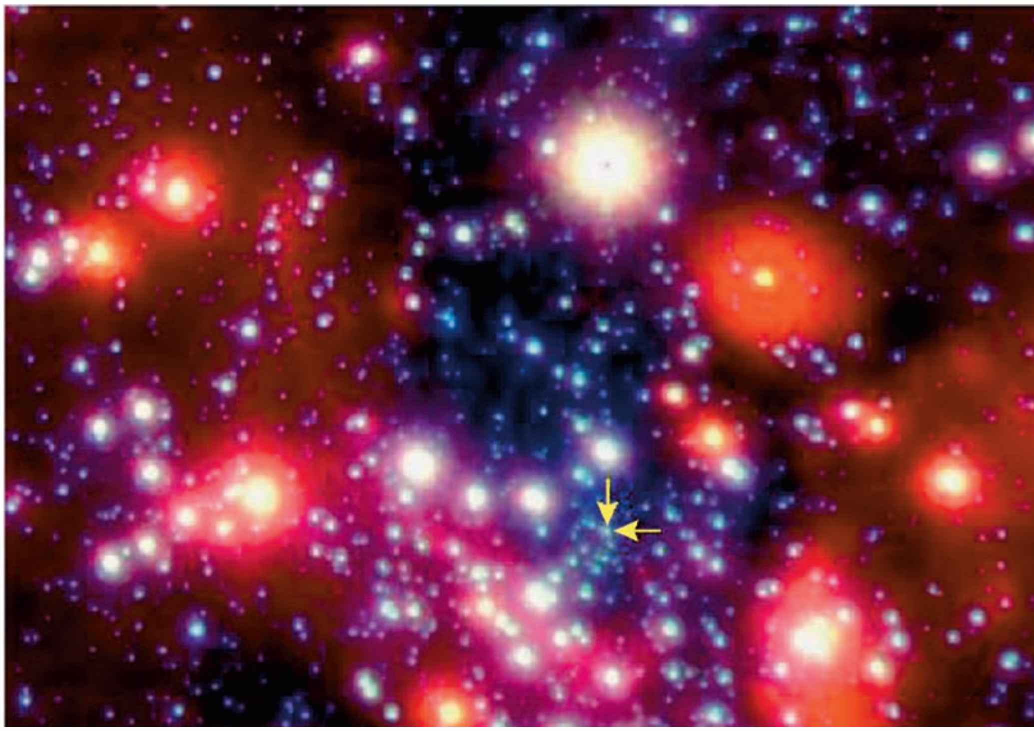
CHAPTER P

Prerequisites



Large distances are measured in *light years*, the distance light travels in one year. Astronomers use the speed of light, approximately 186,000 miles per second, to approximate distances between planets. See page 35 for examples.

Chapter P Overview

Historically, algebra was used to represent problems with symbols (algebraic models) and solve them by reducing the solution to algebraic manipulation of symbols. This technique is still important today. Graphing calculators are used today to approach problems by representing them with graphs (graphical models) and solve them with numerical and graphical techniques of the technology.

We begin with basic properties of real numbers and introduce absolute value, distance formulas, midpoint formulas, and equations of circles. Slope of a line is used to write standard equations for lines, and applications involving linear equations are discussed. Equations and inequalities are solved using both algebraic and graphical techniques.

Bibliography

For students: *Great Jobs for Math Majors*, Stephen Lambert, Ruth J. DeCotis. Mathematical Association of America, 1998.
For teachers: *Algebra in a Technological World, Addenda Series, Grades* 9–12. National Council of Teachers of Mathematics, 1995. *Why Numbers Count—Quantitative Literacy for Tommorrow's America,* Lynn Arthur Steen (Ed.). National Council of Teachers of Mathematics, 1997.

P.1 Real Numbers

What you'll learn about

- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation

... and why

These topics are fundamental in the study of mathematics and science.

Objective

Students will be able to convert between decimals and fractions, write inequalities, apply the basic properties of algebra, and work with exponents and scientific notation.

