4.6 Graphs of Composite Trigonometric Functions

What you'll learn about

- Combining Trigonometric and Algebraic Functions
- Sums and Differences of Sinusoids
- Damped Oscillation

... and why

Function composition extends our ability to model periodic phenomena like heartbeats and sound waves.

Combining Trigonometric and Algebraic Functions

A theme of this text has been "families of functions." We have studied polynomial functions, exponential functions, logarithmic functions, and rational functions (to name a few), and in this chapter we have studied trigonometric functions. Now we consider adding, multiplying, or composing trigonometric functions with functions from these other families.

The notable property that distinguishes the trigonometric function from others we have studied is periodicity. Example 1 shows that when a trigonometric function is combined with a polynomial, the resulting function may or may not be periodic.

EXAMPLE 1 Combining the Sine Function with x^2

Graph each of the following functions for $-2\pi \le x \le 2\pi$, adjusting the vertical window as needed. Which of the functions appear to be periodic?

(a) $y = \sin x + x^2$

(b) $y = x^2 \sin x$

(c) $y = (\sin x)^2$

(d) $y = \sin (x^2)$

SOLUTION We show the graphs and their windows in Figure 4.56 on the next page. Only the graph of $y = (\sin x)^2$ exhibits periodic behavior in the interval $-2\pi \le x \le 2\pi$. (You can widen the window to see further graphical evidence that this is indeed the only periodic function among the four.) ***Now try Exercise 5.***

EXAMPLE 2 Verifying Periodicity Algebraically

Verify algebraically that $f(x) = (\sin x)^2$ is periodic and determine its period graphically.

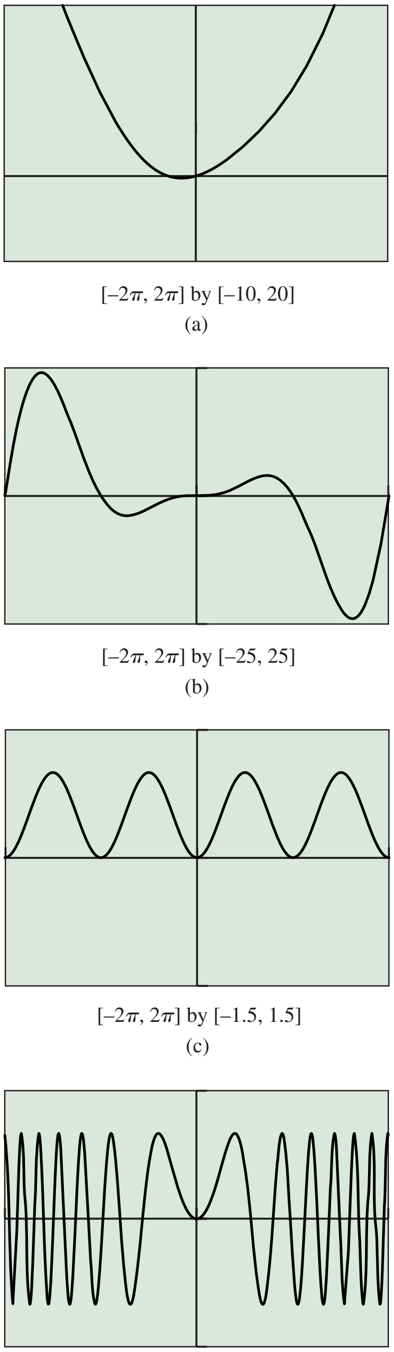
SOLUTION We use the fact that the period of the basic sine function is 2π; that is, $\sin (x + 2\pi) = \sin (x)$ for all x. It follows that

$$\begin{aligned} f(x + 2\pi) &= (\sin (x + 2\pi))^2 \\ &= (\sin (x))^2 \qquad \text{By periodicity of sine} \\ &= f(x) \end{aligned}$$

So $f(x)$ is also periodic, with some period that divides 2π. The graph in Figure 4.56c on the next page shows that the period is actually π. ***Now try Exercise 9.***

Exponent Notation

Example 3 introduces a shorthand notation for powers of trigonometric functions: $(\sin \theta)^n$ can be written as $\sin^n \theta$. (*Caution*: This shorthand notation will probably not be recognized by your calculator.)



