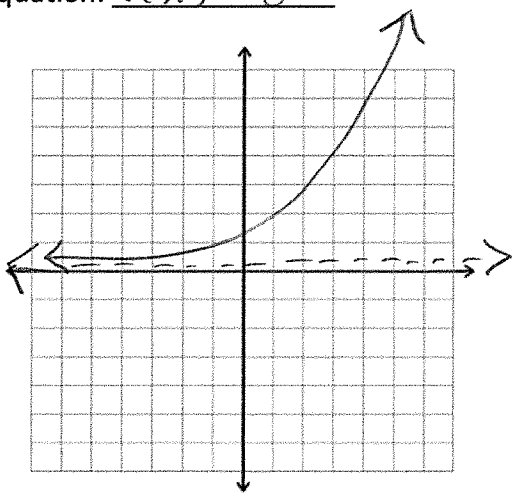
Domain: $[0, +\infty)$
Range: $[0, +\infty)$
~~Positive:~~
~~Negative:~~
Increasing: $(0, +\infty)$
Decreasing: N/A

Family: Absolute Value
Equation: $f(x) = |x|$



Domain: $(-\infty, +\infty)$
Range: $[0, +\infty)$
~~Positive:~~
~~Negative:~~
Increasing: $(0, +\infty)$
Decreasing: $(-\infty, 0)$

Family: Exponential Growth
Equation: $f(x) = e^x$



Domain: $(-\infty, +\infty)$
Range: $(0, +\infty)$
~~Positive:~~
~~Negative:~~
Increasing: $(-\infty, +\infty)$
Decreasing: N/A

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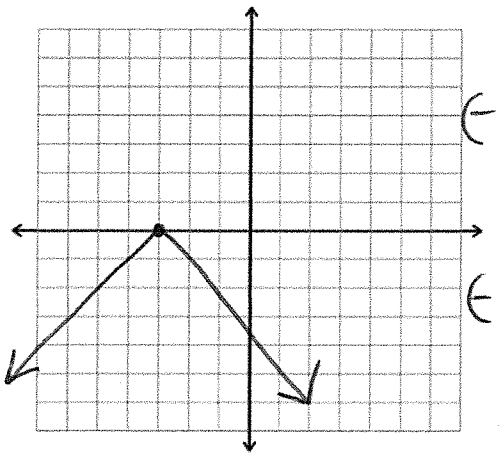
2. For the function $f(x) = a(x - h) + k$, complete the table for the transformation of each variable and describe them.

$a > 1$	$h \geq 0$	$k \leq 0$
vertical stretch (closer to y-axis)	· h is pos. in ( ) comes out neg: · trans. left	· k is neg · trans. down
$0 < a < 1$ compression (closer to x-axis)	$h \leq 0$ · h is neg in ( ) comes out (+) · trans right	$k \geq 0$ · k is pos. · trans UP.
$a < 0$ (a is negative) reflection		

