2.5 Complex Zeros and the Fundamental Theorem of Algebra

What you'll learn about

- Two Major Theorems
- Complex Conjugate Zeros
- Factoring with Real Number Coefficients

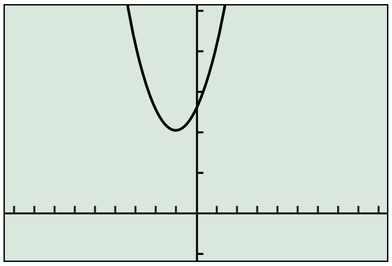
... and why

These topics provide the complete story about the zeros and factors of polynomials with real number coefficients.

Two Major Theorems

In Section 2.3 we learned that a polynomial function of degree n has at most n real zeros. Figure 2.42 shows that the polynomial function $f(x) = x^2 + 2x + 5$ of degree 2 has no real zeros. (Why?) A little arithmetic, however, shows that the complex number $-1 + 2i$ is a zero of f:

$$\begin{aligned} f(-1 + 2i) &= (-1 + 2i)^2 + 2(-1 + 2i) + 5 \\ &= (-3 - 4i) + (-2 + 4i) + 5 \\ &= 0 + 0i \\ &= 0 \end{aligned}$$



[–9.4, 9.4] by [–2, 10]

FIGURE 2.42 The graph of $f(x) = x^2 + 2x + 5$ has no x-intercepts, so f has no real zeros.

The quadratic formula shows that $-1 \pm 2i$ are the two zeros of f and can be used to find the complex zeros for any polynomial function of degree 2. In this section we will learn about complex zeros of polynomial functions of higher degree and how to use these zeros to factor polynomial expressions.

THEOREM Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and nonreal). Some of these zeros may be repeated.

The Factor Theorem extends to the complex zeros of a polynomial function. Thus, k is a complex zero of a polynomial if and only if $x - k$ is a factor of the polynomial, even if k is not a real number. We combine this fact with the Fundamental Theorem of Algebra to obtain the following theorem.

THEOREM Linear Factorization Theorem

If $f(x)$ is a polynomial function of degree $n > 0$, then $f(x)$ has precisely n linear factors and

$$f(x) = a(x - z_1)(x - z_2)\cdots(x - z_n)$$

where a is the leading coefficient of $f(x)$ and $z_1, z_2, \ldots, z_n$ are the complex zeros of $f(x)$. The z_i are not necessarily distinct numbers; some may be repeated.

The Fundamental Theorem of Algebra and the Linear Factorization Theorem are *existence theorems*. They tell us of the existence of zeros and linear factors, but not how to find them.

One connection is lost going from real zeros to complex zeros. If k is a *nonreal* complex zero of a polynomial function $f(x)$, then k is *not* an x-intercept of the graph of f. The other connections hold whether k is real or nonreal:

Fundamental Polynomial Connections in the Complex Case

The following statements about a polynomial function f are equivalent if k is a complex number:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$.
2. k is a zero of the function f.
3. $x - k$ is a factor of $f(x)$.

EXAMPLE 1 **Exploring Fundamental Polynomial Connections**

Write the polynomial function in standard form, and identify the zeros of the function and the x-intercepts of its graph.

(a) $f(x) = (x - 2i)(x + 2i)$

(b) $f(x) = (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i)$

(c) $f(x) = (x - 3)(x - 3)(x - i)(x + i)$

SOLUTION

(a) The quadratic function $f(x) = (x - 2i)(x + 2i) = x^2 + 4$ has two zeros: $x = 2i$ and $x = -2i$. Because the zeros are not real, the graph of f has no x-intercepts.

(b) The cubic function

$$\begin{aligned} f(x) &= (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i) \\ &= (x - 5)(x^2 + 2) \\ &= x^3 - 5x^2 + 2x - 10 \end{aligned}$$

has three zeros: $x = 5$, $x = \sqrt{2}i$, and $x = -\sqrt{2}i$. Of the three, only $x = 5$ is an x-intercept.

