9.4 Sequences

What you'll learn about

- Infinite Sequences
- Limits of Infinite Sequences
- Arithmetic and Geometric Sequences
- Sequences and Graphing Calculators

... and why

Infinite sequences, especially those with finite limits, are involved in some key concepts of calculus.

Infinite Sequences

One of the most natural ways to study patterns in mathematics is to look at an ordered progression of numbers, called a **sequence**. Here are some examples of sequences:

1. 5, 10, 15, 20, 25

2. $2, 4, 8, 16, 32, \ldots, 2^k, \ldots$

3. $\left\{\frac{1}{k}; k = 1, 2, 3, \ldots\right\}$

4. $\{a_1, a_2, a_3, \ldots, a_k, \ldots\}$, which is sometimes abbreviated $\{a_k\}$

The first of these is a **finite sequence**, while the other three are **infinite sequences**. Notice that in (2) and (3) we were able to define a rule that gives the kth number in the sequence (called the ***k*th term**) as a function of k. In (4) we do not have a rule, but notice how we can use subscript notation (a_k) to identify the kth term of a "general" infinite sequence. In this sense, an infinite sequence can be thought of as a *function* that assigns a unique number (a_k) to each natural number k.

EXAMPLE 1 Defining a Sequence Explicitly

Find the first 6 terms and the 100th term of the sequence $\{a_k\}$ in which $a_k = k^2 - 1$.

SOLUTION Since we know the kth term *explicitly* as a function of k, we need only to evaluate the function to find the required terms:

$$a_1 = 1^2 - 1 = 0,\ a_2 = 3,\ a_3 = 8,\ a_4 = 15,\ a_5 = 24,\ a_6 = 35, \quad \text{and}$$
$$a_{100} = 100^2 - 1 = 9999$$

Now try Exercise 1.

Explicit formulas are the easiest to work with, but there are other ways to define sequences. For example, we can specify values for the first term (or terms) of a sequence, then define each of the following terms **recursively** by a formula relating it to previous terms. Example 2 shows how this is done.

Agreement on Sequences

Since we will be dealing primarily with infinite sequences in this book, the word "sequence" will mean an infinite sequence unless otherwise specified.

EXAMPLE 2 Defining a Sequence Recursively

Find the first 6 terms and the 100th term for the sequence defined recursively by the conditions:

$$b_1 = 3$$
$$b_n = b_{n-1} + 2 \text{ for all } n > 1$$

SOLUTION We proceed one term at a time, starting with $b_1 = 3$ and obtaining each succeeding term by adding 2 to the term just before it:

$$b_1 = 3$$
$$b_2 = b_1 + 2 = 5$$
$$b_3 = b_2 + 2 = 7$$
etc.

Eventually it becomes apparent that we are building the sequence of odd natural numbers beginning with 3:

$$\{3, 5, 7, 9, \ldots\}$$

The 100th term is 99 terms beyond the first, which means that we can get there quickly by adding 99 2's to the number 3:

$$b_{100} = 3 + 99 \times 2 = 201$$

Now try Exercise 5.

Limits of Infinite Sequences

Just as we were concerned with the end behavior of functions, we will also be concerned with the end behavior of sequences.

DEFINITION Limit of a Sequence

Let $\{a_n\}$ be a sequence of real numbers, and consider $\lim_{n \to \infty} a_n$. If the limit is a finite number L, the sequence **converges** and L is the **limit of the sequence**. If the limit is infinite or nonexistent, the sequence **diverges**.

EXAMPLE 3 Finding Limits of Sequences

Determine whether the sequence converges or diverges. If it converges, give the limit.

(a) $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$

(b) $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$

(c) $2, 4, 6, 8, 10, \ldots$

(d) $-1, 1, -1, 1, \ldots, (-1)^n, \ldots$

SOLUTION

(a) $\lim_{x \to \infty} \frac{1}{n} = 0$, so the sequence converges to a limit of 0.

(b) Although the nth term is not explicitly given, we can see that $a_n = \frac{n+1}{n}$.

$\lim_{x \to \infty} \frac{n+1}{n} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right) = 1 + 0 = 1$. The sequence converges to a limit of 1.

