

What you'll learn about

- Deriving the Law of Cosines
- Solving Triangles (SAS, SSS)
- Triangle Area and Heron's Formula
- Applications

... and why

The Law of Cosines is an important extension of the Pythagorean Theorem, with many applications.

5.6 The Law of Cosines

Deriving the Law of Cosines

Having seen the Law of Sines, you will probably not be surprised to learn that there is a Law of Cosines. There are many such parallels in mathematics. What you might find surprising is that the Law of Cosines has absolutely no resemblance to the Law of Sines. Instead, it resembles the Pythagorean Theorem. In fact, the Law of Cosines is often called the "generalized Pythagorean Theorem" because it contains that classic theorem as a special case.

Law of Cosines

Let $\triangle ABC$ be any triangle with sides and angles labeled in the usual way (Figure 5.22).

Then

 $a² = b² + c² - 2bc \cos A$ $b² = a² + c² - 2ac \cos B$ $c² = a² + b² - 2ab \cos C$

We derive only the first of the three equations, since the other two are derived in exactly the same way. Set the triangle in a coordinate plane so that the angle that appears in the formula (in this case, A) is at the origin in standard position, with side c along the positive x-axis. Depending on whether angle A is right (Figure 5.23a), acute (Figure 5.23b), or obtuse (Figure 5.23c), the point C will be on the y-axis, in QI, or in QII.



FIGURE 5.23 Three cases for proving the Law of Cosines.

In each of these three cases, C is a point on the terminal side of angle A in standard position, at distance b from the origin. Denote the coordinates of C by (x, y). By our definitions for trigonometric functions of any angle (Section 4.3), we can conclude that

$$\frac{x}{b} = \cos A$$
 and $\frac{y}{b} = \sin A$,

and therefore

$$x = b \cos A$$
 and $y = b \sin A$.



FIGURE 5.22 A triangle with the usual labeling (angles A, B, C; opposite sides a, b, c).

Now set *a* equal to the distance from *C* to *B* using the distance formula:

$$a = \sqrt{(x - c)^2 + (y - 0)^2}$$

Distance formula
$$a^2 = (x - c)^2 + y^2$$

Square both sides.
$$= (b \cos A - c)^2 + (b \sin A)^2$$

Substitution
$$= b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A$$

$$= b^2 (\cos^2 A + \sin^2 A) + c^2 - 2bc \cos A$$

$$= b^2 + c^2 - 2bc \cos A$$

Pythagorean identity

Solving Triangles (SAS, SSS)

While the Law of Sines is the tool we use to solve triangles in the AAS and ASA cases, the Law of Cosines is the required tool for SAS and SSS. (Both methods can be used in the SSA case, but remember that there might be 0, 1, or 2 triangles.)

EXAMPLE 1 Solving a Triangle (SAS)

Solve $\triangle ABC$ given that a = 11, b = 5, and $C = 20^{\circ}$. (See Figure 5.24.)

SOLUTION

$$c^{2} = a^{2} + b^{2} - 2ab \cos C$$

= 11² + 5² - 2(11)(5) cos 20°
= 42.6338...
$$c = \sqrt{42.6338...} \approx 6.5$$

We could now use either the Law of Cosines or the Law of Sines to find one of the two unknown angles. As a general rule, it is better to use the Law of Cosines to find angles, since the arccosine function will distinguish obtuse angles from acute angles.

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$

$$11^{2} = 5^{2} + (6.529...)^{2} - 2(5)(6.529...)\cos A$$

$$\cos A = \frac{5^{2} + (6.529...)^{2} - 11^{2}}{2(5)(6.529...)}$$

$$A = \cos^{-1} \left(\frac{5^{2} + (6.529...)^{2} - 11^{2}}{2(5)(6.529...)}\right)$$

$$\approx 144.8^{\circ}$$

$$B = 180^{\circ} - 144.8^{\circ} - 20^{\circ}$$

$$= 15.2^{\circ}$$

So the six parts of the triangle are:

 $A = 144.8^{\circ}$ a = 11

 $B = 15.2^{\circ}$ b = 5

 $C = 20^{\circ}$ $c \approx 6.5$

Now try Exercise 1.