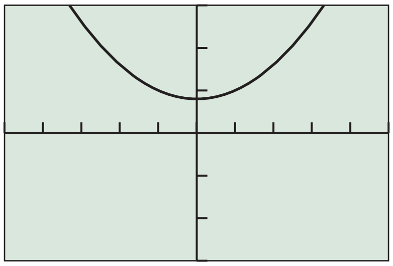
(c) The quartic function

$$\begin{aligned} f(x) &= (x - 3)(x - 3)(x - i)(x + i) \\ &= (x^2 - 6x + 9)(x^2 + 1) \\ &= x^4 - 6x^3 + 10x^2 - 6x + 9 \end{aligned}$$

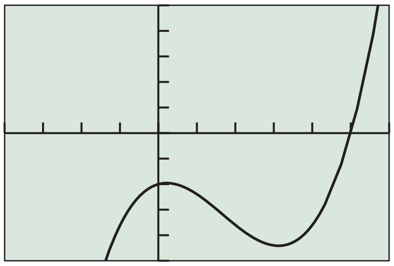
has four zeros: $x = 3$, $x = 3$, $x = i$, and $x = -i$. There are only three distinct zeros. The real zero $x = 3$ is a repeated zero of multiplicity two. Due to this even multiplicity, the graph of f touches but does not cross the x-axis at $x = 3$, the only x-intercept.

Figure 2.43 supports our conclusions regarding x-intercepts.

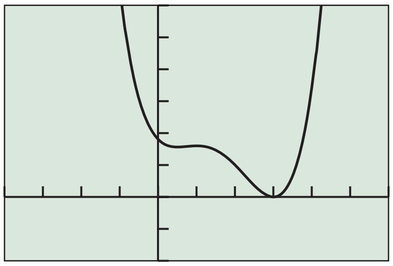
Now try Exercise 1.



[–5, 5] by [–15, 15]
(a)



[–4, 6] by [–25, 25]
(b)



[–4, 6] by [–10, 30]
(c)

FIGURE 2.43 The graphs of (a) $y = x^2 + 4$, (b) $y = x^3 - 5x^2 + 2x - 10$, and (c) $y = x^4 - 6x^3 + 10x^2 - 6x + 9$. (Example 1)

Complex Conjugate Zeros

In Section P.6 we saw that, for quadratic equations $ax^2 + bx + c = 0$ with real coefficients, if the discriminant $b^2 - 4ac$ is negative, the solutions are a conjugate pair of complex numbers. This relationship generalizes to polynomial functions of higher degree in the following way:

THEOREM Complex Conjugate Zeros

Suppose that $f(x)$ is a polynomial function with *real coefficients*. If a and b are real numbers with $b \neq 0$ and $a + bi$ is a zero of $f(x)$, then its complex conjugate $a - bi$ is also a zero of $f(x)$.

EXPLORATION 1 What Can Happen if the Coefficients Are Not Real?

1. Use substitution to verify that $x = 2i$ and $x = -i$ are zeros of $f(x) = x^2 - ix + 2$. Are the conjugates of $2i$ and $-i$ also zeros of $f(x)$?
2. Use substitution to verify that $x = i$ and $x = 1 - i$ are zeros of $g(x) = x^2 - x + (1 + i)$. Are the conjugates of i and $1 - i$ also zeros of $g(x)$?
3. What conclusions can you draw from parts 1 and 2? Do your results contradict the theorem about complex conjugate zeros?

EXAMPLE 2 Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include -3, 4, and $2 - i$.