(c) This time we see that $a_n = 2n$. Since $\lim_{n \to \infty} 2n = \infty$, the sequence diverges.

(d) This sequence oscillates forever between two values and hence has no limit. The sequence diverges.

Now try Exercise 13.

It might help to review the rules for finding the *end behavior asymptotes* of rational functions (page 221 in Section 2.6) because those same rules apply to sequences that are rational functions of n, as in Example 4.

EXAMPLE 4 Finding Limits of Sequences

Determine whether the sequence converges or diverges. If it converges, give the limit.

(a) $\left\{\frac{3n}{n+1}\right\}$

(b) $\left\{\frac{5n^2}{n^3+1}\right\}$

(c) $\left\{\frac{n^3+2}{n^2+n}\right\}$

SOLUTION

(a) Since the degree of the numerator is the same as the degree of the denominator, the limit is the ratio of the leading coefficients.

Thus $\lim_{n\to\infty} \frac{3n}{n+1} = \frac{3}{1} = 3$. The sequence converges to a limit of 3.

(b) Since the degree of the numerator is less than the degree of the denominator, the limit is zero. Thus $\lim_{n\to\infty} \frac{5n^2}{n^3+1} = 0$. The sequence converges to 0.

(c) Since the degree of the numerator is greater than the degree of the denominator, the limit is infinite. Thus $\lim_{n\to\infty} \frac{n^3+2}{n^2+n}$ is infinite. The sequence diverges.

Now try Exercise 15.

Arithmetic and Geometric Sequences

There are all kinds of rules by which we can construct sequences, but two particular types of sequences dominate in mathematical applications: those in which pairs of successive terms all have a common *difference* (**arithmetic** sequences), and those in which pairs of successive terms all have a common quotient, or *ratio* (**geometric** sequences). We will take a closer look at these in this section.

Pronunciation Tip

The word "arithmetic" is probably more familiar to you as a noun, referring to the mathematics you studied in elementary school. In this word, the second syllable ("rith") is accented. When used as an adjective, the third syllable ("met") gets the accent. (For the sake of comparison, a similar shift of accent occurs when going from the noun "analysis" to the adjective "analytic.")

DEFINITION Arithmetic Sequence

A sequence $\{a_n\}$ is an **arithmetic sequence** if it can be written in the form

$$\{a, a+d, a+2d, \dots, a+(n-1)d, \dots\} \text{ for some constant } d.$$

The number d is called the **common difference**.

Each term in an arithmetic sequence can be obtained recursively from its preceding term by adding d:

$$a_n = a_{n-1} + d \text{ (for all } n \geq 2)$$

EXAMPLE 5 Defining Arithmetic Sequences

For each of the following arithmetic sequences, find **(a)** the common difference, **(b)** the tenth term, **(c)** a recursive rule for the nth term, and **(d)** an explicit rule for the nth term.

(1) $-6, -2, 2, 6, 10, \dots$

(2) $\ln 3, \ln 6, \ln 12, \ln 24, \dots$

SOLUTION

(1) **(a)** The difference between successive terms is 4.

(b) $a_{10} = -6 + (10-1)(4) = 30$

(c) The sequence is defined recursively by $a_1 = -6$ and $a_n = a_{n-1} + 4$ for all $n \geq 2$.

(d) The sequence is defined explicitly by $a_n = -6 + (n-1)(4) = 4n - 10$.

(2) **(a)** This sequence might not look arithmetic at first, but $\ln 6 - \ln 3 = \ln \frac{6}{3} \ln 2$ (by a law of logarithms) and the difference between successive terms continues to be $\ln 2$.

(b) $a_{10} = \ln 3 + (10-1)\ln 2 = \ln 3 + 9\ln 2 = \ln(3 \cdot 2^9) = \ln 1536$

(c) The sequence is defined recursively by $a_1 = \ln 3$ and $a_n = a_{n-1} + \ln 2$ for all $n \geq 2$.

(d) The sequence is defined explicitly by $a_n = \ln 3 + (n-1)\ln 2 = \ln(3 \cdot 2^{n-1})$.

Now try Exercise 21.

