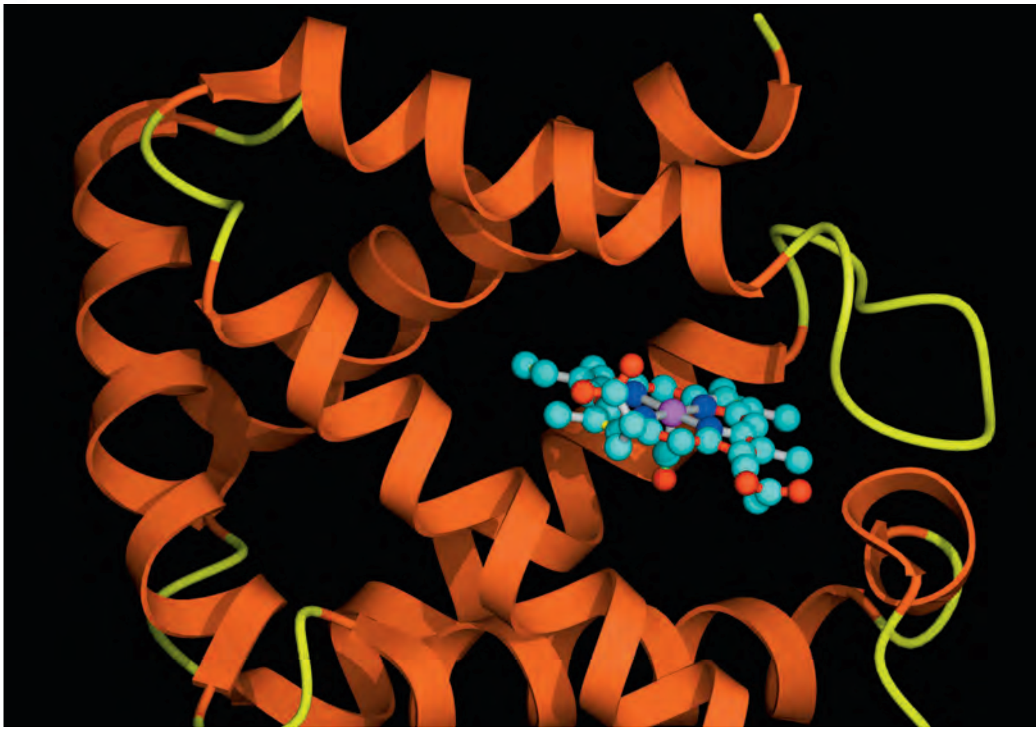
CHAPTER 7

Systems and Matrices



Scientists studying hemoglobin molecules, as represented in the photo, can make new discoveries by viewing the image on a computer. To see all the details, they may need to move the image up or down (translation), turn it around (rotation), or change the size (scaling). In computer graphics these operations are performed using matrix operations. See a related problem involving scaling a triangle on page 539.

Chapter 7 Overview

Many applications of mathematics in science, engineering, business, and other areas involve the use of systems of equations or inequalities in two or more variables as models for the corresponding problems. We investigate several techniques commonly used to solve such systems; and we investigate matrices, which play a central role in several of these techniques. The information age has made the use of matrices widespread because of their use in handling vast amounts of data.

We decompose a rational function into a sum of simpler rational functions using the method of partial fractions. This technique can be used to analyze a rational function, and is used in calculus to integrate rational functions analytically. Finally, we introduce linear programming, a method used to solve problems concerned with decision making in management science.

7.1 Solving Systems of Two Equations

What you'll learn about

- The Method of Substitution
- Solving Systems Graphically
- The Method of Elimination
- Applications

... and why

Many applications in business and science can be modeled using systems of equations.