SOLUTION Because -3 and 4 are real zeros, $x + 3$ and $x - 4$ must be factors. Because the coefficients are real and $2 - i$ is a zero, $2 + i$ must also be a zero. Therefore, $x - (2 - i)$ and $x - (2 + i)$ must both be factors of $f(x)$. Thus,

$$\begin{aligned} f(x) &= (x + 3)(x - 4)[x - (2 - i)][x - (2 + i)] \\ &= (x^2 - x - 12)(x^2 - 4x + 5) \\ &= x^4 - 5x^3 - 3x^2 + 43x - 60 \end{aligned}$$

is a polynomial of the type we seek. Any nonzero real number multiple of $f(x)$ will also be such a polynomial. *Now try Exercise 7.*

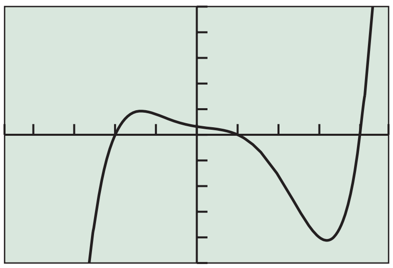
EXAMPLE 3 Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include $x = 1$, $x = 1 + 2i$, $x = 1 - i$.

SOLUTION Because the coefficients are real and $1 + 2i$ is a zero, $1 - 2i$ must also be a zero. Therefore, $x - (1 + 2i)$ and $x - (1 - 2i)$ are both factors of $f(x)$. Likewise, because $1 - i$ is a zero, $1 + i$ must be a zero. It follows that $x - (1 - i)$ and $x - (1 + i)$ are both factors of $f(x)$. Therefore,

$$\begin{aligned} f(x) &= (x - 1)[x - (1 + 2i)][x - (1 - 2i)][x - (1 + i)][x - (1 - i)] \\ &= (x - 1)(x^2 - 2x + 5)(x^2 - 2x + 2) \\ &= (x^3 - 3x^2 + 7x - 5)(x^2 - 2x + 2) \\ &= x^5 - 5x^4 + 15x^3 - 25x^2 + 24x - 10 \end{aligned}$$

is a polynomial of the type we seek. Any nonzero real number multiple of $f(x)$ will also be such a polynomial. *Now try Exercise 13.*



[–4.7, 4.7] by [–125, 125]

FIGURE 2.44 $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$ has three real zeros. (Example 4)

EXAMPLE 4 Factoring a Polynomial with Complex Zeros

Find all zeros of $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$, and write $f(x)$ in its linear factorization.

SOLUTION Figure 2.44 suggests that the real zeros of f are $x = -2$, $x = 1$, and $x = 4$.

Using synthetic division we can verify these zeros and show that $x^2 + 1$ is a factor of f. So $x = i$ and $x = -i$ are also zeros. Therefore,

$$\begin{aligned} f(x) &= x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8 \\ &= (x + 2)(x - 1)(x - 4)(x^2 + 1) \\ &= (x + 2)(x - 1)(x - 4)(x - i)(x + i). \end{aligned}$$

Now try Exercise 29.

Synthetic division can be used with complex number divisors in the same way it is used with real number divisors.

EXAMPLE 5 Finding Complex Zeros

The complex number $z = 1 - 2i$ is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$. Find the remaining zeros of $f(x)$, and write it in its linear factorization.

SOLUTION We use synthetic division to show that $f(1 - 2i) = 0$:

$$\begin{array}{r|rrrrr} 1 - 2i & 4 & 0 & 17 & 14 & 65 \\ & & 4 - 8i & -12 - 16i & -27 - 26i & -65 \\ \hline & 4 & 4 - 8i & 5 - 16i & -13 - 26i & 0 \end{array}$$

Thus $1 - 2i$ is a zero of $f(x)$. The conjugate $1 + 2i$ must also be a zero. We use synthetic division on the quotient found above to find the remaining quadratic factor:

$$\begin{array}{r|rrrr} 1 + 2i & 4 & 4 - 8i & 5 - 16i & -13 - 26i \\ & & 4 + 8i & 8 + 16i & 13 + 26i \\ \hline & 4 & 8 & 13 & 0 \end{array}$$

Finally, we use the quadratic formula to find the two zeros of $4x^2 + 8x + 13$:

$$\begin{aligned} x &= \frac{-8 \pm \sqrt{64 - 208}}{8} \\ &= \frac{-8 \pm \sqrt{-144}}{8} \\ &= \frac{-8 \pm 12i}{8} \\ &= -1 \pm \frac{3}{2}i \end{aligned}$$