$[-2\pi, 2\pi]$ by $[-10, 20]$
(a)

$[-2\pi, 2\pi]$ by $[-25, 25]$
(b)

$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$
(c)

$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$
(d)

FIGURE 4.56 The graphs of the four functions in Example 1. Only graph (c) exhibits periodic behavior over the interval $-2\pi \le x \le 2\pi$.

EXAMPLE 3 Composing $y = \sin x$ and $y = x^3$

Prove algebraically that $f(x) = \sin^3 x$ is periodic and find the period graphically. State the domain and range and sketch a graph showing two periods.

SOLUTION To prove that $f(x) = \sin^3 x$ is periodic, we show that $f(x + 2\pi) = f(x)$ for all x.

$$
\begin{aligned}
f(x + 2\pi) &= \sin^3 (x + 2\pi) \\
&= (\sin (x + 2\pi))^3 && \text{Changing notation} \\
&= (\sin (x))^3 && \text{By periodicity of sine} \\
&= \sin^3(x) && \text{Changing notation} \\
&= f(x)
\end{aligned}
$$

Thus $f(x)$ is periodic with a period that divides 2π. Graphing the function over the interval $-2\pi \le x \le 2\pi$ (Figure 4.57), we see that the period must be 2π.

Since both functions being composed have domain $(-\infty, \infty)$, the domain of f is also $(-\infty, \infty)$. Since cubing all numbers in the interval $[-1, 1]$ gives all numbers in the interval $[-1, 1]$, the range is $[-1, 1]$ (as supported by the graph).

Now try Exercise 13.

Comparing the graphs of $y = \sin^3 x$ and $y = \sin x$ over a single period (Figure 4.58), we see that the two functions have the same zeros and extreme points, but otherwise the graph of $y = \sin^3 x$ is closer to the x-axis than the graph of $y = \sin x$. This is because $|y^3| < |y|$ whenever y is between -1 and 1. In fact, higher odd powers of $\sin x$ yield graphs that are "sucked in" more and more, but always with the same zeros and extreme points.

The absolute value of a periodic function is also a periodic function. We consider two such functions in Example 4.

EXAMPLE 4 Analyzing Nonnegative Periodic Functions

Find the domain, range, and period of each of the following functions. Sketch a graph showing four periods.

(a) $f(x) = |\tan x|$

(b) $g(x) = |\sin x|$

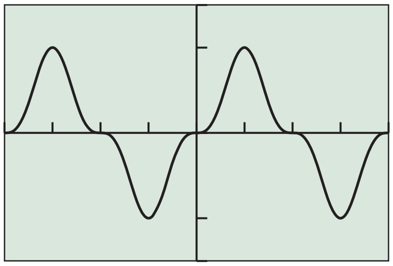
SOLUTION

(a) Whenever $\tan x$ is defined, so is $|\tan x|$. Therefore, the domain of f is the same as the domain of the tangent function, that is, all real numbers except $\pi/2 + n\pi$, $n = 0, \pm 1, \ldots$. Because $f(x) = |\tan x| \ge 0$ and the range of $\tan x$ is $(-\infty, \infty)$, the range of f is $[0, \infty)$. The period of f, like that of $y = \tan x$, is π. The graph of $y = f(x)$ is shown in Figure 4.59.

(b) Whenever $\sin x$ is defined, so is $|\sin x|$. Therefore, the domain of g is the same as the domain of the sine function, that is, all real numbers. Because $g(x) = |\sin x| \ge 0$ and the range of $\sin x$ is $[-1, 1]$ the range of g is $[0, 1]$.