3. Graph the following functions. Find their domain and range.

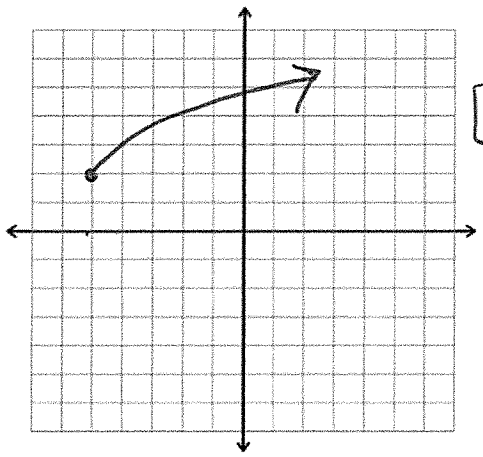
$f(x) = -|x + 3|$



Domain: $(-\infty, +\infty)$

Range: $(-\infty, 0]$

$f(x) = \sqrt{x + 5} + 2$



Domain: $[-5, +\infty)$

Range: $[2, +\infty)$

D: L to R
R: B to T

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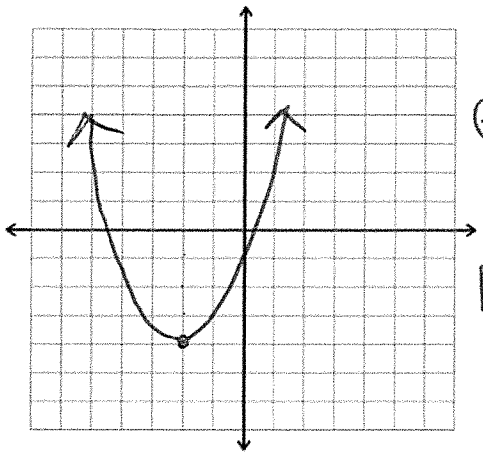
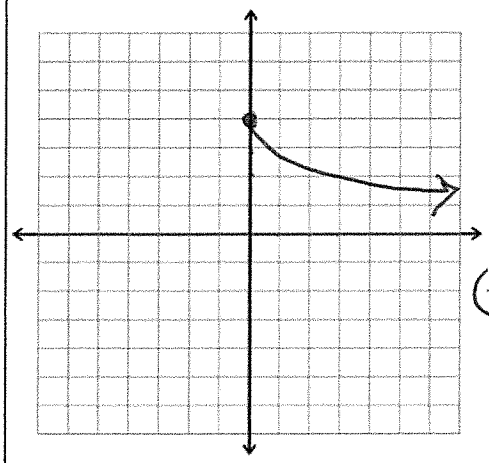
$f(x) = (x+2)^2 - 4$



Domain: $(-\infty, +\infty)$

Range: $[-4, +\infty)$

$f(x) = -\sqrt{x}+4$

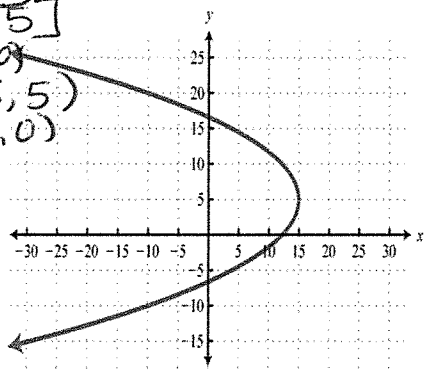


Domain: $[0, +\infty)$

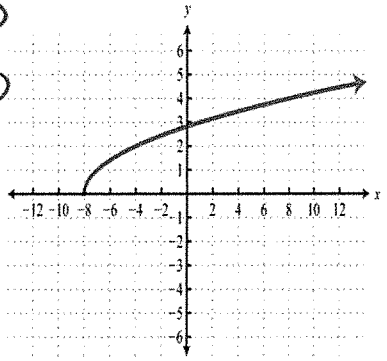
Range: $(-\infty, 4]$

4. For each graph below, determine whether it is a function. Then determine its domain and range in interval notation. Identify **ONE solution to the function (point on the graph)** and **ONE x-intercept.**

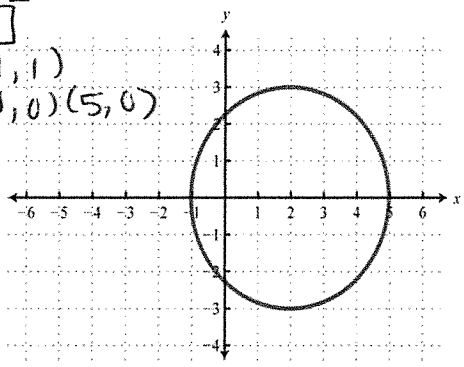
Function? YES/NO
Domain: $(-\infty, 15]$
Range: $(-\infty, +\infty)$
Solution Pt: $(15, 5)$
x- intercept: $(12, 0)$
Parent Fcn:
Quadratic
$f(x) = x^2$



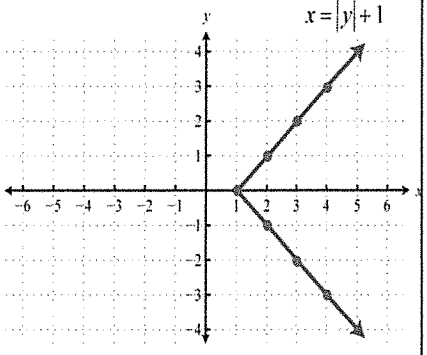
Function? YES/NO
Domain: $[-8, +\infty)$
Range: $[0, +\infty)$
Solution Pt: $(-4, 2)$
x- intercept: $(-8, 0)$
Parent Fcn:
Sq. Root
$f(x) = \sqrt{x}$