Thus the four zeros of $f(x)$ are $1 - 2i$, $1 + 2i$, $-1 + (3/2)i$, and $-1 - (3/2)i$. Because the leading coefficient of $f(x)$ is 4, we obtain

$$f(x) = 4[x - (1 - 2i)][x - (1 + 2i)]\left[x - \left(-1 + \tfrac{3}{2}i\right)\right]\left[x - \left(-1 - \tfrac{3}{2}i\right)\right].$$

If we wish to remove fractions in the factors, we can distribute the 4 to get

$$f(x) = [x - (1 - 2i)][x - (1 + 2i)][2x - (-2 + 3i)][2x - (-2 - 3i)].$$

Now try Exercise 33.

Factoring with Real Number Coefficients

Let $f(x)$ be a polynomial function with real coefficients. The Linear Factorization Theorem tells us that $f(x)$ can be factored into the form

$$f(x) = a(x - z_1)(x - z_2)\cdots(x - z_n),$$

where z_i are complex numbers. Recall, however, that nonreal complex zeros occur in conjugate pairs. The product of $x - (a + bi)$ and $x - (a - bi)$ is

$$\begin{aligned}[x - (a + bi)][x - (a - bi)] &= x^2 - (a - bi)x - (a + bi)x + (a + bi)(a - bi) \\ &= x^2 - 2ax + (a^2 + b^2).\end{aligned}$$

So the quadratic expression $x^2 - 2ax + (a^2 + b^2)$, whose coefficients are real numbers, is a factor of $f(x)$. Such a quadratic expression with real coefficients but no real zeros is **irreducible over the reals**. In other words, if we require that the factors of a polynomial have real coefficients, the factorization can be accomplished with linear factors and irreducible quadratic factors.

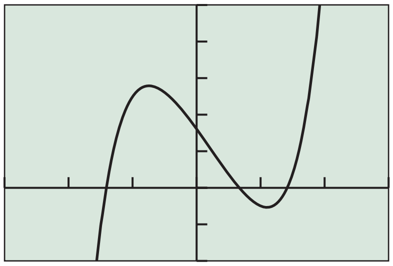
Factors of a Polynomial with Real Coefficients

Every polynomial function with real coefficients can be written as a product of linear factors and irreducible quadratic factors, each with real coefficients.

EXAMPLE 6 Factoring a Polynomial

Write $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$ as a product of linear and irreducible quadratic factors, each with real coefficients.

SOLUTION The Rational Zeros Theorem provides the candidates for the rational zeros of f. The graph of f in Figure 2.45 suggests which candidates to try first. Using synthetic division, we find that $x = 2/3$ is a zero. Thus,



[–3, 3] by [–20, 50]

FIGURE 2.45 $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$ has three real zeros. (Example 6)

$$\begin{aligned} f(x) &= \left(x - \frac{2}{3}\right)(3x^4 + 6x^2 - 24) \\ &= \left(x - \frac{2}{3}\right)(3)(x^4 + 2x^2 - 8) \\ &= (3x - 2)(x^2 - 2)(x^2 + 4) \\ &= (3x - 2)(x - \sqrt{2})(x + \sqrt{2})(x^2 + 4)\end{aligned}$$

Because the zeros of $x^2 + 4$ are complex, any further factorization would introduce nonreal complex coefficients. We have taken the factorization of f as far as possible, subject to the condition that *each factor has real coefficients.*

Now try Exercise 37.

We have seen that if a polynomial function has real coefficients, then its nonreal complex zeros occur in conjugate pairs. *Because a polynomial of odd degree has an odd number of zeros, it must have at least one zero that is real*. This confirms Example 7 of Section 2.3 in light of complex numbers.

Polynomial Function of Odd Degree

Every polynomial function of odd degree with real coefficients has at least one real zero.