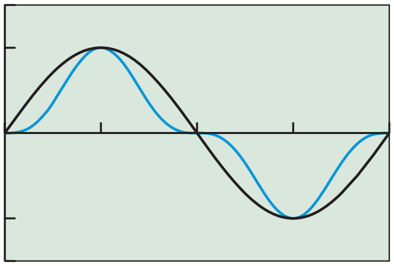
The period of g is only half the period of $y = \sin x$, for reasons that are apparent from viewing the graph. The negative sections of the sine curve below the x-axis are reflected above the x-axis, where they become repetitions of the positive sections. The graph of $y = g(x)$ is shown in Figure 4.60 on the next page.

Now try Exercise 15.



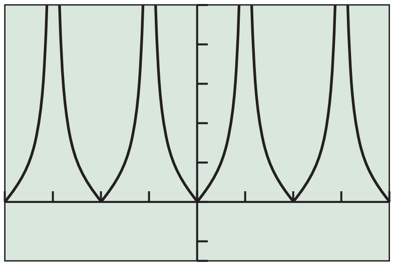
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

FIGURE 4.57 The graph of $f(x) = \sin^3 x$. (Example 3)



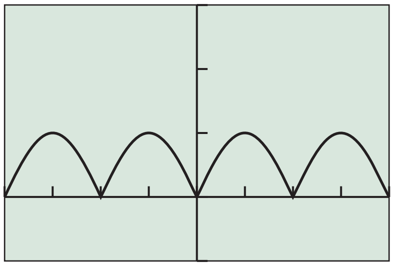
$[0, 2\pi]$ by $[-1.5, 1.5]$

FIGURE 4.58 The graph suggests that $|\sin^3 x| \le |\sin x|$.



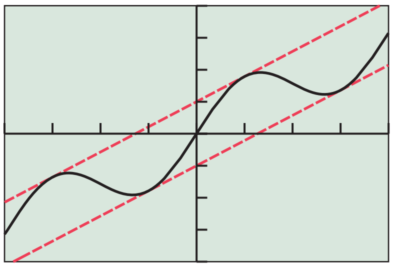
$[-2\pi, 2\pi]$ by $[-1.5, 5]$

FIGURE 4.59 $f(x) = |\tan x|$ has the same period as $y = \tan x$. (Example 4a)



$[-2\pi, 2\pi]$ by $[-1, 3]$

FIGURE 4.60 $g(x) = |\sin x|$ has half the period of $y = \sin x$. (Example 4b)



$[-2\pi, 2\pi]$ by $[-4, 4]$

FIGURE 4.61 The graph of $f(x) = 0.5x + \sin x$ oscillates between the lines $y = 0.5x + 1$ and $y = 0.5x - 1$. Although the wave repeats its shape, it is not periodic. (Example 5)

When a sinusoid is added to a (nonconstant) linear function, the result is *not* periodic. The graph repeats its *shape* at regular intervals, but the function takes on different values over those intervals. The graph will show a curve oscillating between two parallel lines, as in Example 5.

EXAMPLE 5 Adding a Sinusoid to a Linear Function

The graph of $f(x) = 0.5x + \sin x$ oscillates between two parallel lines (Figure 4.61). What are the equations of the two lines?

SOLUTION As $\sin x$ oscillates between -1 and 1, $f(x)$ oscillates between $0.5x - 1$ and $0.5x + 1$. Therefore, the two lines are $y = 0.5x - 1$ and $y = 0.5x + 1$. Graphing the two lines and $f(x)$ in the same window provides graphical support. Of course, the graph should resemble Figure 4.61 if your lines are correct.

Now try Exercise 19.

Sums and Differences of Sinusoids

Section 4.4 introduced you to sinusoids, functions that can be written in the form

$$y = a \sin (b(x - h)) + k$$

and therefore have the shape of a sine curve.

Sinusoids model a variety of physical and social phenomena—such as sound waves, voltage in alternating electrical current, the velocity of air flow during the human respiratory cycle, and many others. Sometimes these phenomena interact in an additive fashion. For example, if y_1 models the sound of one tuning fork and y_2 models the sound of a second tuning fork, then $y_1 + y_2$ models the sound when they are both struck simultaneously. So we are interested in whether the sums and differences of sinusoids are again sinusoids.

