

## What you'll learn about

- More Right Triangle Problems
- Simple Harmonic Motion

## ... and why

These problems illustrate some of the better-known applications of trigonometry.

# 4.8 Solving Problems with Trigonometry

## More Right Triangle Problems

We close this first of two trigonometry chapters by revisiting some of the applications of Section 4.2 (right triangle trigonometry) and Section 4.4 (sinusoids).

An **angle of elevation** is the angle through which the eye moves up from horizontal to look at something above, and an **angle of depression** is the angle through which the eye moves down from horizontal to look at something below. For two observers at different elevations looking at each other, the angle of elevation for one equals the angle of depression for the other. The concepts are illustrated in Figure 4.88 as they might apply to observers at Mount Rushmore or the Grand Canyon.



**FIGURE 4.88** (a) Angle of elevation at Mount Rushmore. (b) Angle of depression at the Grand Canyon.

## **EXAMPLE 1** Using Angle of Depression

The angle of depression of a buoy from the top of the Barnegat Bay lighthouse 130 feet above the surface of the water is  $6^\circ$ . Find the distance *x* from the base of the lighthouse to the buoy.

**SOLUTION** Figure 4.89 models the situation.

In the diagram,  $\theta = 6^{\circ}$  because the angle of elevation from the buoy equals the angle of depression from the lighthouse. We **solve algebraically** using the tangent function:

$$\tan \theta = \tan 6^\circ = \frac{130}{x}$$
$$x = \frac{130}{\tan 6^\circ} \approx 1236.9$$

Interpreting We find that the buoy is about 1237 feet from the base of the lighthouse. Now try Exercise 3.



**FIGURE 4.89** A big lighthouse and a little buoy. (Example 1)



**FIGURE 4.90** A car approaches Altgelt Hall. (Example 2)



**FIGURE 4.91** A large, helium-filled penguin. (Example 3)



FIGURE 4.92 (Example 3)

## **EXAMPLE 2** Making Indirect Measurements

From the top of the 100-ft-tall Altgelt Hall a man observes a car moving toward the building. If the angle of depression of the car changes from 22° to 46° during the period of observation, how far does the car travel?

#### SOLUTION

Solve Algebraically Figure 4.90 models the situation. Notice that we have labeled the acute angles at the car's two positions as  $22^{\circ}$  and  $46^{\circ}$  (because the angle of elevation from the car equals the angle of depression from the building). Denote the distance the car moves as *x*. Denote its distance from the building at the second observation as *d*.

From the smaller right triangle we conclude:

$$\tan 46^\circ = \frac{100}{d}$$
$$d = \frac{100}{\tan 46^\circ}$$

From the larger right triangle we conclude:

$$\tan 22^\circ = \frac{100}{x+d}$$
$$x + d = \frac{100}{\tan 22^\circ}$$
$$x = \frac{100}{\tan 22^\circ} - d$$
$$x = \frac{100}{\tan 22^\circ} - \frac{100}{\tan 46^\circ}$$
$$x \approx 150.9$$

Interpreting our answer, we find that the car travels about 151 feet.

Now try Exercise 7.

## - EXAMPLE 3 Finding Height Above Ground

A large, helium-filled penguin is moored at the beginning of a parade route awaiting the start of the parade. Two cables attached to the underside of the penguin make angles of  $48^{\circ}$  and  $40^{\circ}$  with the ground and are in the same plane as a perpendicular line from the penguin to the ground. (See Figure 4.91.) If the cables are attached to the ground 10 feet from each other, how high above the ground is the penguin?