The function $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$ in Example 6 fits the conditions of this theorem, so we know immediately that we are on the right track in searching for at least one real zero.

QUICK REVIEW 2.5 *(For help, go to Sections P.5, P.6, and 2.4.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, perform the indicated operation, and write the result in the form $a + bi$.

1. $(3 - 2i) + (-2 + 5i)$

2. $(5 - 7i) - (3 - 2i)$

3. $(1 + 2i)(3 - 2i)$

4. $\dfrac{2 + 3i}{1 - 5i}$

In Exercises 5 and 6, factor the quadratic expression.

5. $2x^2 - x - 3$

6. $6x^2 - 13x - 5$

In Exercises 7 and 8, solve the quadratic equation.

7. $x^2 - 5x + 11 = 0$

8. $2x^2 + 3x + 7 = 0$

In Exercises 9 and 10, list all potential rational zeros.

9. $3x^4 - 5x^3 + 3x^2 - 7x + 2$

10. $4x^5 - 7x^2 + x^3 + 13x - 3$

SECTION 2.5 EXERCISES

In Exercises 1–4, write the polynomial in standard form, and identify the zeros of the function and the x-intercepts of its graph.

1. $f(x) = (x - 3i)(x + 3i)$

2. $f(x) = (x + 2)(x - \sqrt{3}i)(x + \sqrt{3}i)$

3. $f(x) = (x - 1)(x - 1)(x + 2i)(x - 2i)$

4. $f(x) = x(x - 1)(x - 1 - i)(x - 1 + i)$

In Exercises 5–12, write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those listed.

5. i and $-i$

6. $1 - 2i$ and $1 + 2i$

7. $1, 3i$, and $-3i$

8. $-4, 1 - i$, and $1 + i$

9. $2, 3$, and i

10. $-1, 2$, and $1 - i$

11. 5 and $3 + 2i$

12. -2 and $1 + 2i$

In Exercises 13–16, write a polynomial function of minimum degree in standard form with real coefficients whose zeros and their multiplicities include those listed.

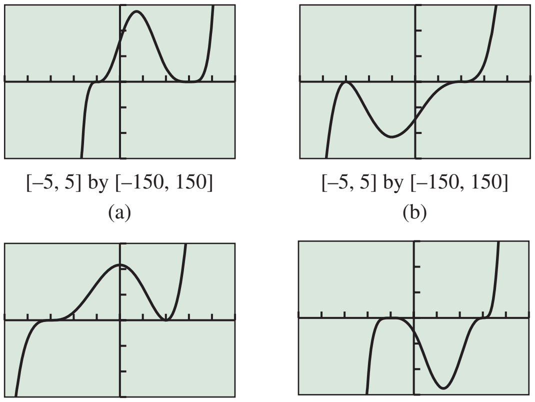
13. 1 (multiplicity 2), -2 (multiplicity 3)

14. -1 (multiplicity 3), 3 (multiplicity 1)

15. 2 (multiplicity 2), $3 + i$ (multiplicity 1)

16. -1 (multiplicity 2), $-2 - i$ (multiplicity 1)

In Exercises 17–20, match the polynomial function graph to the given zeros and multiplicities.



[–5, 5] by [–150, 150]
(a)

[–5, 5] by [–150, 150]
(b)

[–5, 5] by [–150, 150]
(c)

[–5, 5] by [–150, 150]
(d)

17. -3 (multiplicity 2), 2 (multiplicity 3)

18. -3 (multiplicity 3), 2 (multiplicity 2)

19. -1 (multiplicity 4), 3 (multiplicity 3)

20. -1 (multiplicity 3), 3 (multiplicity 4)

In Exercises 21–26, state how many complex and real zeros the function has.

21. $f(x) = x^2 - 2x + 7$

22. $f(x) = x^3 - 3x^2 + x + 1$

23. $f(x) = x^3 - x + 3$

24. $f(x) = x^4 - 2x^2 + 3x - 4$

25. $f(x) = x^4 - 5x^3 + x^2 - 3x + 6$

26. $f(x) = x^5 - 2x^2 - 3x + 6$

In Exercises 27–32, find all of the zeros and write a linear factorization of the function.