Function? YES/NO
Domain: $[-1, 5]$
Range: $[-3, 3]$
Solution Pt: $(-1, 1)$
x- intercept: $(-1, 0)(5, 0)$
Parent Fcn:
~~Function:~~



Function? YES/NO
Domain: $[1, +\infty)$
Range: $(-\infty, +\infty)$
Solution Pt: $(2, 1)$
x- intercept: $(1, 0)$
Parent Fcn:
Abs. Value
$f(x) = |x|$



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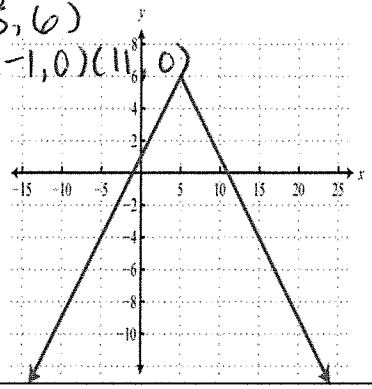
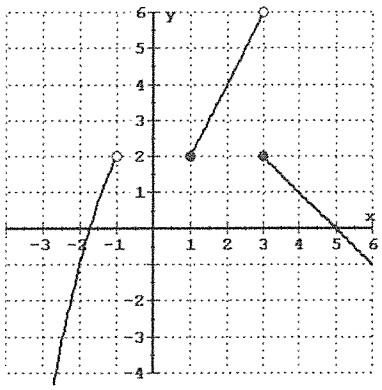
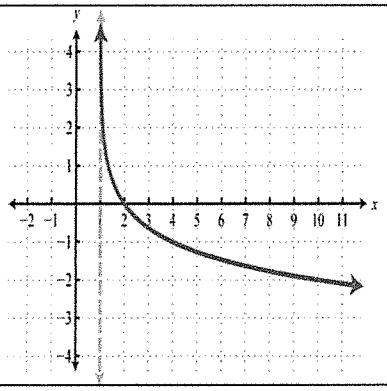
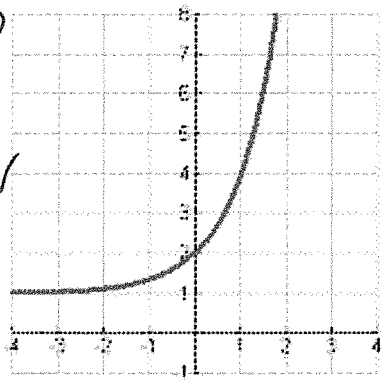
Function? YES/NO Domain: $(-\infty, +\infty)$ Range: $(-\infty, 6]$ Solution Pt: $(5, 6)$ x- intercept: $(-1, 0)(11, 0)$ Parent Function: Abs. Value $f(x) = \lvert x \rvert$	Function? YES/NO Domain: Range: Solution Pt: x- intercept: Parent Function:





5. For each graph, draw one line connecting it to its matching **domain** and then fill in the blanks for the **end behavior** to the right of the graph.

DOMAIN		INTERVALS
$(-\infty, \infty)$	A 	End Behavior: $x \to -\infty, y \to +\infty$ $x \to \infty, y \to +\infty$
$[1, \infty)$	B 	End Behavior: $x \to -\infty, y \to 1$ $x \to \infty, y \to +\infty$