**SOLUTION** We can simplify the drawing to the two right triangles in Figure 4.92 that share the common side *h*.

Model

By the definition of the tangent function,

$$\frac{h}{x} = \tan 48^\circ$$
 and  $\frac{h}{x+10} = \tan 40^\circ$ 

Solve Algebraically

Solving for *h*,

$$h = x \tan 48^{\circ}$$
 and  $h = (x + 10) \tan 40^{\circ}$ 

(continued)

Set these two expressions for *h* equal to each other and solve the equation for *x*:

$$x \tan 48^\circ = (x + 10) \tan 40^\circ$$

$$x \tan 48^\circ = x \tan 40^\circ + 10 \tan 40^\circ$$

$$x \tan 48^\circ - x \tan 40^\circ = 10 \tan 40^\circ$$

$$x(\tan 48^\circ - \tan 40^\circ) = 10 \tan 40^\circ$$

$$x = \frac{10 \tan 40^\circ}{\tan 48^\circ - \tan 40^\circ} \approx 30.90459723$$

We retain the full display for *x* because we are not finished yet; we need to solve for *h*:

$$h = x \tan 48^\circ = (30.90459723) \tan 48^\circ \approx 34.32$$

The penguin is approximately 34 feet above ground level.

Now try Exercise 15.

#### – EXAMPLE 4 Using Trigonometry in Navigation

A U.S. Coast Guard patrol boat leaves Port Cleveland and averages 35 **knots** (nautical mph) traveling for 2 hours on a course of 53° and then 3 hours on a course of 143°. What is the boat's bearing and distance from Port Cleveland?

**SOLUTION** Figure 4.93 models the situation.

Solve Algebraically In the diagram, line *AB* is a transversal that cuts a pair of parallel lines. Thus,  $\beta = 53^{\circ}$  because they are alternate interior angles. Angle  $\alpha$ , as the supplement of a 143° angle, is 37°. Consequently,  $\angle ABC = 90^{\circ}$  and *AC* is the hypotenuse of right  $\triangle ABC$ .

Use distance = rate  $\times$  time to determine distances AB and BC.

$$AB = (35 \text{ knots})(2 \text{ hours}) = 70 \text{ nautical miles}$$

BC = (35 knots)(3 hours) = 105 nautical miles

Solve the right triangle for AC and  $\theta$ .

$$AC = \sqrt{70^2 + 105^2}$$
 Pythagorean Theorem  
 $AC \approx 126.2$   
 $\theta = \tan^{-1}\left(\frac{105}{70}\right)$   
 $\theta \approx 56.3^{\circ}$ 

Interpreting We find that the boat's bearing from Port Cleveland is  $53^\circ + \theta$ , or approximately 109.3°. They are about 126 nautical miles out.

Now try Exercise 17.

## **Simple Harmonic Motion**

Because of their periodic nature, the sine and cosine functions are helpful in describing the motion of objects that oscillate, vibrate, or rotate. For example, the linkage in Figure 4.94 converts the rotary motion of a motor to the back-and-forth motion needed for some machines. When the wheel rotates, the piston moves back and forth.

If the wheel rotates at a constant rate  $\omega$  radians per second, the back-and-forth motion of the piston is an example of *simple harmonic motion* and can be modeled by an equation of the form

$$d = a \cos \omega t, \quad \omega > 0,$$

where a is the radius of the wheel and d is the directed distance of the piston from its center of oscillation.



**FIGURE 4.93** Path of travel for a Coast Guard boat that corners well at 35 knots. (Example 4)



**FIGURE 4.94** A piston operated by a wheel rotating at a constant rate demonstrates simple harmonic motion.

For the sake of simplicity, we will define simple harmonic motion in terms of a point moving along a number line.

#### **Frequency and Period**

Notice that harmonic motion is sinusoidal, with amplitude |a| and period  $2\pi/\omega$ . The frequency is the reciprocal of the period.

#### Simple Harmonic Motion

A point moving on a number line is in simple harmonic motion if its directed distance d from the origin is given by either

 $d = a \sin \omega t$  or  $d = a \cos \omega t$ ,

where *a* and  $\omega$  are real numbers and  $\omega > 0$ . The motion has **frequency**  $\omega/2\pi$ , which is the number of oscillations per unit of time.

#### **EXPLORATION 1** Watching Harmonic Motion

You can watch harmonic motion on your graphing calculator. Set your grapher to parametric mode and set  $X_{1T} = \cos(T)$  and  $Y_{1T} = \sin(T)$ . Set Tmin = 0, Tmax = 25, Tstep = 0.2, Xmin = -1.5, Xmax = 1.5, Xscl = 1, Ymin = -100, Ymax = 100, Yscl = 0.