DEFINITION Geometric Sequence

A sequence $\{a_n\}$ is a **geometric sequence** if it can be written in the form

$$\{a, a \cdot r, a \cdot r^2, \ldots, a \cdot r^{n-1}, \ldots\} \text{ for some nonzero constant } r.$$

The number r is called the **common ratio**.

Each term in a geometric sequence can be obtained recursively from its preceding term by multiplying by r:

$$a_n = a_{n-1} \cdot r \text{ (for all } n \geq 2)$$

EXAMPLE 6 Defining Geometric Sequences

For each of the following geometric sequences, find **(a)** the common ratio, **(b)** the tenth term, **(c)** a recursive rule for the nth term, and **(d)** an explicit rule for the nth term.

(1) 3, 6, 12, 24, 48, . . .

(2) $10^{-3}, 10^{-1}, 10^1, 10^3, 10^5, \ldots$

SOLUTION

(1) **(a)** The ratio between successive terms is 2.

(b) $a_{10} = 3 \cdot 2^{10-1} = 3 \cdot 2^9 = 1536$

(c) The sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1}$ for $n \geq 2$.

(d) The sequence is defined explicitly by $a_n = 3 \cdot 2^{n-1}$.

(2) **(a)** Applying a law of exponents, $\dfrac{10^{-1}}{10^{-3}} = 10^{-1-(-3)} = 10^2$, and the ratio between successive terms continues to be 10^2.

(b) $a_{10} = 10^{-3} \cdot (10^2)^{10-1} = 10^{-3+18} = 10^{15}$

(c) The sequence is defined recursively by $a_1 = 10^{-3}$ and $a_n = 10^2 a_{n-1}$ for $n \geq 2$.

(d) The sequence is defined explicitly by $a_n = 10^{-3}(10^2)^{n-1} = 10^{-3+2n-2} = 10^{2n-5}$.

Now try Exercise 25.

EXAMPLE 7 Constructing Sequences

The second and fifth terms of a sequence are 3 and 24, respectively. Find explicit and recursive formulas for the sequence if it is **(a)** arithmetic and **(b)** geometric.

SOLUTION

(a) If the sequence is arithmetic, then $a_2 = a_1 + d = 3$ and $a_5 = a_1 + 4d = 24$. Subtracting, we have

$$(a_1 + 4d) - (a_1 + d) = 24 - 3$$
$$3d = 21$$
$$d = 7$$

Then $a_1 + d = 3$ implies $a_1 = -4$.

The sequence is defined explicitly by $a_n = -4 + (n - 1) \cdot 7$, or $a_n = 7n - 11$.

The sequence is defined recursively by $a_1 = -4$ and $a_n = a_{n-1} + 7$ for $n \geq 2$.

(b) If the sequence is geometric, then $a_2 = a \cdot r^1 = 3$ and $a_5 = a \cdot r^4 = 24$. Dividing, we have

$$\frac{a_1 \cdot r^4}{a_1 \cdot r^1} = \frac{24}{3}$$
$$r^3 = 8$$
$$r = 2$$

continued

Then $a_1 \cdot r^1 = 3$ implies $a_1 = 1.5$.

The sequence is defined explicitly by $a_n = 1.5(2)^{n-1}$, or $a_n = 3(2)^{n-2}$.

The sequence is defined recursively by $a_1 = 1.5$ and $a_n = 2 \cdot a_{n-1}$.

Now try Exercise 29.

Sequence Graphing

Most graphers enable you to graph in "sequence mode." Check your owner's manual to see how to use this mode.

Sequences and Graphing Calculators

As with other kinds of functions, it helps to be able to represent a sequence geometrically with a graph. There are at least two ways to obtain a sequence graph on a graphing calculator. One way to graph explicitly defined sequences is as scatter plots of points of the form (k, a_k). A second way is to use the sequence graphing mode on a graphing calculator.

EXAMPLE 8 Graphing a Sequence Defined Explicitly

Produce on a graphing calculator a graph of the sequence $\{a_k\}$ in which $a_k = k^2 - 1$.