Motivate

Ask students how real numbers can be classified. Have students discuss ways to display very large or very small numbers without using a lot of zeros.

Representing Real Numbers

A **real number** is any number that can be written as a decimal. Real numbers are represented by symbols such as -8, 0, 1.75, $2.333\ldots$, $0.\overline{36}$, $8/5$, $\sqrt{3}$, $\sqrt[3]{16}$, e, and π.

The set of real numbers contains several important subsets:

The **natural (or counting) numbers**: $\{1, 2, 3, \ldots\}$

The **whole numbers**: $\{0, 1, 2, 3, \ldots\}$

The **integers**: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

The braces $\{\ \}$ are used to enclose the **elements**, or **objects**, of the set. The rational numbers are another important subset of the real numbers. A **rational number** is any number that can be written as a ratio a/b of two integers, where $b \neq 0$. We can use **set-builder notation** to describe the rational numbers:

$$\left\{ \frac{a}{b} \,\middle|\, a, b \text{ are integers, and } b \neq 0 \right\}$$

The vertical bar that follows a/b is read "such that."

The decimal form of a rational number either **terminates** like $7/4 = 1.75$, or is **infinitely repeating** like $4/11 = 0.363636\ldots = 0.\overline{36}$. The bar over the 36 indicates the block of digits that repeats. A real number is **irrational** if it is *not* rational. The decimal form of an irrational number is infinitely nonrepeating. For example, $\sqrt{3} = 1.7320508\ldots$ and $\pi = 3.14159265\ldots$.

Real numbers are approximated with calculators by giving a few of its digits. Sometimes we can find the decimal form of rational numbers with calculators, but not very often.

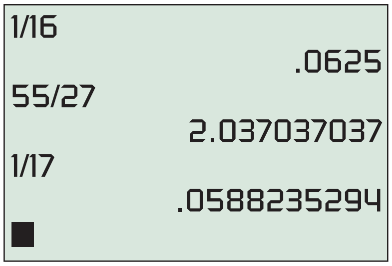


FIGURE P.1 Calculator decimal representations of 1/16, 55/27, and 1/17 with the calculator set in floating decimal mode. (Example 1)

EXAMPLE 1 Examining Decimal Forms of Rational Numbers

Determine the decimal form of 1/16, 55/27, and 1/17.

SOLUTION Figure P.1 suggests that the decimal form of 1/16 terminates and that of 55/27 repeats in blocks of 037.

$$\frac{1}{16} = 0.0625 \qquad \text{and} \qquad \frac{55}{27} = 2.\overline{037}$$

We cannot predict the *exact* decimal form of 1/17 from Figure P.1; however, we can say that $1/17 \approx 0.0588235294$. The symbol $\approx$ is read "*is approximately equal to*." We can use long division (see Exercise 66) to show that

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

Now try Exercise 3.

The real numbers and the points of a line can be matched *one-to-one* to form a **real number line**. We start with a horizontal line and match the real number zero with a point O, the **origin**. **Positive numbers** are assigned to the right of the origin, and **negative numbers** to the left, as shown in Figure P.2.

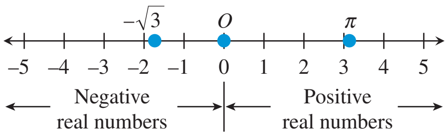


FIGURE P.2 The real number line.

Every real number corresponds to one and only one point on the real number line, and every point on the real number line corresponds to one and only one real number. Between every pair of real numbers on the number line there are infinitely many more real numbers.

The number associated with a point is **the coordinate of the point**. As long as the context is clear, we will follow the standard convention of using the real number for both the name of the point and its coordinate.

Order and Interval Notation

The set of real numbers is **ordered**. This means that we can use inequalities to compare any two real numbers that are not equal and say that one is "less than" or "greater than" the other.

Unordered Systems

Not all number systems are ordered. For example, the complex number system, to be introduced in Section P.6, has no natural ordering.

Order of Real Numbers

Let a and b be any two real numbers.