EXPLORATION 1 Investigating Sinusoids

Graph these functions, one at a time, in the viewing window $[-2\pi, 2\pi]$ by $[-6, 6]$. Which ones appear to be sinusoids?

$$y = 3 \sin x + 2 \cos x \qquad y = 2 \sin x - 3 \cos x$$

$$y = 2 \sin 3x - 4 \cos 2x \qquad y = 2 \sin (5x + 1) - 5 \cos 5x$$

$$y = \cos\left(\frac{7x - 2}{5}\right) + \sin\left(\frac{7x}{5}\right) \qquad y = 3 \cos 2x + 2 \sin 7x$$

What relationship between the sine and cosine functions ensures that their sum or difference will again be a sinusoid? Check your guess on a graphing calculator by constructing your own examples.

The rule turns out to be fairly simple: Sums and differences of sinusoids with the same period are again sinusoids. We state this rule more explicitly as follows.

Sums That Are Sinusoid Functions

If $y_1 = a_1 \sin(b(x - h_1))$ and $y_2 = a_2 \cos(b(x - h_2))$, then

$$y_1 + y_2 = a_1 \sin(b(x - h_1)) + a_2 \cos(b(x - h_2))$$

is a sinusoid with period $2\pi/|b|$.

For the sum to be a sinusoid, the two sinusoids being added together must have the same period, and the sum has the same period as both of them. Also, although the rule is stated in terms of a sine function being added to a cosine function, the fact that every cosine function is a translation of a sine function (and vice versa) makes the rule equally applicable to the sum of two sine functions or the sum of two cosine functions. If they have the same period, their sum is a sinusoid.

EXAMPLE 6 Identifying a Sinusoid

Determine whether each of the following functions is or is not a sinusoid.

(a) $f(x) = 5 \cos x + 3 \sin x$

(b) $f(x) = \cos 5x + \sin 3x$

(c) $f(x) = 2 \cos 3x - 3 \cos 2x$

(d) $f(x) = a \cos\left(\frac{3x}{7}\right) - b \cos\left(\frac{3x}{7}\right) + c \sin\left(\frac{3x}{7}\right)$

SOLUTION

(a) Yes, since both functions in the sum have period 2π.

(b) No, since $\cos 5x$ has period $2\pi/5$ and $\sin 3x$ has period $2\pi/3$.

(c) No, since $2 \cos 3x$ has period $2\pi/3$ and $3 \cos 2x$ has period π.

(d) Yes, since all three functions in the sum have period $14\pi/3$. (The first two sum to a sinusoid with the same period as the third, so adding the third function still yields a sinusoid.)

Now try Exercise 25.

EXAMPLE 7 Expressing the Sum of Sinusoids as a Sinusoid

Let $f(x) = 2 \sin x + 5 \cos x$. From the discussion above, you should conclude that $f(x)$ is a sinusoid.

(a) Find the period of f.

(b) Estimate the amplitude and phase shift graphically (to the nearest hundredth).

(c) Give a sinusoid $a \sin(b(x - h))$ that approximates $f(x)$.

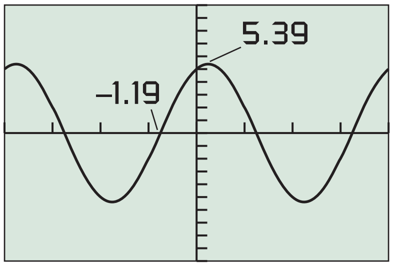
SOLUTION

(a) The period of f is the same as the period of $\sin x$ and $\cos x$, namely 2π.