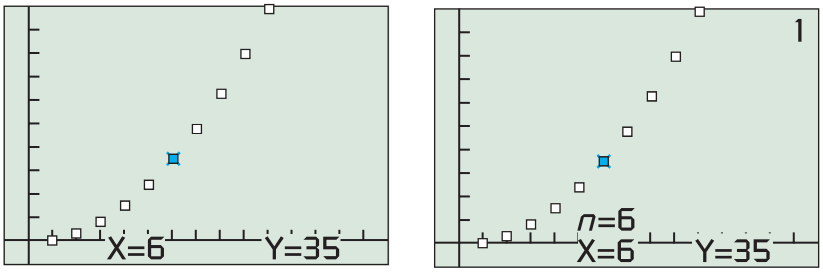
Method 1 (Scatter Plot)

The command seq(K, K, 1, 10) $\rightarrow L_1$ puts the first 10 natural numbers in list L_1. (You could change the 10 if you wanted to graph more or fewer points.)

The command $L_1^2 - 1 \rightarrow L_2$ puts the corresponding terms of the sequence in list L_2. A scatter plot of L_1, L_2 produces the graph in Figure 9.7a.

Method 2 (Sequence Mode)

With your calculator in Sequence mode, enter the sequence $a_k = k^2 - 1$ in the Y = list as $u(n) = n^2 - 1$ with nMin $= 1$, nMax $= 10$, and $u(n\text{Min}) = 0$. (You could change the 10 if you wanted to graph more or fewer points.) Figure 9.7b shows the graph in the same window as Figure 9.7a.



[–1, 15] by [–10, 100]
(a)

[–1, 15] by [–10, 100]
(b)

FIGURE 9.7 The sequence $a_k = k^2 - 1$ graphed (a) as a scatter plot and (b) using the sequence graphing mode. Tracing along the points gives values of a_k for $k = 1, 2, 3, \ldots$. (Example 8)

Now try Exercise 33.

EXAMPLE 9 Generating a Sequence with a Calculator

Using a graphing calculator, generate the specific terms of the following sequences:

(a) (Explicit) $a_k = 3k - 5$ for $k = 1, 2, 3, \ldots$

(b) (Recursive) $a_1 = -2$ and $a_n = a_{n-1} + 3$ for $n = 2, 3, 4, \ldots$

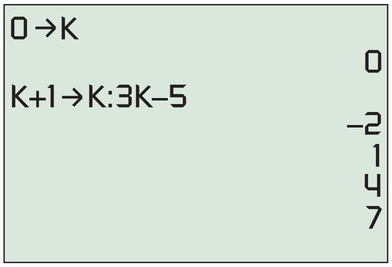


FIGURE 9.8 Typing these two commands (on the left of the viewing screen) will generate the terms of the explicitly defined sequence $a_k = 3k - 5$. (Example 9a)

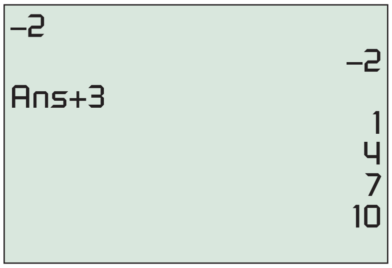


FIGURE 9.9 Typing these two commands (on the left of the viewing screen) will generate the terms of the recursively defined sequence with $a_1 = -2$ and $a_n = a_{n-1} + 3$. (Example 9b)

SOLUTION

(a) On the home screen, type the two commands shown in Figure 9.8. The calculator will then generate the terms of the sequence as you push the ENTER key repeatedly.

(b) On the home screen, type the two commands shown in Figure 9.9. The first command gives the value of a_1. The calculator will generate the remaining terms of the sequence as you push the ENTER key repeatedly.

Notice that these two definitions generate the very same sequence!

Now try Exercises 1 and 5 on your calculator.

A recursive definition of a_n can be made in terms of any combination of preceding terms, just as long as those preceding terms have already been determined. A famous example is the **Fibonacci sequence**, named for Leonardo of Pisa (ca. 1170–1250), who wrote under the name Fibonacci. You can generate it with the two commands shown in Figure 9.10.