Symbol	Definition	Read
$a > b$	$a - b$ is positive	a is greater than b
$a < b$	$a - b$ is negative	a is less than b
$a \geq b$	$a - b$ is positive or zero	a is greater than or equal to b
$a \leq b$	$a - b$ is negative or zero	a is less than or equal to b

The symbols $>$, $<$, $\geq$, and $\leq$ are **inequality symbols**.

Opposites and Number Line

$a < 0 \Rightarrow -a > 0$

If $a < 0$, then a is to the left of 0 on the real number line, and its opposite, $-a$, is to the right of 0. Thus, $-a > 0$.

Geometrically, $a > b$ means that a is to the right of b (equivalently b is to the left of a) on the real number line. For example, since $6 > 3$, 6 is to the right of 3 on the real number line. Note also that $a > 0$ means that $a - 0$, or simply a, is positive and $a < 0$ means that a is negative.

We are able to compare any two real numbers because of the following important property of the real numbers.

Trichotomy Property

Let a and b be any two real numbers. Exactly one of the following is true:

$$a < b, \quad a = b, \quad \text{or} \quad a > b.$$

Inequalities can be used to describe **intervals** of real numbers, as illustrated in Example 2.

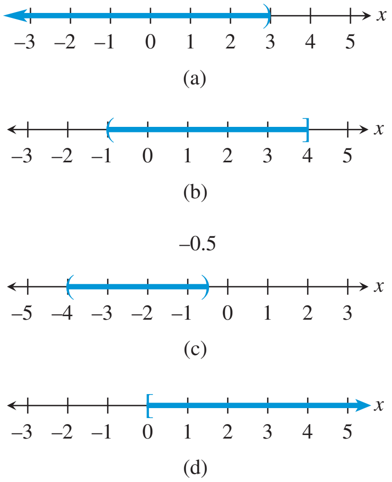


FIGURE P.3 In graphs of inequalities, parentheses correspond to $<$ and $>$ and brackets to $\le$ and $\ge$. (Examples 2 and 3)

EXAMPLE 2 Interpreting Inequalities

Describe and graph the interval of real numbers for the inequality.

(a) $x < 3$ **(b)** $-1 < x \le 4$

SOLUTION

(a) The inequality $x < 3$ describes all real numbers less than 3 (Figure P.3a).

(b) The *double inequality* $-1 < x \le 4$ represents all real numbers between -1 and 4, excluding -1 and including 4 (Figure P.3b). ***Now try Exercise 5.***

EXAMPLE 3 Writing Inequalities

Write an interval of real numbers using an inequality and draw its graph.

(a) The real numbers between -4 and -0.5

(b) The real numbers greater than or equal to zero

SOLUTION

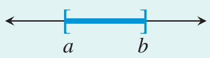
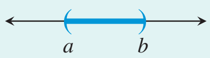
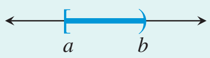
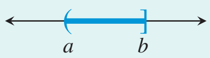
(a) $-4 < x < -0.5$ (Figure P.3c)

(b) $x \ge 0$ (Figure P.3d) ***Now try Exercise 15.***

As shown in Example 2, inequalities define *intervals* on the real number line. We often use $[2, 5]$ to describe the *bounded interval* determined by $2 \le x \le 5$. This interval is **closed** because it contains its *endpoints* 2 and 5. There are four types of **bounded intervals**.

Bounded Intervals of Real Numbers

Let a and b be real numbers with $a < b$.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, b]$	Closed	$a \le x \le b$	
(a, b)	Open	$a < x < b$	
$[a, b)$	Half-open	$a \le x < b$	
$(a, b]$	Half-open	$a < x \le b$	

The numbers a and b are the **endpoints** of each interval.

Interval Notation at $\pm\infty$

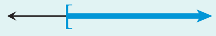
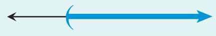
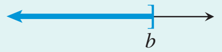
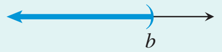
Because $-\infty$ is *not* a real number, we use $(-\infty, 2)$ instead of $[-\infty, 2)$ to describe $x < 2$. Similarly, we use $[-1, \infty)$ instead of $[-1, \infty]$ to describe $x \geq -1$.