The Method of Substitution

Here is an example of a system of two linear equations in the two variables x and y:

$$\begin{aligned} 2x - y &= 10 \\ 3x + 2y &= 1 \end{aligned}$$

A **solution of a system** of two equations in two variables is an ordered pair of real numbers that is a solution of each equation. For example, the ordered pair $(3, -4)$ is a solution to the above system. We can verify this by showing that $(3, -4)$ is a solution of each equation. Substituting $x = 3$ and $y = -4$ into each equation, we obtain

$$\begin{aligned} 2x - y &= 2(3) - (-4) = 6 + 4 = 10, \\ 3x + 2y &= 3(3) + 2(-4) = 9 - 8 = 1. \end{aligned}$$

So, both equations are satisfied.

We have **solved the system of equations** when we have found all its solutions. In Example 1, we use the method of substitution to see that $(3, -4)$ is the only solution of this system.

EXAMPLE 1 Using the Substitution Method

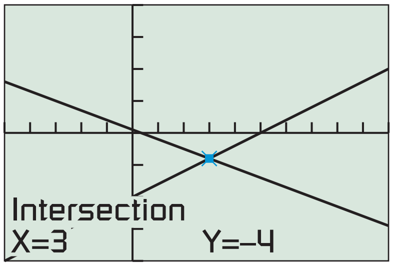
Solve the system

$$\begin{aligned} 2x - y &= 10 \\ 3x + 2y &= 1. \end{aligned}$$

SOLUTION

Solve Algebraically Solving the first equation for y yields $y = 2x - 10$. Then substitute the expression for y into the second equation.

$$\begin{aligned} 3x + 2y &= 1 && \text{Second equation} \\ 3x + 2(2x - 10) &= 1 && \text{Replace } y \text{ by } 2x - 10. \\ 3x + 4x - 20 &= 1 && \text{Distributive property} \\ 7x &= 21 && \text{Collect like terms.} \\ x &= 3 && \text{Divide by 7.} \\ y &= -4 && \text{Use } y = 2x - 10. \end{aligned}$$



[–5, 10] by [–20, 20]

FIGURE 7.1 The two lines $y = 2x - 10$ and $y = -1.5x + 0.5$ intersect in the point $(3, -4)$. (Example 1)

Support Graphically

The graph of each equation is a line. Figure 7.1 shows that the two lines intersect in the single point $(3, -4)$.

Interpret

The solution of the system is $x = 3$, $y = -4$, or the ordered pair $(3, -4)$.

Now try Exercise 5.

The method of substitution can sometimes be applied when the equations in the system are not linear, as illustrated in Example 2.

EXAMPLE 2 Solving a Nonlinear System by Substitution

Find the dimensions of a rectangular garden that has perimeter 100 ft and area 300 ft^2.

SOLUTION

Model

Let x and y be the lengths of adjacent sides of the garden (Figure 7.2). Then

$$2x + 2y = 100 \quad \text{Perimeter is 100.}$$
$$xy = 300. \quad \text{Area is 300.}$$

Solve Algebraically

Solving the first equation for y yields $y = 50 - x$. Then substitute the expression for y into the second equation.

$$\begin{aligned} xy &= 300 && \text{Second equation} \\ x(50 - x) &= 300 && \text{Replace } y \text{ by } 50 - x. \\ 50x - x^2 &= 300 && \text{Distributive property} \\ x^2 - 50x + 300 &= 0 \\ x &= \frac{50 \pm \sqrt{(-50)^2 - 4(300)}}{2} && \text{Quadratic formula} \\ x \approx 6.97 \quad &\text{or} \quad x \approx 43.03 && \text{Evaluate.} \\ y \approx 43.03 \quad &\text{or} \quad y \approx 6.97 && \text{Use } y = 50 - x. \end{aligned}$$

Support Graphically Figure 7.3 shows that the graphs of $y = 50 - x$ and $y = 300/x$ have two points of intersections.

Interpret The two ordered pairs $(6.972\ldots, 43.027\ldots)$ and $(43.027\ldots, 6.972\ldots)$ produce the same rectangle whose dimensions are approximately 7 ft by 43 ft.

Now try Exercise 11.