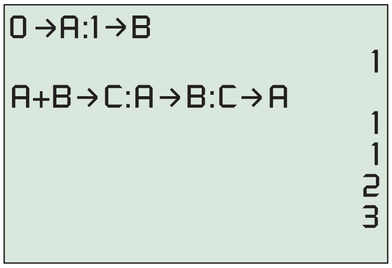


FIGURE 9.10 The two commands on the left will generate the Fibonacci sequence as the ENTER key is pressed repeatedly.

The Fibonacci sequence can be defined recursively using three statements.

The Fibonacci Sequence

The Fibonacci sequence can be defined recursively by

$$a_1 = 1$$
$$a_2 = 1$$
$$a_n = a_{n-2} + a_{n-1}$$

for all positive integers $n \geq 3$.

Fibonacci Numbers

The numbers in the Fibonacci sequence have fascinated professional and amateur mathematicians alike since the thirteenth century. Not only is the sequence, like Pascal's triangle, a rich source of curious internal patterns, but the Fibonacci numbers seem to appear everywhere in nature. If you count the leaflets on a leaf, the leaves on a stem, the whorls on a pine cone, the rows on an ear of corn, the spirals in a sunflower, or the branches from a trunk of a tree, they tend to be Fibonacci numbers. (Check **phyllotaxy** in a biology book.)

QUICK REVIEW 9.4 *(For help, see Section P.1.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1 and 2, evaluate each expression when $a = 3$, $d = 4$, and $n = 5$.

1. $a + (n - 1)d$

2. $\frac{n}{2}(2a + (n - 1)d)$

In Exercises 3 and 4, evaluate each expression when $a = 5$, $r = 4$, and $n = 3$.

3. $a \cdot r^{n-1}$

4. $\frac{a(1 - r^n)}{1 - r}$

In Exercises 5–10, find a_{10}.

5. $a_k = \frac{k}{k + 1}$

6. $a_k = 5 + (k - 1)3$

7. $a_k = 5 \cdot 2^{k-1}$

8. $a_k = \frac{4}{3}\left(\frac{1}{2}\right)^{k-1}$

9. $a_k = 32 - a_{k-1}$ and $a_9 = 17$

10. $a_k = \frac{k^2}{2^k}$

SECTION 9.4 EXERCISES

In Exercises 1–4, find the first 6 terms and the 100th term of the explicitly defined sequence.

1. $u_n = \dfrac{n+1}{n}$ **2.** $v_n = \dfrac{4}{n+2}$

3. $c_n = n^3 - n$ **4.** $d_n = n^2 - 5n$

In Exercises 5–10, find the first 4 terms and the eighth term of the recursively defined sequence.

5. $a_1 = 8$ and $a_n = a_{n-1}, - 4$, for $n \geq 2$

6. $u_1 = -3$ and $u_{k+1} = u_k + 10$, for $k \geq 1$,

7. $b_1 = 2$ and $b_{k+1} = 3b_k$, for $k \geq 1$

8. $v_1 = 0.75$ and $v_n = (-2)v_{n-1}$, for $n \geq 2$

9. $c_1 = 2$, $c_2 = -1$, and $c_{k+2} = c_k + c_{k+1}\ 1$, for $k \geq 1$

10. $c_1 = -2$, $c_2 = 3$, and $c_k = c_{k-2} + c_{k-1}$, for $k \geq 3$

In Exercises 11–20, determine whether the sequence converges or diverges. If it converges, give the limit.

11. $1, 4, 9, 16, \ldots, n^2, \ldots$

12. $\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \ldots, \dfrac{1}{2^n}, \ldots$

13. $\dfrac{1}{1}, \dfrac{1}{4}, \dfrac{1}{9}, \dfrac{1}{16}, \ldots,$

14. $\{3n - 1\}$

15. $\left\{\dfrac{3n-1}{2-3n}\right\}$

16. $\left\{\dfrac{2n-1}{n+1}\right\}$

17. $\{(0.5)^n\}$

18. $\{(1.5)^n\}$

19. $a_1 = 1$ and $a_{n+1} = a_n + 3$ for $n \geq 1$

20. $u_1 = 1$ and $u_{n+1} = \dfrac{u_n}{3}$ for $n \geq 1$

In Exercises 21–24, the sequences are arithmetic. Find

(a) the common difference,

(b) the tenth term,

(c) a recursive rule for the nth term, and

(d) an explicit rule for the nth term.