The interval of real numbers determined by the inequality $x < 2$ can be described by the *unbounded interval* $(-\infty, 2)$. This interval is **open** because it does *not* contain its endpoint 2.

We use the interval notation $(-\infty, \infty)$ to represent the entire set of real numbers. The symbols $-\infty$ (*negative infinity*) and ∞ (*positive infinity*) allow us to use interval notation for unbounded intervals and are *not* real numbers. There are four types of **unbounded intervals**.

Unbounded Intervals of Real Numbers

Let a and b be real numbers.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, \infty)$	Closed	$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$	Closed	$x \leq b$	
$(-\infty, b)$	Open	$x < b$	

Each of these intervals has exactly one endpoint, namely a or b.

EXAMPLE 4 Converting Between Intervals and Inequalities

Convert interval notation to inequality notation or vice versa. Find the endpoints and state whether the interval is bounded, its type, and graph the interval.

(a) $[-6, 3)$ **(b)** $(-\infty, -1)$ **(c)** $-2 \leq x \leq 3$

SOLUTION

(a) The interval $[-6, 3)$ corresponds to $-6 \leq x < 3$ and is bounded and half-open (see Figure P.4a). The endpoints are -6 and 3.

(b) The interval $(-\infty, -1)$ corresponds to $x < -1$ and is unbounded and open (see Figure P.4b). The only endpoint is -1.

(c) The inequality $-2 \leq x \leq 3$ corresponds to the closed, bounded interval $[-2, 3]$ (see Figure P.4c). The endpoints are -2 and 3. *Now try Exercise 29.*

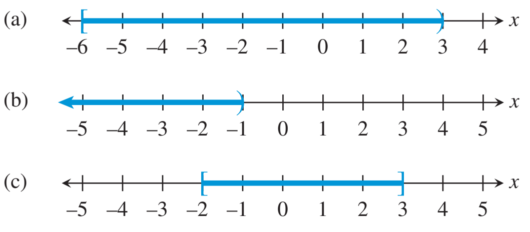


FIGURE P.4 Graphs of the intervals of real numbers in Example 4.

Basic Properties of Algebra

Algebra involves the use of letters and other symbols to represent real numbers. A **variable** is a letter or symbol (for example, x, y, t, θ) that represents an unspecified real number. A **constant** is a letter or symbol (for example, -2, 0, $\sqrt{3}$, π) that represents a specific real number. An **algebraic expression** is a combination of variables and constants involving addition, subtraction, multiplication, division, powers, and roots.

We state some of the properties of the arithmetic operations of addition, subtraction, multiplication, and division, represented by the symbols $+$, $-$, $\times$ (or $\cdot$) and $\div$ (or $/$), respectively. Addition and multiplication are the primary operations. Subtraction and division are defined in terms of addition and multiplication.

Subtraction: $a - b = a + (-b)$

Division: $\dfrac{a}{b} = a\left(\dfrac{1}{b}\right),\ b \neq 0$

Subtraction vs. Negative Numbers

On many calculators, there are two "−" keys, one for subtraction and one for negative numbers or opposites. Be sure you know how to use both keys correctly. Misuse can lead to incorrect results.

In the above definitions, $-b$ is the **additive inverse** or **opposite** of b, and $1/b$ is the **multiplicative inverse** or **reciprocal** of b. Perhaps surprisingly, additive inverses are not always negative numbers. The additive inverse of 5 is the negative number -5. However, the additive inverse of -3 is the positive number 3.

The following properties hold for real numbers, variables, and algebraic expressions.

Properties of Algebra

Let u, v, and w be real numbers, variables, or algebraic expressions.