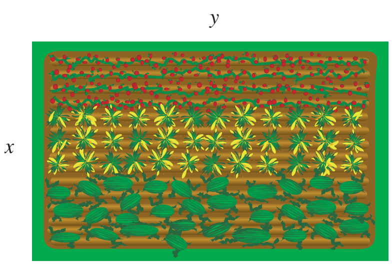
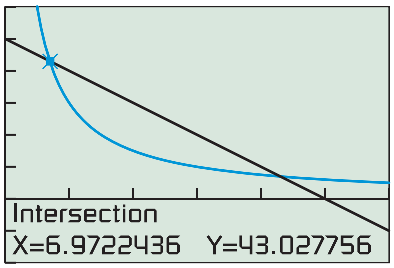


FIGURE 7.2 The rectangular garden in Example 2.

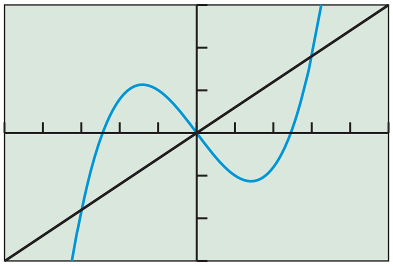


[0, 60] by [–20, 60]

FIGURE 7.3 We can assume $x \geq 0$ and $y \geq 0$ because x and y are lengths. (Example 2)

Rounding at the End

In Example 2, we did *not* round the values found for x until we computed the values for y. For the sake of accuracy, do not round *intermediate results*. Carry all decimals on your calculator computations and then round the final answer(s).



[–5, 5] by [–15, 15]

FIGURE 7.4 The graphs of $y = x^3 - 6x$ and $y = 3x$ have three points of intersection. (Example 3)

EXAMPLE 3 Solving a Nonlinear System Algebraically

Solve the system

$$y = x^3 - 6x$$
$$y = 3x.$$

Support your solution graphically.

SOLUTION

Substituting the value of y from the first equation into the second equation yields

$$\begin{aligned} x^3 - 6x &= 3x \\ x^3 - 9x &= 0 \\ x(x - 3)(x + 3) &= 0 \\ x = 0, x = 3, x &= -3 && \text{Zero factor property} \\ y = 0, y = 9, y &= -9 && \text{Use } y = 3x. \end{aligned}$$

The system of equations has three solutions: $(-3, -9)$, $(0, 0)$, and $(3, 9)$.

Support Graphically The graphs of the two equations in Figure 7.4 suggest that the three solutions found algebraically are correct. ***Now try Exercise 13.***

Solving Systems Graphically

Sometimes the method of substitution leads to an equation in one variable that we are not able to solve using the standard algebraic techniques we have studied in this text. In these cases we can solve the system graphically by finding intersections as illustrated in Exploration 1.

EXPLORATION 1 Solving a System Graphically

Consider the system:

$$y = \ln x$$
$$y = x^2 - 4x + 2$$

1. Draw the graphs of the two equations in the $[0, 10]$ by $[-5, 5]$ viewing window.
2. Use the graph in part 1 to find the coordinates of the points of intersection shown in the viewing window.
3. Use your knowledge about the graphs of logarithmic and quadratic functions to explain why this system has exactly two solutions.

Substituting the expression for y of the first equation of Exploration 1 into the second equation yields

$$\ln x = x^2 - 4x + 2.$$

We have no standard algebraic technique to solve this equation.

The Method of Elimination

Consider a system of two linear equations in x and y. To **solve by elimination**, we rewrite the two equations as two equivalent equations so that one of the variables has opposite coefficients. Then we add the two equations to eliminate that variable.

EXAMPLE 4 Using the Elimination Method

Solve the system

$$\begin{aligned} 2x + 3y &= 5 \\ -3x + 5y &= 21. \end{aligned}$$

SOLUTION

Solve Algebraically Multiply the first equation by 3 and the second equation by 2 to obtain

$$\begin{aligned} 6x + 9y &= 15 \\ -6x + 10y &= 42. \end{aligned}$$

Then add the two equations to eliminate the variable x.

$$19y = 57$$

Next divide by 19 to solve for y.

$$y = 3$$

Finally, substitute $y = 3$ into either of the two original equations to determine that $x = -2$.