21. 6, 10, 14, 18, . . . **22.** −4, 1, 6, 11, . . .

23. −5, −2, 1, 4, . . . **24.** −7, 4, 15, 26, . . .

In Exercises 25–28, the sequences are geometric. Find

(a) the common ratio,

(b) the eighth term,

(c) a recursive rule for the nth term, and

(d) an explicit rule for the nth term.

25. 2, 6, 18, 54, . . . **26.** 3, 6, 12, 24, . . .

27. 1, −2, 4, −8, 16, . . . **28.** −2, 2, −2, 2, . . .

29. The fourth and seventh terms of an arithmetic sequence are −8 and 4, respectively. Find the first term and a recursive rule for the nth term.

30. The fifth and ninth terms of an arithmetic sequence are −5 and −17, respectively. Find the first term and a recursive rule for the nth term.

31. The second and eighth terms of a geometric sequence are 3 and 192, respectively. Find the first term, common ratio, and an explicit rule for the nth term.

32. The third and sixth terms of a geometric sequence are −75 and −9375, respectively. Find the first term, common ratio, and an explicit rule for the nth term.

In Exercises 33–36, graph the sequence.

33. $a_n = 2 - \dfrac{1}{n}$ **34.** $b_n = \sqrt{n} - 3$

35. $c_n = n^2 - 5$ **36.** $d_n = 3 + 2n$

37. Rain Forest Growth The bungy-bungy tree in the Amazon rain forest grows an average 2.3 cm per week. Write a sequence that represents the weekly height of a bungy-bungy over the course of 1 year if it is 7 meters tall today. Display the first four terms and the last two terms.

38. Half-Life (See Section 3.2) Thorium-232 has a half-life of 14 billion years. Make a table showing the half-life decay of a sample of thorium-232 from 16 grams to 1 gram; list the time (in years, starting with $t = 0$) in the first column and the mass (in grams) in the second column. Which type of sequence is each column of the table?

39. Arena Seating The first row of seating in section J of the Athena Arena has 7 seats. In all, there are 25 rows of seats in section J, each row containing two more seats than the row preceding it. How many seats are in section J?

40. Patio Construction Pat designs a patio with a trapezoid-shaped deck consisting of 16 rows of congruent slate tiles. The numbers of tiles in the rows form an arithmetic sequence. The first row contains 15 tiles and the last row contains 30 tiles. How many tiles are used in the deck?

41. Group Activity Pair up with a partner to create a sequence recursively together. Each of you picks five random digits from 1 to 9 (with repetitions, if you wish). Merge your digits to make a list of ten. Now each of you constructs a ten-digit number using exactly the numbers in your list.

Let a_1 = the (positive) difference between your two numbers.

Let a_{n+1} = the sum of the digits of a_n for $n \geq 1$.

This sequence converges, since it is eventually constant. What is the limit? (Remember, you can check your answer in the back of the book.)

42. Group Activity Here is an interesting recursively defined word sequence. Join up with three or four classmates and, without telling it to the others, pick a word from this sentence. Then, with care, count the letters in your word. Move *ahead* that many words in the text to come to a new word. Count the letters in the new word. Move ahead again, and so on. When you come to a point when your next move would take you out of this problem, stop. Share your last word with your friends. Are they all the same?

Standardized Test Questions

43. True or False If the first two terms of a geometric sequence are negative, then so is the third. Justify your answer.

44. True or False If the first two terms of an arithmetic sequence are positive, then so is the third. Justify your answer.

You may use a graphing calculator when solving Exercises 45–48.

45. Multiple Choice The first two terms of an arithmetic sequence are 2 and 8. The fourth term is

(A) 20. **(B)** 26. **(C)** 64. **(D)** 128. **(E)** 256.