Solve Graphically

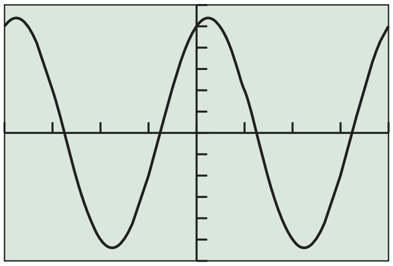
(b) We will learn an algebraic way to find the amplitude and phase shift in the next chapter, but for now we will find this information graphically. Figure 4.62 suggests that f is indeed a sinusoid. That is, for some a and b,

$$2 \sin x + 5 \cos x = a \sin(x - h).$$



$[-2\pi, 2\pi]$ by $[-10, 10]$

FIGURE 4.62 The sum of two sinusoids: $f(x) = 2 \sin x + 5 \cos x$. (Example 7)



$[-2\pi, 2\pi]$ by $[-6, 6]$

FIGURE 4.63 The graphs of $y = 2 \sin x + 5 \cos x$ and $y = 5.39 \sin (x + 1.19)$ appear to be identical. (Example 7)

The *maximum value*, rounded to the nearest hundredth, is 5.39, so the amplitude of f is about 5.39. The *x-intercept* closest to $x = 0$, rounded to the nearest hundredth, is -1.19, so the phase shift of the sine function is about -1.19. We conclude that

$$f(x) = a \sin (x + h) \approx 5.39 \sin (x + 1.19).$$

(c) We support our answer graphically by showing that the graphs of $y = 2 \sin x + 5 \cos x$ and $y = 5.39 \sin (x + 1.19)$ are virtually identical (Figure 4.63).

Now try Exercise 29.

The sum of two sinusoids with different periods, while not a sinusoid, will often be a periodic function. Finding the period of a sum of periodic functions can be tricky. Here is a useful fact to keep in mind. If f is periodic, and if $f(x + s) = f(x)$ for all x in the domain of f, then the period of f divides s exactly. In other words, s is either the period or a multiple of the period.

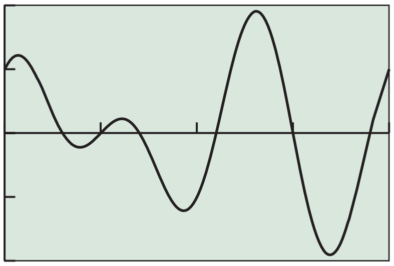
EXAMPLE 8 Showing a Function Is Periodic but Not a Sinusoid

Show that $f(x) = \sin 2x + \cos 3x$ is periodic but not a sinusoid. Graph one period.

SOLUTION Since $\sin 2x$ and $\cos 3x$ have different periods, the sum is not a sinusoid. Next we show that 2π is a candidate for the period of f, that is, $f(x + 2\pi) = f(x)$ for all x.

$$\begin{aligned} f(x + 2\pi) &= \sin (2(x + 2\pi)) + \cos (3(x + 2\pi)) \\ &= \sin (2x + 4\pi) + \cos (3x + 6\pi) \\ &= \sin 2x + \cos 3x \\ &= f(x) \end{aligned}$$

This means either that 2π is the period of f or that the period is an exact divisor of 2π. Figure 4.64 suggests that the period is not smaller than 2π, so it must be 2π. The graph shows that indeed f is not a sinusoid. ***Now try Exercise 35.***

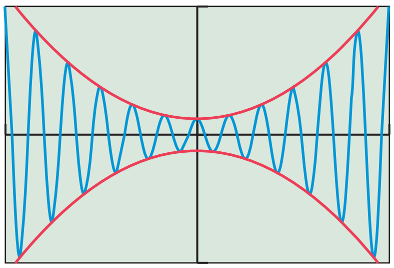


$[0, 2\pi]$ by $[-2, 2]$

FIGURE 4.64 One period of $f(x) = \sin 2x + \cos 3x$. (Example 8)

Damped Oscillation

Because the values of $\sin bt$ and $\cos bt$ oscillate between -1 and 1, something interesting happens when either of these functions is multiplied by another function. For example, consider the function $y = (x^2 + 5) \cos 6x$, graphed in Figure 4.65. The (blue) graph of the function oscillates between the (red) graphs of $y = x^2 + 5$ and $y = -(x^2 + 5)$. The "squeezing" effect that can be seen near the origin is called **damping**.