27. $f(x) = x^3 + 4x - 5$

28. $f(x) = x^3 - 10x^2 + 44x - 69$

29. $f(x) = x^4 + x^3 + 5x^2 - x - 6$

30. $f(x) = 3x^4 + 8x^3 + 6x^2 + 3x - 2$

31. $f(x) = 6x^4 - 7x^3 - x^2 + 67x - 105$

32. $f(x) = 20x^4 - 148x^3 + 269x^2 - 106x - 195$

In Exercises 33–36, using the given zero, find all of the zeros and write a linear factorization of $f(x)$.

33. $1 + i$ is a zero of $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$.

34. $4i$ is a zero of $f(x) = x^4 + 13x^2 - 48$.

35. $3 - 2i$ is a zero of $f(x) = x^4 - 6x^3 + 11x^2 + 12x - 26$.

36. $1 + 3i$ is a zero of $f(x) = x^4 - 2x^3 + 5x^2 + 10x - 50$.

In Exercises 37–42, write the function as a product of linear and irreducible quadratic factors all with real coefficients.

37. $f(x) = x^3 - x^2 - x - 2$

38. $f(x) = x^3 - x^2 + x - 6$

39. $f(x) = 2x^3 - x^2 + 2x - 3$

40. $f(x) = 3x^3 - 2x^2 + x - 2$

41. $f(x) = x^4 + 3x^3 - 3x^2 + 3x - 4$

42. $f(x) = x^4 - 2x^3 + x^2 - 8x - 12$

In Exercises 43 and 44, use *Archimedes' Principle,* which states that when a sphere of radius r with density d_S is placed in a liquid of density $d_L = 62.5$ lb/ft^3, it will sink to a depth h where

$$\frac{\pi}{3}(3rh^2 - h^3)d_L = \frac{4}{3}\pi r^3 d_S.$$

Find an approximate value for h if:

43. $r = 5$ ft and $d_S = 20$ lb/ft^3.

44. $r = 5$ ft and $d_S = 45$ lb/ft^3.

In Exercises 45–48, answer yes or no. If yes, include an example. If no, give a reason.

45. **Writing to Learn** Is it possible to find a polynomial of degree 3 with real number coefficients that has -2 as its only real zero?

46. **Writing to Learn** Is it possible to find a polynomial of degree 3 with real coefficients that has $2i$ as its only nonreal zero?

47. **Writing to Learn** Is it possible to find a polynomial $f(x)$ of degree 4 with real coefficients that has zeros -3, $1 + 2i$, and $1 - i$?

48. **Writing to Learn** Is it possible to find a polynomial $f(x)$ of degree 4 with real coefficients that has zeros $1 + 3i$ and $1 - i$?

In Exercises 49 and 50, find the unique polynomial with real coefficients that meets these conditions.

49. Degree 4; zeros at $x = 3$, $x = -1$, and $x = 2 - i$; $f(0) = 30$

50. Degree 4; zeros at $x = 1 - 2i$ and $x = 1 + i$; $f(0) = 20$

51. Sally's distance D from a motion detector is given by the data in Table 2.16.

(a) Find a cubic regression model, and graph it together with a scatter plot of the data.

(b) Describe Sally's motion.

(c) Use the cubic regression model to estimate when Sally changes direction. How far is she from the motion detector when she changes direction?

Table 2.16 Motion Detector Data

t (sec)	D (m)	t (sec)	D (m)
0.0	3.36	4.5	3.59
0.5	2.61	5.0	4.15
1.0	1.86	5.5	3.99
1.5	1.27	6.0	3.37
2.0	0.91	6.5	2.58
2.5	1.14	7.0	1.93
3.0	1.69	7.5	1.25
3.5	2.37	8.0	0.67
4.0	3.01		

52. Jacob's distance D from a motion detector is given by the data in Table 2.17.

(a) Find a quadratic regression model, and graph it together with a scatter plot of the data.