EXAMPLE 2 Solving a Triangle (SSS) Solve $\triangle ABC$ if a = 9, b = 7, and c = 5. (See Figure 5.25.)



FIGURE 5.24 A triangle with two sides and an included angle known. (Example 1)



FIGURE 5.25 A triangle with three sides known. (Example 2)

SOLUTION We use the Law of Cosines to find two of the angles. The third angle can be found by subtraction from 180°.

$$a^{2} = b^{2} + c^{2} - 2bc \cos A \qquad b^{2} = a^{2} + c^{2} - 2ac \cos B$$

$$9^{2} = 7^{2} + 5^{2} - 2(7)(5)\cos A \qquad 7^{2} = 9^{2} + 5^{2} - 2(9)(5)\cos B$$

$$70 \cos A = -7 \qquad 90 \cos B = 57$$

$$A = \cos^{-1}(-0.1) \qquad B = \cos^{-1}(57/90)$$

$$\approx 95.7^{\circ} \qquad \approx 50.7^{\circ}$$

Then $C = 180^{\circ} - 95.7^{\circ} - 50.7^{\circ} = 33.6^{\circ}$.
Now try Exercise 3.

Triangle Area and Heron's Formula

The same parts that determine a triangle also determine its area. If the parts happen to be two sides and an included angle (SAS), we get a simple area formula in terms of those three parts that does not require finding an altitude.

Observe in Figure 5.23 (used in explaining the Law of Cosines) that each triangle has base *c* and altitude $y = b \sin A$. Applying the standard area formula, we have

$$\Delta \operatorname{Area} = \frac{1}{2}(base)(height) = \frac{1}{2}(c)(b\sin A) = \frac{1}{2}bc\sin A$$

This is actually three formulas in one, as it does not matter which side we use as the base.

Area of a Triangle

$$\Delta \text{ Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

• **EXAMPLE 3** Finding the Area of a Regular Polygon

Find the area of a regular octagon (8 equal sides, 8 equal angles) inscribed inside a circle of radius 9 inches.

SOLUTION Figure 5.26 shows that we can split the octagon into 8 congruent triangles. Each triangle has two 9-inch sides with an included angle of $\theta = 360^{\circ}/8 = 45^{\circ}$. The area of each triangle is

$$\Delta$$
 Area = (1/2)(9)(9) sin 45° = (81/2) sin 45° = 81 $\sqrt{2}/4$.

Therefore, the area of the octagon is

Area = 8
$$\Delta$$
 Area = $162\sqrt{2} \approx 229$ square inches

Now try Exercise 31.

There is also an area formula that can be used when the three sides of the triangle are known.

Although Heron proved this theorem using only classical geometric methods, we prove it, as most people do today, by using the tools of trigonometry.



FIGURE 5.26 A regular octagon inscribed inside a circle of radius 9 inches. (Example 3)

Heron's Formula

The formula is named after Heron of Alexandria, whose proof of the formula is the oldest on record, but ancient Arabic scholars claimed to have known it from the works of Archimedes of Syracuse centuries earlier. Archimedes (c. 287–212 B.C.E.) is considered to be the greatest mathematician of all antiquity.

THEOREM Heron's Formula

Let a, b, and c be the sides of $\triangle ABC$, and let s denote the **semiperimeter**. (a + b + c)/2. Then the area of $\triangle ABC$ is given by Area = $\sqrt{s(s-a)(s-b)(s-c)}$. Proof Area $=\frac{1}{2}ab\sin C$ $4(\text{Area}) = 2ab \sin C$ $16(\text{Area})^2 = 4a^2b^2\sin^2 C$ $= 4a^2b^2(1 - \cos^2 C)$ Pythagorean identity $=4a^{2}b^{2}-4a^{2}b^{2}\cos^{2}C$ $= 4a^{2}b^{2} - (2ab\cos C)^{2}$ $= 4a^{2}b^{2} - (a^{2} + b^{2} - c^{2})^{2}$ Law of Cosines $= (2ab - (a^{2} + b^{2} - c^{2}))(2ab + (a^{2} + b^{2} - c^{2}))$ Difference of squares $= (c^{2} - (a^{2} - 2ab + b^{2}))((a^{2} + 2ab + b^{2}) - c^{2})$ $= (c^{2} - (a - b)^{2})((a + b)^{2} - c^{2})$ = (c - (a - b))(c + (a - b))((a + b) - c)((a + b) + c)Difference of squares = (c - a + b)(c + a - b)(a + b - c)(a + b + c)= (2s - 2a)(2s - 2b)(2s - 2c)(2s)2s = a + b + c $16(\text{Area})^2 = 16s(s-a)(s-b)(s-c)$ $(Area)^2 = s(s - a)(s - b)(s - c)$ Area = $\sqrt{s(s-a)(s-b)(s-c)}$