1. **Commutative property**
   Addition: $u + v = v + u$
   Multiplication: $uv = vu$
2. **Associative property**
   Addition:
   $(u + v) + w = u + (v + w)$
   Multiplication: $(uv)w = u(vw)$
3. **Identity property**
   Addition: $u + 0 = u$
   Multiplication: $u \cdot 1 = u$
4. **Inverse property**
   Addition: $u + (-u) = 0$
   Multiplication: $u \cdot \dfrac{1}{u} = 1,\ u \neq 0$
5. **Distributive property**
   Multiplication over addition:
   $u(v + w) = uv + uw$
   $(u + v)w = uw + vw$
   Multiplication over subtraction:
   $u(v - w) = uv - uw$
   $(u - v)w = uw - vw$

The left-hand sides of the equations for the distributive property show the **factored form** of the algebraic expressions, and the right-hand sides show the **expanded form**.

EXAMPLE 5 Using the Distributive Property

(a) Write the expanded form of $(a + 2)x$.

(b) Write the factored form of $3y - by$.

SOLUTION

(a) $(a + 2)x = ax + 2x$

(b) $3y - by = (3 - b)y$

Now try Exercise 37.

Here are some properties of the additive inverse together with examples that help illustrate their meanings.

Properties of the Additive Inverse

Let u and v be real numbers, variables, or algebraic expressions.

Property	Example
1. $-(-u) = u$	$-(-3) = 3$
2. $(-u)v = u(-v) = -(uv)$	$(-4)3 = 4(-3) = -(4 \cdot 3) = -12$
3. $(-u)(-v) = uv$	$(-6)(-7) = 6 \cdot 7 = 42$
4. $(-1)u = -u$	$(-1)5 = -5$
5. $-(u + v) = (-u) + (-v)$	$-(7 + 9) = (-7) + (-9) = -16$

Integer Exponents

Exponential notation is used to shorten products of factors that repeat. For example,

$$(-3)(-3)(-3)(-3) = (-3)^4 \quad \text{and} \quad (2x + 1)(2x + 1) = (2x + 1)^2.$$

Exponential Notation

Let a be a real number, variable, or algebraic expression and n a positive integer. Then

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}},$$

where n is the **exponent**, a is the **base**, and a^n is the ***n*th power of *a***, read as "a to the nth power."

The two exponential expressions in Example 6 have the same value but have different bases. Be sure you understand the difference.

Understanding Notation

$(-3)^2 = 9$

$-3^2 = -9$

Be careful!

EXAMPLE 6 Identifying the Base

(a) In $(-3)^5$, the base is -3.

(b) In -3^5, the base is 3.

Now try Exercise 43.

Here are the basic properties of exponents together with examples that help illustrate their meanings.

Properties of Exponents

Let u and v be real numbers, variables, or algebraic expressions and m and n be integers. All bases are assumed to be nonzero.

Property	Example
1. $u^m u^n = u^{m+n}$	$5^3 \cdot 5^4 = 5^{3+4} = 5^7$
2. $\dfrac{u^m}{u^n} = u^{m-n}$	$\dfrac{x^9}{x^4} = x^{9-4} = x^5$
3. $u^0 = 1$	$8^0 = 1$
4. $u^{-n} = \dfrac{1}{u^n}$	$y^{-3} = \dfrac{1}{y^3}$
5. $(uv)^m = u^m v^m$	$(2z)^5 = 2^5 z^5 = 32z^5$
6. $(u^m)^n = u^{mn}$	$(x^2)^3 = x^{2 \cdot 3} = x^6$
7. $\left(\dfrac{u}{v}\right)^m = \dfrac{u^m}{v^m}$	$\left(\dfrac{a}{b}\right)^7 = \dfrac{a^7}{b^7}$

To simplify an expression involving powers means to rewrite it so that each factor appears only once, all exponents are positive, and exponents and constants are combined as much as possible.