If your calculator allows you to change style to graph a moving ball, choose that style. When you graph the function, you will see the ball moving along the *x*-axis between -1 and 1 in simple harmonic motion. If your grapher does not have the moving ball option, wait for the grapher to finish graphing, then press TRACE and keep your finger pressed on the right arrow key to see the tracer move in simple harmonic motion.

- 1. For each value of *T*, the parametrization gives the point  $(\cos (T), \sin (T))$ . What well-known curve should this parametrization produce?
- 2. Why does the point seem to go back and forth on the *x*-axis when it should be following the curve identified in part (1)? [*Hint:* Check that viewing window again!]
- **3.** Why does the point slow down at the extremes and speed up in the middle? [*Hint:* Remember that the grapher is really following the curve identified in part (1).]
- 4. How can you tell that this point moves in simple harmonic motion?

### - **EXAMPLE 5** Calculating Harmonic Motion

In a mechanical linkage like the one shown in Figure 4.94, a wheel with an 8-cm radius turns with an angular velocity of  $8\pi$  radians/sec.

- (a) What is the frequency of the piston?
- (b) What is the distance from the starting position (t = 0) exactly 3.45 seconds after starting?

**SOLUTION** Imagine the wheel to be centered at the origin and let P(x, y) be a point on its perimeter (Figure 4.95). As the wheel rotates and P goes around, the motion of the piston follows the path of the x-coordinate of P along the x-axis. The angle determined by P at any time t is  $8\pi t$ , so its x-coordinate is  $8 \cos 8\pi t$ . Therefore, the sinusoid  $d = 8 \cos 8\pi t$  models the motion of the piston.



**FIGURE 4.95** Modeling the path of a piston by a sinusoid. (Example 5)



[0, 1] by [-10, 10]

**FIGURE 4.96** A sinusoid with frequency 4 models the motion of the piston in Example 5.

- (a) The frequency of d = 8 cos 8πt is 8π/2π, or 4. The piston makes four complete back-and-forth strokes per second. The graph of d as a function of t is shown in Figure 4.96. The four cycles of the sinusoidal graph in the interval [0, 1] model the four cycles of the motor or the four strokes of the piston. Note that the sinusoid has a period of 1/4, the reciprocal of the frequency.
- (b) We must find the distance between the positions at t = 0 and t = 3.45.

The initial position at t = 0 is

$$d(0)=8.$$

The position at t = 3.45 is

$$d(3.45) = 8 \cos(8\pi \cdot 3.45) \approx 2.47.$$

The distance between the two positions is approximately 8 - 2.47 = 5.53.

Interpreting our answer, we conclude that the piston is approximately 5.53 cm from its starting position after 3.45 seconds. *Now try Exercise 27.* 

### - **EXAMPLE 6** Calculating Harmonic Motion

A mass oscillating up and down on the bottom of a spring (assuming perfect elasticity and no friction or air resistance) can be modeled as harmonic motion. If the weight is displaced a maximum of 5 cm, find the modeling equation if it takes 2 seconds to complete one cycle. (See Figure 4.97.)



FIGURE 4.97 The mass and spring in Example 6.

**SOLUTION** We have our choice between the two equations  $d = a \sin \omega t$  or  $d = a \cos \omega t$ . Assuming that the spring is at the origin of the coordinate system when t = 0, we choose the equation  $d = a \sin \omega t$ .

Because the maximum displacement is 5 cm, we conclude that the amplitude a = 5. Because it takes 2 seconds to complete one cycle, we conclude that the period is 2 and the frequency is 1/2. Therefore,

$$\frac{\omega}{2\pi} = \frac{1}{2},$$
$$\omega = \pi.$$

Putting it all together, our modeling equation is  $d = 5 \sin \pi t$ .

Now try Exercise 29.