(b) Describe Jacob's motion.

(c) Use the quadratic regression model to estimate when Jacob changes direction. How far is he from the motion detector when he changes direction?

Table 2.17 Motion Detector Data

t (sec)	D (m)	t (sec)	D (m)
0.0	4.59	4.5	1.70
0.5	3.92	5.0	2.25
1.0	3.14	5.5	2.84
1.5	2.41	6.0	3.39
2.0	1.73	6.5	4.02
2.5	1.21	7.0	4.54
3.0	0.90	7.5	5.04
3.5	0.99	8.0	5.59
4.0	1.31		

Standardized Test Questions

53. **True or False** There is at least one polynomial with real coefficients with $1 - 2i$ as its only nonreal zero. Justify your answer.

54. **True or False** A polynomial of degree 3 with real coefficients must have two nonreal zeros. Justify your answer.

In Exercises 55–58, you may use a graphing calculator to solve the problem.

55. **Multiple Choice** Let z be a nonreal complex number and $\bar{z}$ its complex conjugate. Which of the following is not a real number?

(A) $z + \bar{z}$ **(B)** $z\bar{z}$ **(C)** $(z + \bar{z})^2$ **(D)** $(z\bar{z})^2$ **(E)** z^2

56. **Multiple Choice** Which of the following cannot be the number of real zeros of a polynomial of degree 5 with real coefficients?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

57. **Multiple Choice** Which of the following cannot be the number of nonreal zeros of a polynomial of degree 5 with real coefficients?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

(E) None of the above

58. **Multiple Choice** Assume that $1 + 2i$ is a zero of the polynomial f with real coefficients. Which of the following statements is not true?

(A) $x - (1 + 2i)$ is a factor of $f(x)$.

(B) $x^2 - 2x + 5$ is a factor of $f(x)$.

(C) $x - (1 - 2i)$ is a factor of $f(x)$.

(D) $1 - 2i$ is a zero of f.

(E) The number of nonreal complex zeros of f could be 1.

Explorations

59. Group Activity The Powers of $1 + i$

(a) Selected powers of $1 + i$ are displayed in Table 2.18. Find a pattern in the data, and use it to extend the table to power 7, 8, 9, and 10.

(b) Compute $(1 + i)^7$, $(1 + i)^8$, $(1 + i)^9$, nd $(1 +a i)^{10}$ using the fact that $(1 + i)^6 = -8i$.

(c) Compare your results from parts (a) and (b) and reconcile, if needed.

Table 2.18 Powers of $1 + i$

Power	Real Part	Imaginary Part
0	1	0
1	1	1
2	0	2
3	−2	2
4	−4	0
5	−4	−4
6	0	−8

60. Group Activity The Square Roots of i
Let a and b be real numbers such that $(a + bi)^2 = i$.

(a) Expand the left-hand side of the given equation.

(b) Think of the right-hand side of the equation as $0 + 1i$, and separate the real and imaginary parts of the equation to obtain two equations.

(c) Solve for a and b.

(d) Check your answer by substituting them in the original equation.

(e) What are the two square roots of i?

61. Verify that the complex number i is a zero of the polynomial $f(x) = x^3 - ix^2 + 2ix + 2$.

62. Verify that the complex number $-2i$ is a zero of the polynomial $f(x) = x^3 - (2 - i)x^2 + (2 - 2i)x - 4$.

Extending the Ideas

In Exercises 63 and 64, verify that $g(x)$ is a factor of $f(x)$. Then find $h(x)$ so that $f = g \cdot h$.

63. $g(x) = x - i$; $f(x) = x^3 + (3 - i)x^2 - 4ix - 1$

64. $g(x) = x - 1 - i$; $f(x) = x^3 - (1 + i)x^2 + x - 1 - i$

65. Find the three cube roots of 8 by solving $x^3 = 8$.

66. Find the three cube roots of -64 by solving $x^3 = -64$.