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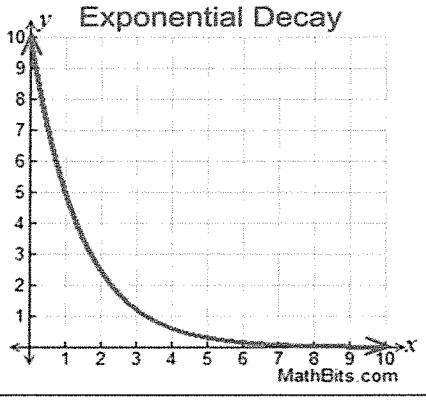
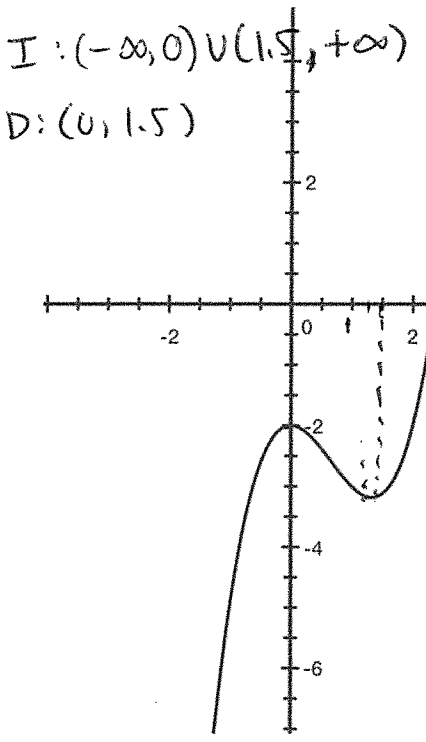
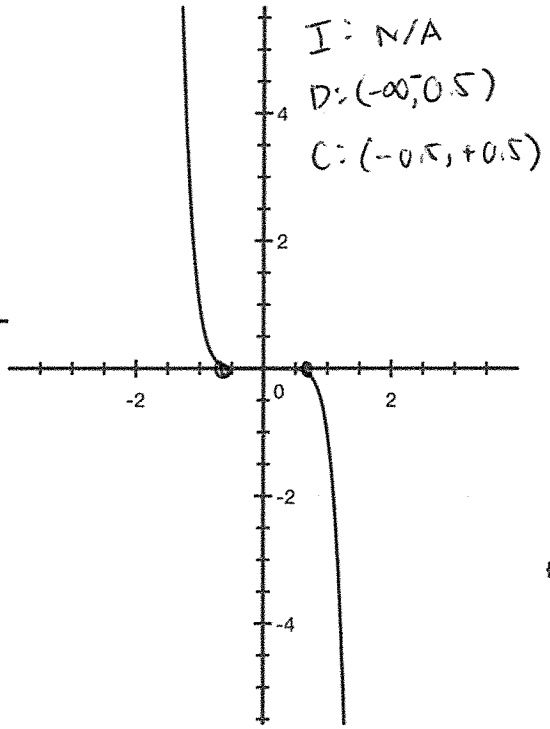
$(-\infty, \infty)$ →		End Behavior: $x \to -\infty, y \to +\infty$ $x \to \infty, y \to 0$

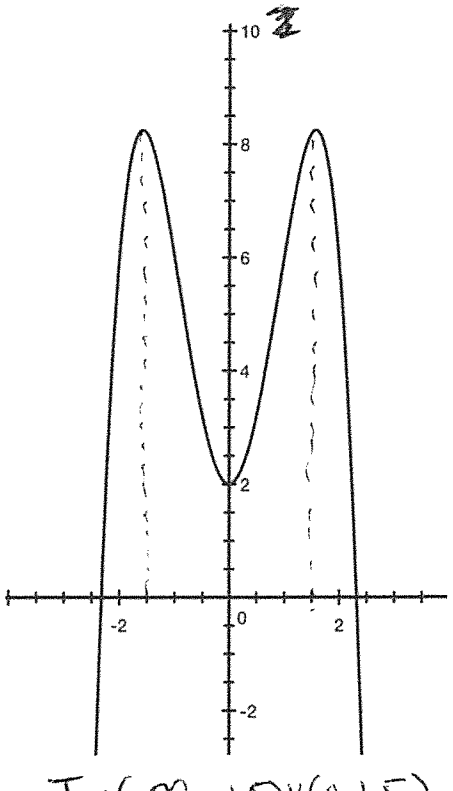
Exp. Decay & Growth have asymptotes. they have special end behaviors.

* JUST LOOK AT X-AXIS!

6. Identify the intervals of which the function is **increasing, decreasing or co**nstants.







I: (−∞, −1.5) ∪ (0, 1.5)

D: (−1.5, 0) ∪ (1.5, +∞)

UNIT 3: QUADRATICS

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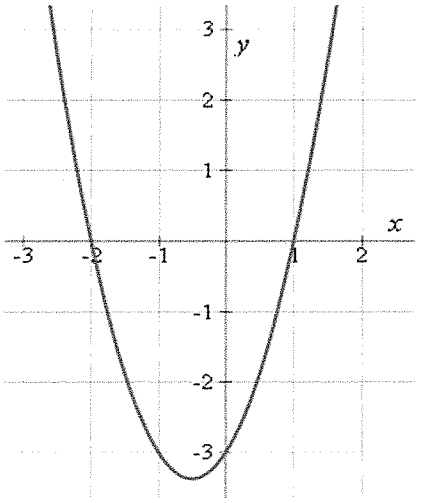
UNIT 3: Quadratics