## **Chapter Opener Problem** (from page 319)

**Problem:** If we know that the musical note A above middle C has a pitch of 440 Hertz, how can we model the sound produced by it at 80 decibels?

**Solution:** Sound is modeled by simple harmonic motion, with frequency perceived as pitch and measured in cycles per second, and amplitude perceived as loudness and measured in decibels. So for the musical note A with a pitch of 440 hertz, we have frequency  $= \omega/2\pi = 440$  and therefore  $\omega = 2\pi 440 = 880\pi$ .

If this note is played at a loudness of 80 decibels, we have |a| = 80. Using the simple harmonic motion model  $d = a \sin \omega t$ , we have

 $d = 80 \sin 880\pi t.$ 

## **QUICK REVIEW 4.8** (For help, go to Sections 4.1, 4.2, and 4.3.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, find the lengths *a*, *b*, and *c*.



In Exercises 5 and 6, find the complement and supplement of the angle.

In Exercises 7 and 8, state the bearing that describes the direction.

7. NE (northeast)

**5.** 32°

8. SSW (south-southwest)

In Exercises 9 and 10, state the amplitude and period of the sinusoid.

9. 
$$-3 \sin 2(x - 1)$$
  
10.  $4 \cos 4(x + 2)$ 

## SECTION 4.8 EXERCISES

In Exercises 1–43, solve the problem using your knowledge of geometry and the techniques of this section. Sketch a figure if one is not provided.

1. Finding a Cathedral

**Height** The angle of elevation of the top of the Ulm Cathedral from a point 300 ft away from the base of its steeple on level ground is 60°. Find the height of the cathedral.



- **2. Finding a Monument Height** From a point 100 ft from its base, the angle of elevation of the top of the Arch of Septimus Severus, in Rome, Italy, is 34°13′12″. How tall is this monument?
- **3. Finding a Distance** The angle of depression from the top of the Smoketown Lighthouse 120 ft above the surface of the water to a buoy is 10°. How far is the buoy from the lighthouse?



- **4. Finding a Baseball Stadium Dimension** The top row of the red seats behind home plate at Cincinnati's Riverfront Stadium is 90 ft above the level of the playing field. The angle of depression to the base of the left field wall is 14°. How far is the base of the left field wall from a point on level ground directly below the top row?
- **5. Finding a Guy-Wire Length** A guy wire connects the top of an antenna to a point on level ground 5 ft from the base of the antenna. The angle of elevation formed by this wire is 80°. What are the length of the wire and the height of the antenna?



- 6. Finding a Length A wire stretches from the top of a vertical pole to a point on level ground 16 ft from the base of the pole. If the wire makes an angle of 62° with the ground, find the height of the pole and the length of the wire.
- **7. Height of Eiffel Tower** The angle of elevation of the top of the TV antenna mounted on top of the Eiffel Tower in Paris is measured to be 80°1'12" at a point 185 ft from the base of the tower. How tall is the tower plus TV antenna?
- 8. Finding the Height of Tallest Chimney The world's tallest smokestack at the International Nickel Co., Sudbury, Ontario, casts a shadow that is approximately 1580 ft long when the Sun's angle of elevation (measured from the horizon) is 38°. How tall is the smokestack?



- **9. Cloud Height** To measure the height of a cloud, you place a bright searchlight directly below the cloud and shine the beam straight up. From a point 100 ft away from the searchlight, you measure the angle of elevation of the cloud to be 83°12'. How high is the cloud?
- **10. Ramping Up** A ramp leading to a freeway overpass is 470 ft long and rises 32 ft. What is the average angle of inclination of the ramp to the nearest tenth of a degree?

- **11. Antenna Height** A guy wire attached to the top of the KSAM radio antenna is anchored at a point on the ground 10 m from the antenna's base. If the wire makes an angle of 55° with level ground, how high is the KSAM antenna?
- **12. Building Height** To determine the height of the Louisiana-Pacific (LP) Tower, the tallest building in Conroe, Texas, a surveyor stands at a point on the ground, level with the base of the LP building. He measures the point to be 125 ft from the building's base and the angle of elevation to the top of the building to be 29°48'. Find the height of the building.
- **13.** Navigation The *Paz Verde*, a whalewatch boat, is located at point *P*, and *L* is the nearest point on the Baja California shore. Point *Q* is located 4.25 mi down the shoreline from *L* and  $\overline{PL} \perp \overline{LQ}$ . Determine the distance that the *Paz Verde* is from the shore if  $\angle PQL = 35^{\circ}$ .