$[-2\pi, 2\pi]$ by $[-40, 40]$

FIGURE 4.65 The graph of $y = (x^2 + 5) \cos 6x$ shows **damped** oscillation.

Damped Oscillation

The graph of $y = f(x) \cos bx$ (or $y = f(x) \sin bx$) oscillates between the graphs of $y = f(x)$ and $y = -f(x)$. When this reduces the amplitude of the wave, it is called **damped oscillation**. The factor $f(x)$ is called the **damping factor**.

EXAMPLE 9 Identifying Damped Oscillation

For each of the following functions, determine if the graph shows damped oscillation. If it does, identify the damping factor and tell where the damping occurs.

(a) $f(x) = 2^{-x} \sin 4x$

(b) $f(x) = 3 \cos 2x$

(c) $f(x) = -2x \cos 2x$

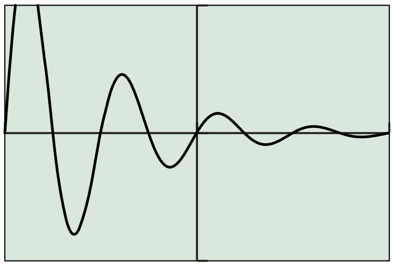
SOLUTION The graphs are shown in Figure 4.66.

(a) This is damped oscillation. The damping factor is 2^{-x} and the damping occurs as $x \to \infty$.

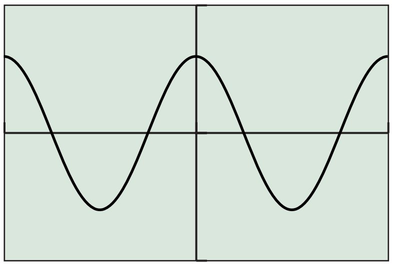
(b) This wave has a constant amplitude of 3. No damping occurs.

(c) This is damped oscillation. The damping factor is $-2x$. The damping occurs as $x \to 0$.

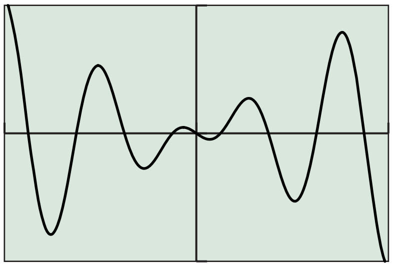
Now try Exercise 43.



$[-\pi, \pi]$ by $[-5, 5]$
(a)



$[-\pi, \pi]$ by $[-5, 5]$
(b)



$[-2\pi, 2\pi]$ by $[-12, 12]$
(c)

FIGURE 4.66 The graphs of functions (a), (b), and (c) in Example 9. The wave in graph (b) does not exhibit damped oscillation.

EXAMPLE 10 A Damped Oscillating Spring

Dr. Sanchez's physics class collected data for an air table glider that oscillates between two springs. The class determined from the data that the equation

$$y = 0.22e^{-0.065t} \cos 2.4t$$

modeled the displacement y of the spring from its original position as a function of time t.

(a) Identify the damping factor and tell where the damping occurs.

(b) Approximately how long does it take for the spring to be damped so that $-0.1 \le y \le 0.1$?

SOLUTION The graph is shown in Figure 4.67.