Moving Factors

Be sure you understand how exponent property 4 permits us to move factors from the numerator to the denominator and vice versa:

$$\frac{v^{-m}}{u^{-n}} = \frac{u^n}{v^m}$$

EXAMPLE 7 Simplifying Expressions Involving Powers

(a) $(2ab^3)(5a^2b^5) = 10(aa^2)(b^3b^5) = 10a^3b^8$

(b) $\dfrac{u^2v^{-2}}{u^{-1}v^3} = \dfrac{u^2u^1}{v^2v^3} = \dfrac{u^3}{v^5}$

(c) $\left(\dfrac{x^2}{2}\right)^{-3} = \dfrac{(x^2)^{-3}}{2^{-3}} = \dfrac{x^{-6}}{2^{-3}} = \dfrac{2^3}{x^6} = \dfrac{8}{x^6}$

Now try Exercise 47.

Scientific Notation

Any positive number can be written in **scientific notation**,

$$c \times 10^m, \text{ where } 1 \le c < 10 \text{ and } m \text{ is an integer.}$$

This notation provides a way to work with very large and very small numbers. For example, the distance between the Earth and the Sun is about 93,000,000 miles. In scientific notation,

$$93{,}000{,}000 \text{ mi} = 9.3 \times 10^7 \text{ mi.}$$

The *positive exponent* 7 indicates that moving the decimal point in 9.3 to the right 7 places produces the decimal form of the number.

The mass of an oxygen molecule is about

$$0.000\,000\,000\,000\,000\,000\,000\,053 \text{ gram.}$$

In scientific notation,

$$0.000\,000\,000\,000\,000\,000\,000\,053 \text{ g} = 5.3 \times 10^{-23} \text{ g.}$$

The *negative exponent* -23 indicates that moving the decimal point in 5.3 to the left 23 places produces the decimal form of the number.

EXAMPLE 8 Converting to and from Scientific Notation

(a) $2.375 \times 10^8 = 237{,}500{,}000$

(b) $0.000000349 = 3.49 \times 10^{-7}$

Now try Exercises 57 and 59.

EXAMPLE 9 Using Scientific Notation

Simplify $\dfrac{(360{,}000)(4{,}500{,}000{,}000)}{18{,}000}$, without using a calculator.

SOLUTION

$$\begin{aligned}\frac{(360{,}000)(4{,}500{,}000{,}000)}{18{,}000} &= \frac{(3.6 \times 10^5)(4.5 \times 10^9)}{1.8 \times 10^4}\\ &= \frac{(3.6)(4.5)}{1.8} \times 10^{5+9-4}\\ &= 9 \times 10^{10}\\ &= 90{,}000{,}000{,}000\end{aligned}$$

Now try Exercise 63.

Using a Calculator

Figure P.5 shows two ways to perform the computation. In the first, the numbers are entered in decimal form. In the second, the numbers are entered in scientific notation. The calculator uses "9E10" to stand for 9×10^{10}.

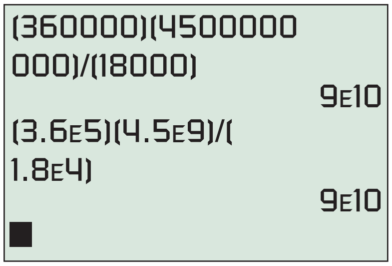


FIGURE P.5 Be sure you understand how your calculator displays scientific notation. (Example 9)

QUICK REVIEW P.1

1. List the positive integers between -3 and 7.

2. List the integers between -3 and 7.

3. List all negative integers greater than -4.

4. List all positive integers less than 5.

In Exercises 5 and 6, use a calculator to evaluate the expression. Round the value to two decimal places.

5. **(a)** $4(-3.1)^3 - (-4.2)^5$ **(b)** $\dfrac{2(-5.5) - 6}{7.4 - 3.8}$

6. **(a)** $5[3(-1.1)^2 - 4(-0.5)^3]$ **(b)** $5^{-2} + 2^{-4}$

In Exercises 7 and 8, evaluate the algebraic expression for the given values of the variables.

7. $x^3 - 2x + 1, x = -2, 1.5$

8. $a^2 + ab + b^2, a = -3, b = 2$

In Exercises 9 and 10, list the possible remainders.