The solution of the original system is $(-2, 3)$. *Now try Exercise 19.*

EXAMPLE 5 Finding No Solution

Solve the system

$$\begin{aligned} x - 3y &= -2 \\ 2x - 6y &= 4. \end{aligned}$$

SOLUTION We use the elimination method.

Solve Algebraically

$$\begin{aligned} -2x + 6y &= 4 && \text{Multiply first equation by } -2. \\ 2x - 6y &= 4 && \text{Second equation} \\ 0 &= 8 && \text{Add.} \end{aligned}$$

The last equation is true for *no* values of x and y. The system has no solution.

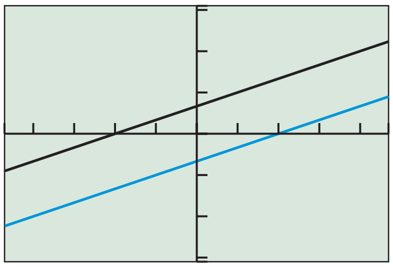
Support Graphically

Figure 7.5 suggests that the two lines that are the graphs of the two equations in the system are parallel. Solving for y in each equation yields

$$\begin{aligned} y &= \frac{1}{3}x + \frac{2}{3} \\ y &= \frac{1}{3}x - \frac{2}{3}. \end{aligned}$$

The two lines have the same slope of 1/3 and are therefore parallel.

Now try Exercise 23.



[–4.7, 4.7] by [–3.1, 3.1]

FIGURE 7.5 The graph of the two lines in Example 5 in this square viewing window appear to be parallel.

An easy way to determine the *number of solutions* of a system of two linear equations in two variables is to look at the graphs of the two lines. There are three possibilities. The two lines can intersect in a single point, producing exactly *one* solution as in Examples 1 and 4. The two lines can be parallel, producing *no* solution as in Example 5. The two lines can be the same, producing infinitely many solutions as illustrated in Example 6.

EXAMPLE 6 Finding Infinitely Many Solutions

Solve the system

$$4x - 5y = 2$$
$$-12x + 15y = -6.$$

SOLUTION

$$12x - 15y = 6 \quad \text{Multiply first equation by 3.}$$
$$-12x + 15y = -6 \quad \text{Second equation}$$
$$0 = 0 \quad \text{Add.}$$

The last equation is true for all values of x and y. Thus, every ordered pair that satisfies one equation satisfies the other equation. The system has infinitely many solutions.

Another way to see that there are infinitely many solutions is to solve each equation for y. Both equations yield

$$y = \frac{4}{5}x - \frac{2}{5}.$$

The two lines are the same. *Now try Exercise 25.*

Applications

Table 7.1 shows the personal consumption expenditures (in billions of dollars) for dentists and health insurance in the United States for several years.

Table 7.1 U.S. Personal Consumption Expenditures

Year	Dentists (billions)	Health Insurance (billions)
2001	66.8	89.4
2002	72.2	96.6
2003	74.6	112.8
2004	80.2	129.5
2005	85.0	141.3
2006	91.1	146.7
2007	95.8	153.2

Source: U.S. Department of Commerce Bureau of Economic Analysis, Table 2.5.5. Personal Consumption Expenditures by Type of Expenditure, Last Revised on August 6, 2008.

EXAMPLE 7 Estimating Personal Expenditures with Linear Models

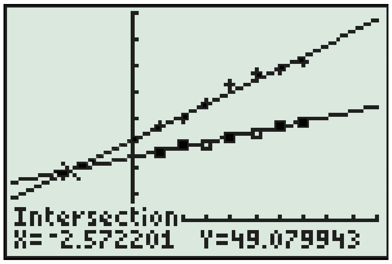
(a) Find linear regression equations for the U.S. personal consumption expenditures for dentists and health insurance in Table 7.1. Superimpose their graphs on a scatter plot of the data.