- 14. **Recreational Hiking** While hiking on a level path toward Colorado's front range, Otis Evans determines that the angle of elevation to the top of Long's Peak is 30°. Moving 1000 ft closer to the mountain, Otis determines the angle of elevation to be 35°. How much higher is the top of Long's Peak than Otis's elevation?
- **15. Civil Engineering** The angle of elevation from an observer to the bottom edge of the Delaware River drawbridge observation deck located 200 ft from the observer is 30°. The angle of elevation from the observer to the top of the observation deck is 40°. What is the height of the observation deck?



**16. Traveling Car** From the top of a 100-ft building a man observes a car moving toward him. If the angle of depression of the car changes from 15° to 33° during the period of observation, how far does the car travel?



17. Navigation The Coast Guard cutter Angelica travels at 30 knots from its home port of Corpus Christi on a course of 95° for 2 hr and then changes to a course of 185° for 2 hr. Find the distance and the bearing from the Corpus Christi port to the boat.



- **18.** Navigation The *Cerrito Lindo* travels at a speed of 40 knots from Fort Lauderdale on a course of 65° for 2 hr and then changes to a course of 155° for 4 hr. Determine the distance and the bearing from Fort Lauderdale to the boat.
- **19. Land Measure** The angle of depression is 19° from a point 7256 ft above sea level on the north rim of the Grand Canyon level to a point 6159 ft above sea level on the south rim. How wide is the canyon at that point?
- **20. Ranger Fire Watch** A ranger spots a fire from a 73-ft tower in Yellowstone National Park. She measures the angle of depression to be 1°20′. How far is the fire from the tower?
- **21. Civil Engineering** The bearing of the line of sight to the east end of the Royal Gorge footbridge from a point 325 ft due north of the west end of the footbridge across the Royal Gorge is  $117^{\circ}$ . What is the length *l* of the bridge?



**22. Space Flight** The angle of elevation of a space shuttle from Cape Canaveral is 17° when the shuttle is directly over a ship 12 mi downrange. What is the altitude of the shuttle when it is directly over the ship?



**23. Architectural Design** A barn roof is constructed as shown in the figure. What is the height of the vertical center span?



- **24. Recreational Flying** A hot-air balloon over Park City, Utah, is 760 ft above the ground. The angle of depression from the balloon to an observer is 5.25°. Assuming the ground is relatively flat, how far is the observer from a point on the ground directly under the balloon?
- **25.** Navigation A shoreline runs north-south, and a boat is due east of the shoreline. The bearings of the boat from two points on the shore are 110° and 100°. Assume the two points are 550 ft apart. How far is the boat from the shore?



- 26. Navigation Milwaukee, Wisconsin, is directly west of Grand Haven, Michigan, on opposite sides of Lake Michigan. On a foggy night, a law enforcement boat leaves from Milwaukee on a course of 105° at the same time that a small smuggling craft steers a course of 195° from Grand Haven. The law enforcement boat averages 23 knots and collides with the smuggling craft. What was the smuggling boat's average speed?
- 27. Mechanical Design *Refer to Figure 4.94.* The wheel in a piston linkage like the one shown in the figure has a radius of 6 in. It turns with an angular velocity of  $16\pi$  rad/sec. The initial position is the same as that shown in Figure 4.94.
  - (a) What is the frequency of the piston?
  - (b) What equation models the motion of the piston?
  - (c) What is the distance from the initial position 2.85 sec after starting?
- **28. Mechanical Design** Suppose the wheel in a piston linkage like the one shown in Figure 4.94 has a radius of 18 cm and turns with an angular velocity of  $\pi$  rad/sec.
  - (a) What is the frequency of the piston?
  - (b) What equation models the motion of the piston?
  - (c) How many cycles does the piston make in 1 min?