EXAMPLE 4 Using Heron's Formula

Find the area of a triangle with sides 13, 15, 18.

SOLUTION First we compute the semiperimeter: s = (13 + 15 + 18)/2 = 23.

Then we use Heron's Formula

rea =
$$\sqrt{23}(23 - 13)(23 - 15)(23 - 18)$$

= $\sqrt{23 \cdot 10 \cdot 8 \cdot 5} = \sqrt{9200} - 20\sqrt{23}$.

The approximate area is 96 square units.

A

Now try Exercise 21.

Applications

We end this section with a few applications.

EXAMPLE 5 Measuring a Baseball Diamond

The bases on a baseball diamond are 90 feet apart, and the front edge of the pitcher's rubber is 60.5 feet from the back corner of home plate. Find the distance from the center of the front edge of the pitcher's rubber to the far corner of first base.

(continued)



FIGURE 5.27 The diamond-shaped part of a baseball diamond. (Example 5)

Platonic Solids

The regular tetrahedron in Example 6 is one of only 5 *regular* solids (solids with faces that are congruent polygons having equal angles and equal sides). The others are the cube (6 square faces), the octahedron (8 triangular faces), the dodecahedron (12 pentagonal faces), and the icosahedron (20 triangular faces). Referred to as the *Platonic* solids, Plato did not discover them, but they are featured in his cosmology as being the stuff of which everything in the universe is made. The Platonic universe itself is a dodecahedron, a favorite symbol of the Pythagoreans.



FIGURE 5.28 The measure of $\angle ABC$ is the same as the measure of any dihedral angle formed by two of the tetrahedron's faces. (Example 6)

SOLUTION Figure 5.27 shows first base as *A*, the pitcher's rubber as *B*, and home plate as *C*. The distance we seek is side *c* in $\triangle ABC$.

By the Law of Cosines,

$$c^{2} = 60.5^{2} + 90^{2} - 2(60.5)(90) \cos 45^{\circ}$$

$$c = \sqrt{60.5^{2} + 90^{2} - 2(60.5)(90) \cos 45^{\circ}}$$

$$\approx 63.7$$

The distance from first base to the pitcher's rubber is about 63.7 feet. Now try Exercise 37.

• **EXAMPLE 6** Measuring a Dihedral Angle (Solid Geometry)

A regular tetrahedron is a solid with four faces, each of which is an equilateral triangle. Find the measure of the *dihedral angle* formed along the common edge of two intersecting faces of a regular tetrahedron with edges of length 2.

SOLUTION Figure 5.28 shows the tetrahedron. Point *B* is the midpoint of edge *DE*, and *A* and *C* are the endpoints of the opposite edge. The measure of $\angle ABC$ is the same as the measure of the dihedral angle formed along edge *DE*, so we will find the measure of $\angle ABC$.

Because both $\triangle ADB$ and $\triangle CDB$ are $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangles, *AB* and *BC* both have length $\sqrt{3}$. If we apply the Law of Cosines to $\triangle ABC$, we obtain

$$2^{2} = (\sqrt{3})^{2} + (\sqrt{3})^{2} - 2\sqrt{3}\sqrt{3}\cos\left(\angle ABC\right)$$
$$\cos\left(\angle ABC\right) = \frac{1}{3}$$
$$\angle ABC = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.53^{\circ}$$

The dihedral angle has the same measure as $\angle ABC$, approximately 70.53°. (We chose sides of length 2 for computational convenience, but in fact this is the measure of a dihedral angle in a regular tetrahedron of any size.)