(b) Use the models in part (a) to estimate when the U.S. personal consumption expenditures for dentists was the same as that for health insurance and the corresponding amount.

SOLUTION

(a) Let $x = 0$ stand for 2000, $x = 1$ for 2001, and so forth. We use a graphing calculator to find linear regression equations for the amount of expenditures for dentists, y_D, and the amount of expenditures for health insurance, y_{HI}:

$$y_D \approx 4.8286x + 61.5$$
$$y_{HI} \approx 11.4321x + 78.4857$$



[–5, 10] by [–25, 200]

FIGURE 7.6 The scatter plot and regression equations for the data in Table 7.1. Dentist (□), health insurance (+). (Example 7)

Figure 7.6 shows the two regression equations together with a scatter plot of the two sets of data.

(b) Figure 7.6 shows that the graphs of y_D and y_{HI} intersect at approximately $(-2.57, 49.08)$. $x = -3$ stands for 1997, so Figure 7.6 suggests that the personal consumption expenditures for dentists and for health insurance were both about 49.08 billion sometime during 1997. *Now try Exercise 45.*

Suppliers will usually increase production, x, if they can get higher prices, p, for their products. So, as one variable increases, the other also increases. Normal mathematical practice would be to use p as the independent variable and x as the dependent variable. However, most economists put x on the horizontal axis and p on the vertical axis. In keeping with this practice, we write $p = f(x)$ for a **supply curve**. On one hand, as the price increases (vertical axis) so does the willingness for suppliers to increase production x (horizontal axis).

On the other hand, the demand, x, for a product by consumers will decrease as the price, p, goes up. So, as one variable increases, the other decreases. Again economists put x (demand) on the horizontal axis and p (price) on the vertical axis, even though it seems as though p should be the dependent variable. In keeping with this practice, we write $p = g(x)$ for a **demand curve**.

Finally, a point where the supply curve and demand curve intersect is an **equilibrium point**. The corresponding price is the **equilibrium price**.

EXAMPLE 8 Determining the Equilibrium Price

Nibok Manufacturing has determined that production and price of a new tennis shoe should be geared to the equilibrium point for this system of equations.

$$p = 160 - 5x \qquad \text{Demand curve}$$
$$p = 35 + 20x \qquad \text{Supply curve}$$

The price, p, is in dollars and the number of shoes, x, is in millions of pairs. Find the equilibrium point.

SOLUTION

We use substitution to solve the system.

$$160 - 5x = 35 + 20x$$
$$25x = 125$$
$$x = 5$$

Substitute this value of x into the demand curve and solve for p.

$$p = 160 - 5x$$
$$p = 160 - 5(5) = 135$$

The equilibrium point is $(5, 135)$. The equilibrium price is \$135, the price for which supply and demand will be equal at 5 million pairs of tennis shoes.

Now try Exercise 43.

QUICK REVIEW 7.1 *(For help, go to Sections P.4 and P.5.)*

In Exercises 1 and 2, solve for y in terms of x.

1. $2x + 3y = 5$ **2.** $xy + x = 4$

In Exercises 3–6, solve the equation algebraically.

3. $3x^2 - x - 2 = 0$ **4.** $2x^2 + 5x - 10 = 0$

5. $x^3 = 4x$ **6.** $x^3 + x^2 = 6x$

7. Write an equation for the line through the point $(-1, 2)$ and parallel to the line $4x + 5y = 2$.

8. Write an equation for the line through the point $(-1, 2)$ and perpendicular to the line $4x + 5y = 2$.

9. Write an equation equivalent to $2x + 3y = 5$ with coefficient of x equal to -4.

10. Find the points of intersection of the graphs of $y = 3x$ and $y = x^3 - 6x$ graphically.

SECTION 7.1 EXERCISES

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1 and 2, determine whether the ordered pair is a solution of the system.