**29. Vibrating Spring** A mass on a spring oscillates back and forth and completes one cycle in 0.5 sec. Its maximum displacement is 3 cm. Write an equation that models this motion.



- **30. Tuning Fork** A point on the tip of a tuning fork vibrates in harmonic motion described by the equation  $d = 14 \sin \omega t$ . Find  $\omega$  for a tuning fork that has a frequency of 528 vibrations per second.
- **31. Ferris Wheel Motion** The Ferris wheel shown in this figure makes one complete turn every 20 sec. A rider's height, *h*, above the ground can be modeled by the equation  $h = a \sin \omega t + k$ , where *h* and *k* are given in feet and *t* is given in seconds.



- (a) What is the value of *a*?
- (**b**) What is the value of *k*?
- (c) What is the value of  $\omega$ ?
- **32.** Ferris Wheel Motion Jacob and Emily ride a Ferris wheel at a carnival in Billings, Montana. The wheel has a 16-m diameter and turns at 3 rpm with its lowest point 1 m above the ground. Assume that Jacob and Emily's height *h* above the ground is a sinusoidal function of time *t* (in seconds), where t = 0 represents the lowest point of the wheel.
  - (a) Write an equation for h.
  - (b) Draw a graph of h for  $0 \le t \le 30$ .
  - (c) Use *h* to estimate Jacob and Emily's height above the ground at t = 4 and t = 10.
- **33.** Monthly Temperatures in Charleston The monthly normal mean temperatures for the last 30 years in Charleston, SC, are shown in Table 4.3. A scatter plot suggests that the mean monthly temperatures follow a sinusoidal curve over time. Assume that the sinusoid has equation  $y = a \sin(b(t h)) + k$ .
  - (a) Given that the period is 12 months, find *b*.
  - (b) Assuming that the high and low temperatures in the table determine the range of the sinusoid, find *a* and *k*.
  - (c) Find a value of *h* that will put the minimum at t = 1 and the maximum at t = 7.
  - (d) Superimpose a graph of your sinusoid on a scatter plot of the data. How good is the fit?

(e) Use your sinusoidal model to predict dates in the year when the mean temperature in Charleston will be 70°. (Assume that t = 0 represents January 1.)

Table 4.3 Temperat	ture Data for Charleston, SC
Month	Temperature
1	48
2	51
3	58
4	64
5	72
6	78
7	82
8	81
9	76
10	66
11	58
12	51

Source: National Climatic Data Center, as reported in the World Almanac and Book of Facts 2009.



**34. Writing to Learn** For the Ferris wheel in Exercise 31, which equation correctly models the height of a rider who begins the ride at the bottom of the wheel when t = 0?

(a) 
$$h = 25 \sin \frac{\pi t}{10}$$
  
(b)  $h = 25 \sin \frac{\pi t}{10} + 8$   
(c)  $h = 25 \sin \frac{\pi t}{10} + 33$   
(d)  $h = 25 \sin \left(\frac{\pi t}{10} + \frac{3\pi}{2}\right) + 33$ 

 $\pi t$ 

Explain your thought process, and use of a graphing utility in choosing the correct modeling equation.

- **35. Monthly Sales** Owing to startup costs and seasonal variations, Gina found that the monthly profit in her bagel shop during the first year followed an up-and-down pattern that could be modeled by  $P = 2t 7 \sin(\pi t/3)$ , where *P* was measured in hundreds of dollars and *t* was measured in months after January 1.
  - (a) In what month did the shop first begin to make money?
  - (b) In what month did the shop enjoy its greatest profit in that first year?
- **36. Weight Loss** Courtney tried several different diets over a two-year period in an attempt to lose weight. She found that her weight *W* followed a fluctuating curve that could be modeled by the function  $W = 220 1.5t + 9.81 \sin (\pi t/4)$ , where *t* was measured in months after January 1 of the first year and *W* was measured in pounds.
  - (a) What was Courtney's weight at the start and at the end of two years?
  - (b) What was her maximum weight during the two-year period?
  - (c) What was her minimum weight during the two-year period?