9. When the positive integer n is divided by 7

10. When the positive integer n is divided by 13

SECTION P.1 EXERCISES

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, find the decimal form for the rational number. State whether it repeats or terminates.

1. $-37/8$ **2.** $15/99$

3. $-13/6$ **4.** $5/37$

In Exercises 5–10, describe and graph the interval of real numbers.

5. $x \le 2$ **6.** $-2 \le x < 5$

7. $(-\infty, 7)$ **8.** $[-3, 3]$

9. x is negative.

10. x is greater than or equal to 2 and less than or equal to 6.

In Exercises 11–16, use an inequality to describe the interval of real numbers.

11. $[-1, 1)$ **12.** $(-\infty, 4]$

13.

14.

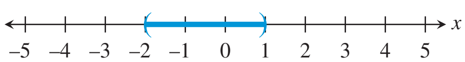
15. x is between -1 and 2.

16. x is greater than or equal to 5.

In Exercises 17–22, use interval notation to describe the interval of real numbers.

17. $x > -3$ **18.** $-7 < x < -2$

19.



20.

21. x is greater than -3 and less than or equal to 4.

22. x is positive.

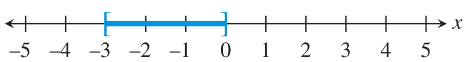
In Exercises 23–28, use words to describe the interval of real numbers.

23. $4 < x \le 9$ **24.** $x \ge -1$

25. $[-3, \infty)$ **26.** $(-5, 7)$

27.

28.



In Exercises 29–32, convert to inequality notation. Find the endpoints and state whether the interval is bounded or unbounded and its type.

29. $(-3, 4]$ **30.** $(-3, -1)$

31. $(-\infty, 5)$ **32.** $[-6, \infty)$

In Exercises 33–36, use both inequality and interval notation to describe the set of numbers. State the meaning of any variables you use.

33. Writing to Learn Bill is at least 29 years old.

34. Writing to Learn No item at Sarah's Variety Store costs more than \$2.00.

35. Writing to Learn The price of a gallon of gasoline varies from \$1.099 to \$1.399.

36. Writing to Learn Salary raises at the State University of California at Chico will average between 2% and 6.5%.

In Exercises 37–40, use the distributive property to write the factored form or the expanded form of the given expression.

37. $a(x^2 + b)$ **38.** $(y - z^3)c$

39. $ax^2 + dx^2$ **40.** $a^3z + a^3w$

In Exercises 41 and 42, find the additive inverse of the number.

41. $6 - \pi$ **42.** -7

In Exercises 43 and 44, identify the base of the exponential expression.

43. -5^2 **44.** $(-2)^7$

45. Group Activity Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

(a) $(3x)y = 3(xy)$ **(b)** $a^2b = ba^2$

(c) $a^2b + (-a^2b) = 0$ **(d)** $(x + 3)^2 + 0 = (x + 3)^2$

(e) $a(x + y) = ax + ay$

46. Group Activity Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

(a) $(x + 2)\dfrac{1}{x + 2} = 1$ **(b)** $1 \cdot (x + y) = x + y$

(c) $2(x - y) = 2x - 2y$

(d) $2x + (y - z) = 2x + (y + (-z))$
$= (2x + y) + (-z) =$
$(2x + y) - z$

(e) $\dfrac{1}{a}(ab) = \left(\dfrac{1}{a}a\right)b = 1 \cdot b = b$

In Exercises 47–52, simplify the expression. Assume that the variables in the denominators are nonzero.

47. $\dfrac{x^4y^3}{x^2y^5}$ **48.** $\dfrac{(3x^2)^2y^4}{3y^2}$

49. $\left(\dfrac{4}{x^2}\right)^2$ **50.** $\left(\dfrac{2}{xy}\right)^{-3}$

51. $\dfrac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}}$ **52.** $\left(\dfrac{4a^3b}{a^2b^3}\right)\left(\dfrac{3b^2}{2a^2b^4}\right)$

The data in Table P.1 give the expenditures in millions of dollars for U.S. public schools for the 2005–2006 school year.