1. $5x - 2y = 8$
$2x - 3y = 1$

(a) $(0, 4)$ **(b)** $(2, 1)$

(c) $(-2, -9)$

2. $y = x^2 - 6x + 5$
$y = 2x - 7$

(a) $(2, -3)$ **(b)** $(1, -5)$

(c) $(6, 5)$

In Exercises 3–12, solve the system by substitution.

3. $x + 2y = 5$
$y = -2$

4. $x = 3$
$x - y = 20$

5. $3x + y = 20$
$x - 2y = 10$

6. $2x - 3y = -23$
$x + y = 0$

7. $2x - 3y = -7$
$4x + 5y = 8$

8. $3x + 2y = -5$
$2x - 5y = -16$

9. $x - 3y = 6$
$-2x + 6y = 4$

10. $3x - y = -2$
$-9x + 3y = 6$

11. $y = x^2$
$y - 9 = 0$

12. $x = y + 3$
$x - y^2 = 3y$

In Exercises 13–18, solve the system algebraically. Support your answer graphically.

13. $y = 6x^2$
$7x + y = 3$

14. $y = 2x^2 + x$
$2x + y = 20$

15. $y = x^3 - x^2$
$y = 2x^2$

16. $y = x^3 + x^2$
$y = -x^2$

17. $x^2 + y^2 = 9$
$x - 3y = -1$

18. $x^2 + y^2 = 16$
$4x + 7y = 13$

In Exercises 19–26, solve the system by elimination.

19. $x - y = 10$
$x + y = 6$

20. $2x + y = 10$
$x - 2y = -5$

21. $3x - 2y = 8$
$5x + 4y = 28$

22. $4x - 5y = -23$
$3x + 4y = 6$

23. $2x - 4y = -10$
$-3x + 6y = -21$

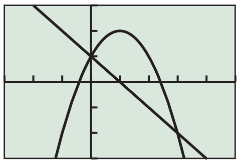
24. $2x - 4y = 8$
$-x + 2y = -4$

25. $2x - 3y = 5$
$-6x + 9y = -15$

26. $2x - y = 3$
$-4x + 2y = 5$

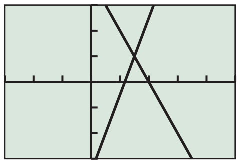
In Exercises 27–30, use the graph to estimate any solutions of the system. Confirm by substitution.

27. $y = 1 + 2x - x^2$
$y = 1 - x$



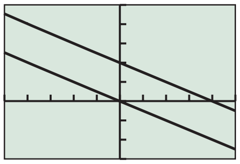
[–3, 5] by [–3, 3]

28. $6x - 2y = 7$
$2x + y = 4$



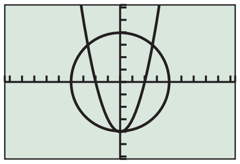
[–3, 5] by [–3, 3]

29. $x + 2y = 0$
$0.5x + y = 2$



[–5, 5] by [–3, 5]

30. $x^2 + y^2 = 16$
$y + 4 = x^2$



[–9.4, 9.4] by [–6.2, 6.2]

In Exercises 31–34, use graphs to determine the number of solutions for the system.

31. $3x + 5y = 7$
$4x - 2y = -3$

32. $3x - 9y = 6$
$2x - 6y = 1$

33. $2x - 4y = 6$
$3x - 6y = 9$

34. $x - 7y = 9$
$3x + 4y = 1$

In Exercises 35–42, solve the system graphically. Support your answer numerically using substitution and a calculator.

35. $y = \ln x$
$1 = 2x + y$

36. $y = 3 \cos x$
$1 = 2x - y$

37. $y = x^3 - 4x$
$4 = x - 2y$

38. $y = x^2 - 3x - 5$
$1 = 2x - y$

39. $x^2 + y^2 = 4$
$x + 2y = 2$

40. $x^2 + y^2 = 4$
$x - 2y = 2$

41. $x^2 + y^2 = 9$
$y = x^2 - 2$

42. $x^2 + y^2 = 9$
$y = 2 - x^2$

In Exercises 43 and 44, find the equilibrium point for the given demand and supply curve.