## **Standardized Test Questions**

- **37. True or False** Higher frequency sound waves have shorter periods. Justify your answer.
- **38. True or False** A car traveling at 30 miles per hour is traveling faster than a ship traveling at 30 knots. Justify your answer.

You may use a graphing calculator when answering these questions.

**39. Multiple Choice** To get a rough idea of the height of a building, John paces off 50 feet from the base of the building, then measures the angle of elevation from the ground to the top of the building at that point to be 58°. About how tall is the building?

(A) 31 feet	<b>(B)</b> 42 feet	(C) 59 feet
<b>(D)</b> 80 feet	(E) 417 feet	

**40. Multiple Choice** A boat leaves harbor and travels at 20 knots on a bearing of 90°. After two hours, it changes course to a bearing of 150° and continues at the same speed for another hour.

After the entire 3-hour trip, how far is it from the harbor?

(A) 50 nautical miles	<b>(B)</b> 53 nautical miles
(C) 57 nautical miles	(D) 60 nautical miles

- (E) 67 nautical miles
- **41. Multiple Choice** At high tide at 8:15 P.M., the water level on the side of a pier is 9 feet from the top. At low tide 6 hours and 12 minutes later, the water level is 13 feet from the top. At which of the following times in that interval is the water level 10 feet from the top of the pier?

(A) 9:15 p.m.	<b>(B)</b> 9:48 p.m.	(C) 9:52 p.m.
<b>(D)</b> 10:19 p.m.	<b>(E)</b> 11:21 P.M.	

<b>42</b> .	Multiple	Choice	The l	loudness	of a m	usical	tone is
	determined	by which o	charact	eristic of	f its sou	nd wa	ave?

(A) Amplitude	(B) Frequency	(C) Period
(D) Phase shift	(E) Pitch	

## **Explorations**

**43. Group Activity** The data for displacement versus time on a tuning fork, shown in Table 4.4, were collected using a CBL and a microphone.

Ì	Table 4.4	Tuning Fork Data				
	Time	Displacement	Time	Displacement		
	0.00091	-0.080	0.00362	0.217		
	0.00108	0.200	0.00379	0.480		
	0.00125	0.480	0.00398	0.681		
	0.00144	0.693	0.00416	0.810		
	0.00162	0.816	0.00435	0.827		
	0.00180	0.844	0.00453	0.749		
	0.00198	0.771	0.00471	0.581		
	0.00216	0.603	0.00489	0.346		
	0.00234	0.368	0.00507	0.077		
	0.00253	0.099	0.00525	-0.164		
	0.00271	-0.141	0.00543	-0.320		
	0.00289	-0.309	0.00562	-0.354		
	0.00307	-0.348	0.00579	-0.248		
	0.00325	-0.248	0.00598	-0.035		
	0.00344	-0.041				

- (a) Graph a scatter plot of the data in the [0, 0.0062] by [-0.5, 1] viewing window.
- (b) Select the equation that appears to be the best fit of these data.
  - i.  $y = 0.6 \sin (2464x 2.84) + 0.25$
  - **ii.**  $y = 0.6 \sin(1210x 2) + 0.25$

**iii.**  $y = 0.6 \sin (2440x - 2.1) + 0.15$ 

(c) What is the approximate frequency of the tuning fork?

**44. Writing to Learn** Human sleep-awake cycles at three different ages are described by the accompanying graphs. The portions of the graphs above the horizontal lines represent times awake, and the portions below represent times asleep.



- (a) What is the period of the sleep-awake cycle of a newborn? of a four-year-old? of an adult?
- (b) Which of these three sleep-awake cycles is the closest to being modeled by a function  $y = a \sin bx$ ?

#### Using Trigonometry in Geometry In a

*regular polygon* all sides have equal length and all angles have equal measure. In Exercises 45 and 46, consider the regular seven-sided polygon whose sides are 5 cm.