Now try Exercise 43.

EXPLORATION 1 Estimating Acreage of a Plot of Land

Jim and Barbara are house hunting and need to estimate the size of an irregular adjacent lot that is described by the owner as "a little more than an acre." With Barbara stationed at a corner of the plot, Jim starts at another corner and walks a straight line toward her, counting his paces. They then shift corners and Jim paces again, until they have recorded the dimensions of the lot (in paces) as in Figure 5.29. They later measure Jim's pace as 2.2 feet. What is the approximate acreage of the lot?

- 1. Use Heron's Formula to find the area in square paces.
- 2. Convert the area to square feet, using the measure of Jim's pace.
- **3.** There are 5280 feet in a mile. Convert the area to square miles.
- 4. There are 640 square acres in a square mile. Convert the area to acres.
- 5. Is there good reason to doubt the owner's estimate of the acreage of the lot?
- 6. Would Jim and Barbara be able to modify their system to estimate the area of an irregular lot with five straight sides?



FIGURE 5.29 Dimensions (in paces) of an irregular plot of land. (Exploration 1)



Chapter Opener Problem (from page 403)

Problem: Because deer require food, water, cover for protection from weather and predators, and living space for healthy survival, there are natural limits to the number of deer that a given plot of land can support. Deer populations in national parks average 14 animals per square kilometer. If a triangular region with sides of 3 kilometers, 4 kilometers, and 6 kilometers has a population of 50 deer, how close is the population on this land to the average national park population?

Solution: We can find the area of the land region

by using Heron's Formula with

$$s = (3 + 4 + 6)/2 = 13/2$$

and

Area =
$$\sqrt{s(s-a)(s-b)(s-c)}$$

= $\sqrt{\frac{13}{2}\left(\frac{13}{2}-3\right)\left(\frac{13}{2}-4\right)\left(\frac{13}{2}-6\right)}$
= $\sqrt{\frac{13}{2}\left(\frac{7}{2}\right)\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)} \approx 5.3,$

so the area of the land region is 5.3 km^2 .

If this land were to support 14 deer/km², it would have $(5.3...km^2)(14 \text{ deer/km}^2) = 74.7 \approx 75 \text{ deer}$. Thus, the land supports 25 deer less than the average.

QUICK REVIEW 5.6 (For help, go to Sections 2.4 and 4.7.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, find an angle between 0° and 180° that is a solution to the equation.

1.	$\cos A = 3/5$	2.	$\cos C = -0.23$
3.	$\cos A = -0.68$	4.	$3\cos C = 1.92$

In Exercises 5 and 6, solve the equation (in terms of x and y) for (a) $\cos A$ and (b) $A, 0 \le A \le 180^{\circ}$.

5.
$$9^2 = x^2 + y^2 - 2xy \cos A$$
 6. $y^2 = x^2 + 25 - 10 \cos A$

In Exercises 7–10, find a quadratic polynomial with real coefficients that satisfies the given condition.

- 7. Has two positive zeros
- 8. Has one positive and one negative zero
- 9. Has no real zeros
- **10.** Has exactly one positive zero

SECTION 5.6 EXERCISES

In Exercises 1–4, solve the triangle.



In Exercises 5–16, solve the triangle.

5. $A = 55^{\circ}$, b = 12, c = 7 **6.** $B = 35^{\circ}$, a = 43, c = 19 **7.** a = 12, b = 21, $C = 95^{\circ}$ **8.** b = 22, c = 31, $A = 82^{\circ}$ **9.** a = 1, b = 5, c = 4 **10.** a = 1, b = 5, c = 8 **11.** a = 3.2, b = 7.6, c = 6.4 **12.** a = 9.8, b = 12, c = 23 **13.** $A = 42^{\circ}$, a = 7, b = 10 **14.** $A = 57^{\circ}$, a = 11, b = 10 **15.** $A = 63^{\circ}$, a = 8.6, b = 11.1**16.** $A = 71^{\circ}$, a = 9.3, b = 8.5

In Exercises 17–20, find the area of the triangle.