43. $p = 200 - 15x$
$p = 50 + 25x$

44. $p = 15 - \dfrac{7}{100}x$
$p = 2 + \dfrac{3}{100}x$

45. Medical Research Expenditures Table 7.2 shows expenditures (in billions of dollars) for public medical research costs for several years. Let $x = 0$ stand for 2000, $x = 1$ stand for 2001, and so forth.

(a) Find the quadratic regression equation and superimpose its graph on a scatter plot of the data.

(b) Find the logistic regression equation and superimpose its graph on a scatter plot of the data.

(c) When will the two models predict expenditures of 42 billion dollars?

(d) Writing to Learn Explain the long-range implications of using the quadratic regression equation to predict future expenditures.

(e) Writing to Learn Explain the long-range implications of using the logistic regression equation to predict future expenditures.

Table 7.2 National Public Expenditures on Medical Research

Year	Expenditures (billions)
2000	23.1
2001	26.0
2002	29.5
2003	32.2
2004	35.4
2005	36.9
2006	37.8

Source: U.S. Government Census Bureau, Table 125. National Health Expenditures by Type: 1990 to 2006.

46. Personal Income Table 7.3 gives the total personal income (in billions of dollars) for residents of the states of Iowa and Nevada for several years. Let $x = 0$ stand for 2000, $x = 1$ stand for 2001, and so forth.

(a) Find the linear regression equation for the Iowa data and superimpose its graph on a scatter plot of the Iowa data.

(b) Find the linear regression equation for the Nevada data and superimpose its graph on a scatter plot of the Nevada data.

(c) When will Nevada's personal income be about 14 billion dollars greater than Iowa's?

Table 7.3 Total Personal Income

Year	Iowa (billions)	Nevada (billions)
2002	82.4	66.6
2006	97.2	96.5
2007	104.2	101.8
2008	110.1	104.9

Source: U.S. Total Personal Income by State, U.S. Department of Commerce Bureau of Economic Analysis, data released March 24, 2009.

47. Population Table 7.4 gives the population in thousands of the states of Florida and Indiana in selected years. Let $x = 0$ stand for 1990, $x = 1$ stand for 1991, and so forth.

(a) Find the linear regression equation for Florida's data and superimpose its graph on a scatter plot of Florida's data.

(b) Find the linear regression equation for Indiana's data and superimpose its graph on a scatter plot of Indiana's data.

(c) Using the models in parts (a) and (b), when were the populations of the two states about the same?

Table 7.4 Population

Year	Florida (thousands)	Indiana (thousands)
1990	12,938	5,544
2000	15,983	6,081
2002	16,668	6,151
2003	16,959	6,185
2004	17,343	6,219
2005	17,736	6,257
2006	18,058	6,303
2007	18,251	6,345

Source: U.S. Census Bureau, Current Population Reports, P25-1106, "Table CO-EST2001-12-00—Time Series of Intercensal State Population Estimates: April 1, 1990 to April 1, 2000"; published April 11, 2002.

48. Group Activity Describe all possibilities for the number of solutions to a system of two equations in two variables if the graphs of the two equations are **(a)** a line and a circle, and **(b)** a circle and a parabola.

49. Garden Problem Find the dimensions of a rectangle with a perimeter of 200 m and an area of 500 m^2.

50. Cornfield Dimensions Find the dimensions of a rectangular cornfield with a perimeter of 220 yd and an area of 3000 yd^2.

51. Rowing Speed Hank can row a boat 1 mi upstream (against the current) in 24 min. He can row the same distance downstream in 13 min. If both the rowing speed and current speed are constant, find Hank's rowing speed and the speed of the current.

52. Airplane Speed An airplane flying with the wind from Los Angeles to New York City takes 3.75 hr. Flying against the wind, the airplane takes 4.4 hr for the return trip. If the air distance between Los Angeles and New York is 2500 mi and the airplane speed and wind speed are constant, find the airplane speed and the wind speed.