- **45.** Find the length of the *apothem*, the segment from the center of the seven-sided polygon to the midpoint of a side.
- **46.** Find the radius of the circumscribed circle of the regular sevensided polygon.
- **47.** A *rhombus* is a quadrilateral with all sides equal in length. Recall that a rhombus is also a parallelogram. Find length *AC* and length *BD* in the rhombus shown here.



## **Extending the Ideas**

- **48.** A roof has two sections, one with a  $50^{\circ}$  elevation and the other with a  $20^{\circ}$  elevation, as shown in the figure.
  - (a) Find the height BE.
  - (b) Find the height *CD*.
  - (c) Find the length AE + ED, and double it to find the length of the roofline.



- **49. Steep Trucking** The *percentage grade* of a road is its slope expressed as a percentage. A tractor-trailer rig passes a sign that reads, "6% grade next 7 miles." What is the average angle of inclination of the road?
- **50. Television Coverage** Many satellites travel in *geosynchronous orbits*, which means that the satellite stays over the same point on the Earth. A satellite that broadcasts cable television is in geosynchronous orbit 100 mi above the Earth. Assume that the Earth is a sphere with radius 4000 mi, and find the arc length of coverage area for the cable television satellite on the Earth's surface.

**51. Group Activity** A musical note like that produced with a tuning fork or pitch meter is a pressure wave. Typically, frequency is measured in hertz (1 Hz = 1 cycle per second). Table 4.5 gives frequency (in Hz) of several musical notes. The time-vs.-pressure tuning fork data in Table 4.6 was collected using a CBL and a microphone.

Table 4.5   Tunin	ıg Fork Data
Note	Frequency (Hz)
С	262
C <sup>♯</sup> or D <sup>↓</sup>	277
D	294
D <sup>♯</sup> or E <sup>♭</sup>	311
E	330
F	349
F <sup>♯</sup> or G <sup>♭</sup>	370
G	392
G <sup>♯</sup> or A <sup>↓</sup>	415
А	440
$A^{\sharp}$ or $B^{\flat}$	466
В	494
C (next octave)	524

Tuning For	k Data	
Pressure	Time (sec)	Pressure
1.29021	0.0049024	-1.06632
1.50851	0.0051520	0.09235
1.51971	0.0054112	1.44694
1.51411	0.0056608	1.51411
1.47493	0.0059200	1.51971
0.45619	0.0061696	1.51411
-0.89280	0.0064288	1.43015
-1.51412	0.0066784	0.19871
-1.15588	0.0069408	-1.06072
-0.04758	0.0071904	-1.51412
1.36858	0.0074496	-0.97116
1.50851	0.0076992	0.23229
1.51971	0.0079584	1.46933
1.51411	0.0082080	1.51411
1.45813	0.0084672	1.51971
0.32185	0.0087168	1.50851
-0.97676	0.0089792	1.36298
-1.51971		
	Pressure 1.29021 1.50851 1.51971 1.51411 1.47493 0.45619 -0.89280 -1.51412 -1.5588 -0.04758 1.50851 1.51971 1.51411 1.45813 0.32185 -0.97676 -1.51971	Pressure         Time (sec)           1.29021         0.0049024           1.50851         0.0051520           1.51971         0.0054112           1.51971         0.0059200           0.45619         0.0061696           -0.89280         0.0064288           -1.51412         0.006784           -1.5588         0.0071904           1.36858         0.0071904           1.36858         0.007920           1.51971         0.007992           1.51971         0.007984           1.51411         0.0082080           1.45813         0.0084672           0.32185         0.0087168           -0.97676         0.0089792           -1.51971         0.0087168

(a) Graph a scatter plot of the data.

- (b) Determine a, b, and h so that the equation  $y = a \sin(b(t - h))$  is a model for the data.
- (c) Determine the frequency of the sinusoid in part (b), and use Table 4.5 to identify the musical note produced by the tuning fork.
- (d) Identify the musical note produced by the tuning fork used in Exercise 43.