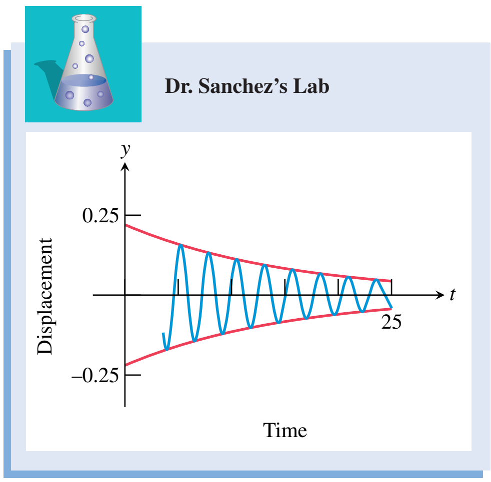
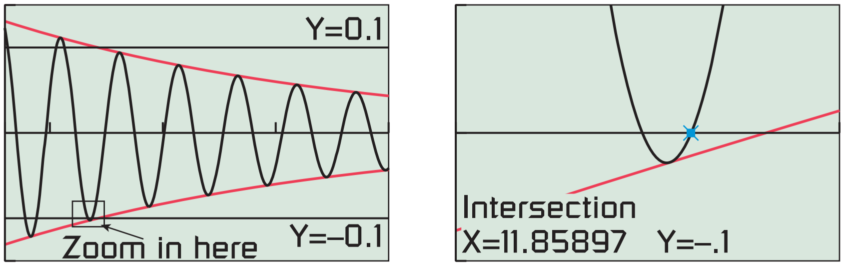


FIGURE 4.67 Damped oscillation in the physics lab. (Example 10)

(a) The damping factor is $0.22e^{-0.065t}$. The damping occurs as $t \to \infty$.

(b) We want to find how soon the curve $y = 0.22e^{-0.065t} \cos 2.4t$ falls entirely between the lines $y = -0.1$ and $y = 0.1$. By zooming in on the region indicated in Figure 4.68a and using grapher methods, we find that it takes approximately 11.86 seconds until the graph of $y = 0.22e^{-0.065t} \cos 2.4t$ lies entirely between $y = -0.1$ and $y = 0.1$ (Figure 4.68b).

Now try Exercise 71.



[8, 25] by [–0.15, 0.15]
(a)

[11, 12.4] by [–0.11, –0.09]
(b)

FIGURE 4.68 The damped oscillation in Example 10 eventually gets to be less than 0.1 in either direction.

QUICK REVIEW 4.6 *(For help, go to Sections 1.2 and 1.4.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–6, state the domain and range of the function.

1. $f(x) = 3 \sin 2x$ **2.** $f(x) = -2 \cos 3x$

3. $f(x) = \sqrt{x - 1}$ **4.** $f(x) = \sqrt{x}$

5. $f(x) = |x| - 2$ **6.** $f(x) = |x + 2| + 1$

In Exercises 7 and 8, describe the end behavior of the function, that is, the behavior as $|x| \to \infty$.

7. $f(x) = 5e^{-2x}$ **8.** $f(x) = -0.2(5^{-0.1x})$

In Exercises 9 and 10, form the compositions $f \circ g$ and $g \circ f$. State the domain of each function.

9. $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$

10. $f(x) = x^2$ and $g(x) = \cos x$

SECTION 4.6 EXERCISES

In Exercises 1–8, graph the function for $-2\pi \le x \le 2\pi$, adjusting the vertical window as needed. State whether or not the function appears to be periodic.

1. $f(x) = (\sin x)^2$ **2.** $f(x) = (1.5 \cos x)^2$

3. $f(x) = x^2 + 2 \sin x$ **4.** $f(x) = x^2 - 2 \cos x$

5. $f(x) = x \cos x$ **6.** $f(x) = x^2 \cos x$

7. $f(x) = (\sin x + 1)^3$ **8.** $f(x) = (2 \cos x - 4)^2$

In Exercises 9–12, verify algebraically that the function is periodic and determine its period graphically. Sketch a graph showing two periods.

9. $f(x) = \cos^2 x$ **10.** $f(x) = \cos^3 x$

11. $f(x) = \sqrt{\cos^2 x}$ **12.** $f(x) = |\cos^3 x|$

In Exercises 13–18, state the domain and range of the function and sketch a graph showing four periods.