17. $A = 47^{\circ}$,	b = 32 ft, c	= 19 ft
18. $A = 52^{\circ}$,	b = 14 m, c	c = 21 m
19. $B = 101^{\circ}$,	a = 10 cm,	c = 22 cm
20. $C = 112^{\circ}$,	a = 1.8 in.,	b = 5.1 in.

In Exercises 21–28, decide whether a triangle can be formed with the given side lengths. If so, use Heron's Formula to find the area of the triangle.

21. a = 4, b = 5, c = 8 **22.** a = 5, b = 9, c = 7 **23.** a = 3, b = 5, c = 8 **24.** a = 23, b = 19, c = 12 **25.** a = 19.3, b = 22.5, c = 31 **26.** a = 8.2, b = 12.5, c = 28 **27.** a = 33.4, b = 28.5, c = 22.3**28.** a = 18.2, b = 17.1, c = 12.3

- **29.** Find the radian measure of the largest angle in the triangle with sides of 4, 5, and 6.
- **30.** A parallelogram has sides of 18 and 26 ft, and an angle of 39°. Find the shorter diagonal.

- **31.** Find the area of a regular hexagon inscribed in a circle of radius 12 inches.
- **32.** Find the area of a regular nonagon (9 sides) inscribed in a circle of radius 10 inches.
- **33.** Find the area of a regular hexagon circumscribed about a circle of radius 12 inches. [*Hint*: Start by finding the distance from a vertex of the hexagon to the center of the circle.]
- **34.** Find the area of a regular nonagon (9 sides) circumscribed about a circle of radius 10 inches.

35. Measuring Distance Indirectly

Juan wants to find the distance between two points A and Bon opposite sides of a building. A ---He locates a point C that is 110 ft from A and 160 ft from B, as illustrated in the figure. If the angle at C is 54°, find distance AB. 110 ft



36. Designing a Baseball Field

- (a) Find the distance from the center of the front edge of the pitcher's rubber to the far corner of second base. How does this distance compare with the distance from the pitcher's rubber to first base? (See Example 5.)
- **(b)** Find $\angle B$ in $\triangle ABC$.



- **37. Designing a Softball Field** In softball, adjacent bases are 60 ft apart. The distance from the center of the front edge of the pitcher's rubber to the far corner of home plate is 40 ft.
 - (a) Find the distance from the center of the pitcher's rubber to the far corner of first base.
 - (b) Find the distance from the center of the pitcher's rubber to the far corner of second base.
 - (c) Find $\angle B$ in $\triangle ABC$.



38. Surveyor's Calculations Tony must find the distance from *A* to *B* on opposite sides of a lake. He locates a point *C* that is 860 ft from *A* and 175 ft from *B*. He measures the angle at *C* to be 78° . Find distance *AB*.



- **39. Construction Engineering** A manufacturer is designing the roof truss that is modeled in the figure shown.
 - (a) Find the measure of $\angle CAE$.
 - (b) If AF = 12 ft, find the length DF.
 - (c) Find the length *EF*.



- **40. Navigation** Two airplanes flying together in formation take off in different directions. One flies due east at 350 mph, and the other flies east-northeast at 380 mph. How far apart are the two airplanes 2 hr after they separate, assuming that they fly at the same altitude?
- **41. Football Kick** The player waiting to receive a kickoff stands at the 5 yard line (point *A*) as the ball is being kicked 65 yd up the field from the opponent's 30 yard line. The kicked ball travels 73 yd at an angle of 8° to the right of the receiver, as shown in the figure (point *B*). Find the distance the receiver runs to catch the ball.



42. Group Activity Architectural

Design Building Inspector Julie Wang checks a building in the shape of a regular octagon, each side 20 ft long. She checks that the contractor has located the corners of the foundation correctly by measuring several of the diagonals. Calculate what the lengths of diagonals *HB*, *HC*, and *HD* should be.



43. Connecting Trigonometry and Geometry $\angle CAB$ is $\frac{1}{1}$ ft

inscribed in a rectangular box whose sides are 1, 2, and 3 ft long as shown. Find the measure of $\angle CAB$.



44. Group Activity Connecting Trigonometry and Geometry

> A cube has edges of length 2 ft. Point *A* is the midpoint of an edge. Find the measure of $\angle ABC$.