46. Multiple Choice Which of the following sequences is divergent?

(A) $\left\{\frac{n + 100}{n}\right\}$ **(B)** $\{\sqrt{n}\}$ **(C)** $\{\pi^{-n}\}$ **(D)** $\left\{\frac{2n + 2}{n + 1}\right\}$ **(E)** $\{n^{-2}\}$

47. Multiple Choice A geometric sequence $\{a_n\}$ begins 2, 6, ... What is $\frac{a_6}{a_2}$?

(A) 3 **(B)** 4 **(C)** 9 **(D)** 12 **(E)** 81

48. Multiple Choice Which of the following rules for $n \geq 1$ will define a geometric sequence if $a_1 \neq 0$?

(A) $a_{n+1} = a_n + 3$ **(B)** $a_{n+1} = a_n - 3$

(C) $a_{n+1} = a_n \div 3$ **(D)** $a_{n+1} = a_n^3$ **(E)** $a_{n+1} = a_n \cdot 3^{n-1}$

Explorations

49. Rabbit Populations Assume that 2 months after birth, each male-female pair of rabbits begins producing one new male-female pair of rabbits each month. Further assume that the rabbit colony begins with one newborn male-female pair of rabbits and no rabbits die for 12 months. Let a_n represent the number of *pairs* of rabbits in the colony after $n - 1$ months.

(a) Writing to Learn Explain why $a_1 = 1$, $a_2 = 1$, and $a_3 = 2$.

(b) Find $a_4, a_5, a_6, \ldots, a_{13}$.

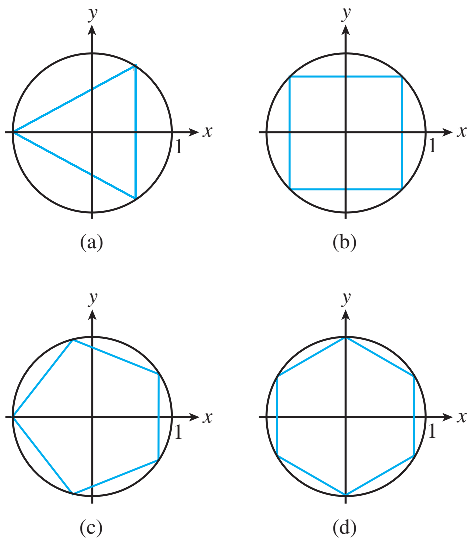
(c) Writing to Learn Explain why the sequence $\{a_n\}$, $1 \leq n \leq 13$, is a model for the size of the rabbit colony for a 1-year period.

50. Fibonacci Sequence Compute the first seven terms of the sequence whose nth term is

$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1 + \sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1 - \sqrt{5}}{2}\right)^n.$$

How do these seven terms compare with the first seven terms of the Fibonacci sequence?

51. Connecting Geometry and Sequences In the following sequence of diagrams, regular polygons are inscribed in unit circles with at least one side of each polygon perpendicular to the positive x-axis.



(a) Prove that the perimeter of each polygon in the sequence is given by $a_n = 2n \sin(\pi/n)$, where n is the number of sides in the polygon.

(b) Investigate the value of a_n for $n = 10, 100, 1000$, and 10,000. What conclusion can you draw?

52. Recursive Sequence The population of Centerville was 525,000 in 1992 and is growing annually at the rate of 1.75%. Write a recursive sequence $\{P_n\}$ for the population. State the first term P_1 for your sequence.

53. Writing to Learn If $\{a_n\}$ is a geometric sequence with all positive terms, explain why $\{\log a_n\}$ must be arithmetic.

54. Writing to Learn If $\{b_n\}$ is an arithmetic sequence, explain why $\{10^{b_n}\}$ must be geometric.

Extending the Ideas

55. A Sequence of Matrices Write out the first seven terms of the "geometric sequence" with the first term the matrix $\begin{bmatrix} 1 & 1 \end{bmatrix}$ and the common ratio the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. How is this sequence of matrices related to the Fibonacci sequence?

56. Another Sequence of Matrices Write out the first seven terms of the "geometric sequence" which has for its first term the matrix $\begin{bmatrix} 1 & a \end{bmatrix}$ and for its common ratio the matrix $\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$. How is this sequence of matrices related to the arithmetic sequence?