Table P.1 U.S. Public Schools

Category	Amount (in millions)
Current expenditures	449,595
Capital outlay	57,375
Interest on school debt	14,347
Total	528,735

Source: National Center for Education Statistics, U.S. Department of Education, as reported in The World Almanac and Book of Facts 2009.

In Exercises 53–56, write the amount of expenditures in dollars obtained from the category in scientific notation.

53. Current expenditures

54. Capital outlay

55. Interest on school debt

56. Total

In Exercises 57 and 58, write the number in scientific notation.

57. The mean distance from Jupiter to the Sun is about 483,900,000 miles.

58. The electric charge, in coulombs, of an electron is about $-0.000\,000\,000\,000\,000\,000\,16$.

In Exercises 59–62, write the number in decimal form.

59. 3.33×10^{-8} **60.** 6.73×10^{11}

61. The distance that light travels in 1 year (*one light year*) is about 5.87×10^{12} mi.

62. The mass of a neutron is about 1.6747×10^{-24} g.

In Exercises 63 and 64, use scientific notation to simplify.

63. $\dfrac{(1.3 \times 10^{-7})(2.4 \times 10^{8})}{1.3 \times 10^{9}}$ without using a calculator

64. $\dfrac{(3.7 \times 10^{-7})(4.3 \times 10^{6})}{2.5 \times 10^{7}}$

Explorations

65. Investigating Exponents For positive integers m and n, we can use the definition to show that $a^m a^n = a^{m+n}$.

(a) Examine the equation $a^m a^n = a^{m+n}$ for $n = 0$ and explain why it is reasonable to define $a^0 = 1$ for $a \neq 0$.

(b) Examine the equation $a^m a^n = a^{m+n}$ for $n = -m$ and explain why it is reasonable to define $a^{-m} = 1/a^m$ for $a \neq 0$.

66. Decimal Forms of Rational Numbers Here is the third step when we divide 1 by 17. (The first two steps are not shown, because the quotient is 0 in both cases.)

$$\begin{array}{r} 0.05 \\ 17\overline{)1.00} \\ \underline{85} \\ 15 \end{array}$$

By convention we say that 1 is the first remainder in the long division process, 10 is the second, and 15 is the third remainder.

(a) Continue this long division process until a remainder is repeated, and complete the following table:

Step	Quotient	Remainder
1	0	1
2	0	10
3	5	15
⋮	⋮	⋮

(b) Explain why the digits that occur in the quotient between the pair of repeating remainders determine the infinitely repeating portion of the decimal representation. In this case

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

(c) Explain why this procedure will always determine the infinitely repeating portion of a rational number whose decimal representation does not terminate.

Standardized Test Questions

67. True or False The additive inverse of a real number must be negative. Justify your answer.

68. True or False The reciprocal of a positive real number must be less than 1. Justify your answer.

In Exercises 69–72, solve these problems without using a calculator.

69. Multiple Choice Which of the following inequalities corresponds to the interval $[-2, 1)$?

(A) $x \leq -2$ **(B)** $-2 \leq x \leq 1$

(C) $-2 < x < 1$ **(D)** $-2 < x \leq 1$

(E) $-2 \leq x < 1$

70. Multiple Choice What is the value of $(-2)^4$?

(A) 16 **(B)** 8

(C) 6 **(D)** -8

(E) -16

71. Multiple Choice What is the base of the exponential expression -7^2?

(A) -7 **(B)** 7

(C) -2 **(D)** 2

(E) 1

72. Multiple Choice Which of the following is the simplified form of $\frac{x^6}{x^2}, x \neq 0$?

(A) x^{-4} **(B)** x^2

(C) x^3 **(D)** x^4

(E) x^8

Extending the Ideas

The **magnitude** of a real number is its distance from the origin.

73. List the whole numbers whose magnitudes are less than 7.

74. List the natural numbers whose magnitudes are less than 7.

75. List the integers whose magnitudes are less than 7.