Standardized Test Questions

- **45.** True or False If $\triangle ABC$ is any triangle with sides and angles labeled in the usual way, then $b^2 + c^2 > 2bc \cos A$. Justify your answer.
- **46. True or False** If *a*, *b*, and θ are two sides and an included angle of a parallelogram, the area of the parallelogram is *ab* sin θ . Justify your answer.

You may use a graphing calculator when answering these questions.

47. Multiple Choice What is the area of a regular dodecagon (12-sided figure) inscribed in a circle of radius 12?

(A) 427 (B) 432 (C) 437 (D) 442 (E) 447

48. Multiple Choice The area of a triangle with sides 7, 8, and 9 is

(A) $6\sqrt{15}$. (B) $12\sqrt{5}$. (C) $16\sqrt{3}$. (D) $17\sqrt{3}$. (E) $18\sqrt{3}$.

- **49. Multiple Choice** Two boats start at the same point and speed away along courses that form a 110° angle. If one boat travels at 24 miles per hour and the other boat travels at 32 miles per hour, how far apart are the boats after 30 minutes?
 - (A) 21 miles (B) 22 miles (C) 23 miles
 - **(D)** 24 miles **(E)** 25 miles
- **50. Multiple Choice** What is the measure of the smallest angle in a triangle with sides 12, 17, and 25?

(A) 21° (B) 22° (C) 23° (D) 24° (E) 25°

Explorations

51. Find the area of a regular polygon with *n* sides inscribed inside a circle of radius *r*. (Express your answer in terms of *n* and *r*.)

52. (a) Prove the identity:
$$\frac{\cos A}{a} = \frac{b^2 + c^2 - a^2}{2abc}$$
.

(b) Prove the (tougher) identity:

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$

[*Hint*: Use the identity in part (a), along with its other variations.]

53. Navigation Two ships leave a common port at 8:00 A.M. and travel at a constant rate of speed. Each ship keeps a log showing its distance from port and its distance from the other ship. Portions of the logs from later that morning for both ships are shown in the following tables.



Time	Naut mi from port	Naut mi from ship B	Time	Naut mi from port	Naut mi from ship A
9:00	15.1	8.7	9:00	12.4	8.7
10:00	30.2	17.3	11:00	37.2	26.0

- (a) How fast is each ship traveling? (Express your answer in knots, which are nautical miles per hour.)
- (b) What is the angle of intersection of the courses of the two ships?
- (c) How far apart are the ships at 12:00 noon if they maintain the same courses and speeds?

Extending the Ideas

54. Prove that the area of a triangle can be found with the formula

$$\Delta \text{ Area} = \frac{a^2 \sin B \sin C}{2 \sin A}$$

55. A **segment** of a circle is the region enclosed between a chord of a circle and the arc intercepted by the chord. Find the area of a segment intercepted by a 7-inch chord in a circle of radius 5 inches.



CHAPTER 5 Key Ideas

Properties, Theorems, and Formulas

Reciprocal Identities 404 Quotient Identities 404 Pythagorean Identities 405 Cofunction Identities 406 Odd-Even Identities 407 Sum/Difference Identities 422–424 Double-Angle Identities 428 Power-Reducing Identities 428 Half-Angle Identities 430 Law of Sines 434 Law of Cosines 442 Triangle Area 444 Heron's Formula 445

Procedures

Strategies for Proving an Identity 413-415

CHAPTER 5 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1 and 2, write the expression as the sine, cosine, or tangent of an angle.

1. $2 \sin 100^{\circ} \cos 100^{\circ}$ **2.** $\frac{2 \tan 40^{\circ}}{1 - \tan^2 40^{\circ}}$ In Exercises 3 and 4, simplify the expression to a single term. Support your answer graphically.

3.
$$(1 - 2\sin^2\theta)^2 + 4\sin^2\theta\cos^2\theta$$

4. $1 - 4\sin^2x\cos^2x$

In Exercises 5–22, prove the identity.

5. $\cos 3x = 4 \cos^3 x - 3 \cos x$ 6. $\cos^2 2x - \cos^2 x = \sin^2 x - \sin^2 2x$ 7. $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$