1. Simplify the following radicals.
   a. $\sqrt{24} < \frac{\sqrt{4}}{\sqrt{6}} = 2\sqrt{6}$
   b. $\sqrt{75} = 5\sqrt{3}$
   c. $\sqrt{-100} = 10i$
   d. $\sqrt{45} = 3\sqrt{5}$

2. Find the **y-intercepts** of the following quadratic functions.
   a. $f(x) = x^2 + 74x - 45$ $(0, -45)$
   b. $f(x) = x^2 + 4x + 23$ $(0, 23)$
   c. $f(x) = 2x^2 - 3x - 4$ $(0, -4)$

3. **NC** The graph to the right can be represented by which of the following equations:
   a. $y = (x-2)(x+1)$
   b. $y = (x-1)(x+2)$
   c. $y = (x+1)(x+2)$
   d. $y = (x-1)(x-2)$



4. Factor the following functions:
   a. $f(x) = x^2 + 2x - 3$ $(x-1)(x+3)$
   b. $f(x) = x^2 + 8x + 15$ $(x+5)(x+3)$
   c. $f(x) = x^2 - x - 20$ $(x-5)(x+4)$
   d. $f(x) = x^2 + 4x + 23$ NOT FACTORABLE

5. Don't forget that you will have to use the quadratic formula on the final. Please state the quadratic formula and use it to solve for x in the following equations.

$3x^2 - 3x - 4 = 0$

$x = \frac{+3 \pm \sqrt{(-3)^2 - 4(3)(-4)}}{2(3)}$

$= \frac{3 \pm \sqrt{9 + 48}}{6}$

$= \boxed{\frac{3 \pm \sqrt{57}}{6}}$

$11x^2 + 6x + 9 = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(11)(9)}}{2(11)}$

$= \frac{-6 \pm \sqrt{36 - 396}}{22}$

$= \frac{-6 \pm \sqrt{-360}}{22} < \frac{\sqrt{36}}{\sqrt{10}}\, i$

$= \frac{-6 \pm 6i\sqrt{10}}{22} \Rightarrow \boxed{\frac{-3 \pm 3i\sqrt{10}}{11}}$

6. How many times does each of the following functions intersect the x-axis?
   a. $y = 3x^2 + \frac{2}{3}x - \frac{1}{3}$ crosses 2 times
   b. $f(x) = \frac{4}{3}x^2 - 4x + 3$ crosses 1 time

graph the following and see where they intersect the x-axis, & how many times - if any.

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c. $y = 2x^2 - \frac{1}{2}x + \frac{3}{2}$

crosses 0 times

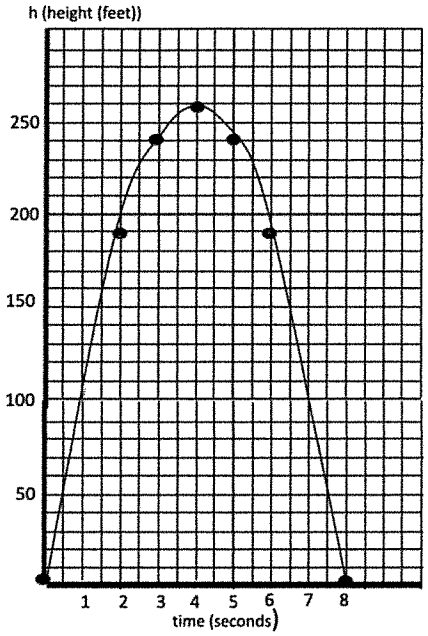
$(-2-i)(4+i)$
$-8-2i-4i-i^2$
$-8-6i+1$
$\boxed{9-6i}$

$(6-5i)(3+2i)$
$18+12i-15i-10i^2$
$18-3i+10$
$\boxed{28-3i}$

$6-2i-11-4i$
$\boxed{-5-6i}$

7. Simplify each expression. Write your answer in $a + bi$ format.
   a. $(6-2i)-(11+4i) \Rightarrow -5-6i$
   b. $(-2-i)(4+i) \Rightarrow 9-6i$
   c. $(6-5i)(3+2i) \Rightarrow 28-3i$