53. Food Prices At Philip's convenience store the total cost of one medium and one large soda is \$1.74. The large soda costs \$0.16 more than the medium soda. Find the cost of each soda.

54. Nut Mixture A 5-lb nut mixture is worth \$2.80 per pound. The mixture contains peanuts worth \$1.70 per pound and cashews worth \$4.55 per pound. How many pounds of each type of nut are in the mixture?

55. Connecting Algebra and Functions Determine a and b so that the graph of $y = ax + b$ contains the two points $(-1, 4)$ and $(2, 6)$.

56. Connecting Algebra and Functions Determine a and b so that the graph of $ax + by = 8$ contains the two points $(2, -1)$ and $(-4, -6)$.

57. Rental Van Pedro has two plans to choose from to rent a van.

Company A: a flat fee of \$40 plus 10 cents a mile.

Company B: a flat fee of \$25 plus 15 cents a mile.

(a) How many miles can Pedro drive in order to be charged the same amount by the two companies?

(b) Writing to Learn Give reasons why Pedro might choose one plan over the other. Explain.

58. Salary Package Stephanie is offered two different salary options to sell major household appliances.

Plan A: a \$300 weekly salary plus 5% of her sales.

Plan B: a \$600 weekly salary plus 1% of her sales.

(a) What must Stephanie's sales be to earn the same amount on the two plans?

(b) Writing to Learn Give reasons why Stephanie might choose one plan over the other. Explain.

Standardized Test Questions

59. True or False Let a and b be real numbers. The following system of equations can have exactly two solutions:

$$\begin{aligned} 2x + 5y &= a \\ 3x - 4y &= b \end{aligned}$$

Justify your answer.

60. True or False If the resulting equation after using elimination correctly on a system of two linear equations in two variables is $7 = 0$, then the system has infinitely many solutions. Justify your answer.

In Exercises 61–64, solve the problem without using a calculator.

61. Multiple Choice Which of the following is a solution of the system $2x - 3y = 12$
$x + 2y = -1$?

(A) $(-3, 1)$ **(B)** $(-1, 0)$ **(C)** $(3, -2)$
(D) $(3, 2)$ **(E)** $(6, 0)$

62. Multiple Choice Which of the following cannot be the number of solutions of a system of two equations in two variables whose graphs are a circle and a parabola?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

63. Multiple Choice Which of the following cannot be the number of solutions of a system of two equations in two variables whose graphs are parabolas?

(A) 1 **(B)** 2 **(C)** 4

(D) 5 **(E)** Infinitely many

64. Multiple Choice Which of the following is the number of solutions of a system of two linear equations in two variables if the resulting equation after using elimination correctly is $4 = 4$?

(A) 0 **(B)** 1 **(C)** 2

(D) 3 **(E)** Infinitely many

Explorations

65. An Ellipse and a Line Consider the system of equations

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$
$$x + y = 1.$$

(a) Solve the equation $x^2/4 + y^2/9 = 1$ for y in terms of x to determine the two implicit functions determined by the equation.

(b) Solve the system of equations graphically.

(c) Use substitution to confirm the solutions found in part (b).

66. A Hyperbola and a Line Consider the system of equations

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$
$$x - y = 0.$$

(a) Solve the equation $x^2/4 - y^2/9 = 1$ for y in terms of x to determine the two implicit functions determined by the equation.

(b) Solve the system of equations graphically.

(c) Use substitution to confirm the solutions found in part (b).

Extending the Ideas

In Exercises 67 and 68, use the elimination method to solve the system of equations.

67. $x^2 - 2y = -6$
$x^2 + y = 4$

68. $x^2 + y^2 = 1$
$x^2 - y^2 = 1$

In Exercises 69 and 70, $p(x)$ is the demand curve. The total revenue if x units are sold is $R = px$. Find the number of units sold that gives the maximum revenue.

69. $p = 100 - 4x$

70. $p = 80 - x^2$