8. Using the graph at the right, It shows the **height *h*** in feet of a small rocket ***t* seconds** after it is launched. The path of the rocket is given by the equation: $h(x) = -16x^2 + 128t$.



   a. How long is the rocket in the air? 8 s
   b. What is the greatest height the rocket reaches? 260 ft
   c. About how high is the rocket after 1 second? 100 ft
   d. After 2 seconds, about how high is the rocket? 190 ft
   e. After 6 seconds, about how high is the rocket? 190 ft

9. A quarterback throws a football up into the air with an initial height of 6 feet. The height, *h*, of the ball at *t* seconds can be represented by the equation $h(t) = -16t^2 + 20t + 6$.
   a. How many seconds will it take to reach the maximum height of the football?

$x = \frac{-b}{2a} = \frac{-20}{2(-16)} = \frac{-20}{-32} = \boxed{5/8\ s}$

   b. What is the maximum height of the football?

$h(t) = -16(5/8)^2 + 20(5/8) + 6$
$\boxed{h(t) = 12.25\ ft}$

10. A squirrel is 24 feet up in a tree and tosses a nut out of the tree. The nuts height, *h*, at time *t* seconds can be represented by the equation $h(t) = -16t^2 + 8t + 24$.
   a. How many seconds will it take for the nut to reach maximum height?

$x = \frac{-b}{2a} = \frac{-8}{2(-16)} = \frac{-8}{-32} = \boxed{0.25\ s}$

   b. What is the maximum height of the nut?

$h(t) = -16(0.25)^2 + 8(0.25) + 24$
$\boxed{h(t) = 25\ ft}$

11. Solve the following for x.
   a. $\sqrt{3x-5} - 5 = 2 \Rightarrow 18$
   b. $(x-4)^2 = 9 \Rightarrow x = 7, 1$
   c. $4x^2 + 72 = 0 \Rightarrow x = \pm 3i\sqrt{2}$

$\sqrt{3x-5} - 5 = 2$
$(\sqrt{3x-5})^2 = (7)^2$
$3x-5 = 49$
$3x = 54$
$\boxed{x = 18}$

$\sqrt{(x-4)^2} = \sqrt{9}$
$x-4 = \pm 3$
$x = 4 \pm 3$
$x = 4+3 = 7$
$x = 4-3 = 1$
$\boxed{x = 7, 1}$

$4x^2 + 72 = 0$
$4x^2 = -72$
$\sqrt{x^2} = \sqrt{-18} < \frac{\sqrt{9}}{\sqrt{2}}$
$\boxed{x = \pm 3i\sqrt{2}}$

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d. $(\sqrt{20-5x})^2 = (x-5)^2$

$20-5x = (x-5)(x-5)$

$20-5x = x^2-10x+25$

$+5x \quad +5x$

$20 = x^2-5x+25$

$-20 \quad -20$

$0 = x^2-5x+5$

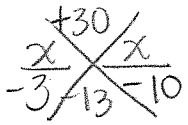
Quadratics

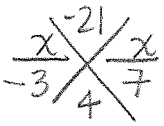
SKIP 11 d.

UNIT 4: RATIONALS

1. Simplify the following rational expressions and state the excluded value.

a. $\frac{x^2-13x+30}{x^2+4x-21}$





$= \frac{(x-3)(x-10)}{(x-3)(x+7)}$

$x \neq 3, -7$

$= \frac{(x-10)}{(x+7)}$

b. $\frac{x+4}{x^2+9x+20}$

$\frac{x}{5} \quad \frac{20}{9} \quad \frac{x}{4}$

$= \frac{x+4}{(x+4)(x+5)} = \frac{1}{x+5}$

$x \neq -4, -5$

2. Solve the following rational expressions (find x).

a. $\frac{5}{2x-3} = \frac{7}{3x}$

$7(2x-3) = 15x$

$14x - 21 = 15x$

$-14x \quad -14x$

$-21 = x$

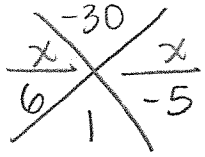
b. $\frac{x+3}{3} = \frac{8}{x-2}$

$(x+3)(x-2) = 24$

$x^2 + x - 6 = 24$

$-24 \quad -24$

$x^2 + x - 30 = 0$



$(x+6)(x-5) = 0$

$x = -6, 5$