13. $y = \cos^2 x$ **14.** $y = |\cos x|$

15. $y = |\cot x|$ **16.** $y = \cos |x|$

17. $y = -\tan^2 x$ **18.** $y = -\sin^2 x$

The graph of each function in Exercises 19–22 oscillates between two parallel lines, as in Example 5. Find the equations of the two lines and graph the lines and the function in the same viewing window.

19. $y = 2x + \cos x$ **20.** $y = 1 - 0.5x + \cos 2x$

21. $y = 2 - 0.3x + \cos x$ **22.** $y = 1 + x + \cos 3x$

In Exercises 23–28, determine whether $f(x)$ is a sinusoid.

23. $f(x) = \sin x - 3 \cos x$

24. $f(x) = 4 \cos x + 2 \sin x$

25. $f(x) = 2 \cos \pi x + \sin \pi x$

26. $f(x) = 2 \sin x - \tan x$

27. $f(x) = 3 \sin 2x - 5 \cos x$

28. $f(x) = \pi \sin 3x - 4\pi \sin 2x$

In Exercises 29–34, find a, b, and h so that $f(x) \approx a \sin (b(x - h))$.

29. $f(x) = 2 \sin 2x - 3 \cos 2x$

30. $f(x) = \cos 3x + 2 \sin 3x$

31. $f(x) = \sin \pi x - 2 \cos \pi x$

32. $f(x) = \cos 2\pi x + 3 \sin 2\pi x$

33. $f(x) = 2 \cos x + \sin x$

34. $f(x) = 3 \sin 2x - \cos 2x$

In Exercises 35–38, the function is periodic but not a sinusoid. Find the period graphically and sketch a graph showing one period.

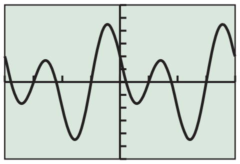
35. $y = 2 \cos x + \cos 3x$

36. $y = 2 \sin 2x + \cos 3x$

37. $y = \cos 3x - 4 \sin 2x$

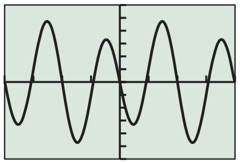
38. $y = \sin 2x + \sin 5x$

In Exercises 39–42, match the function with its graph.



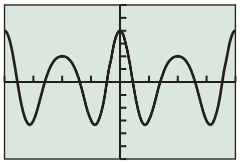
$[-2\pi, 2\pi]$ by $[-6, 6]$

(a)



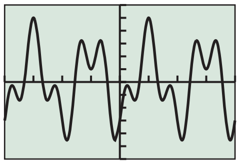
$[-2\pi, 2\pi]$ by $[-6, 6]$

(b)



$[-2\pi, 2\pi]$ by $[-6, 6]$

(c)



$[-2\pi, 2\pi]$ by $[-6, 6]$

(d)

39. $y = 2 \cos x - 3 \sin 2x$

40. $y = 2 \sin 5x - 3 \cos 2x$

41. $y = 3 \cos 2x + \cos 3x$

42. $y = \sin x - 4 \sin 2x$

In Exercises 43–48, tell whether the function exhibits damped oscillation. If so, identify the damping factor and tell whether the damping occurs as $x \to 0$ or as $x \to \infty$.

43. $f(x) = e^{-x} \sin 3x$ **44.** $f(x) = x \sin 4x$

45. $f(x) = \sqrt{5} \cos 1.2x$ **46.** $f(x) = \pi^2 \cos \pi x$

47. $f(x) = x^3 \sin 5x$ **48.** $f(x) = \left(\frac{2}{3}\right)^x \sin\left(\frac{2x}{3}\right)$

In Exercises 49–52, graph both f and plus or minus its damping factor in the same viewing window. Describe the behavior of the function f for $x > 0$. What is the end behavior of f?

49. $f(x) = 1.2^{-x} \cos 2x$ **50.** $f(x) = 2^{-x} \sin 4x$

51. $f(x) = x^{-1} \sin 3x$ **52.** $f(x) = e^{-x} \cos 3x$

In Exercises 53–56, find the period and graph the function over two periods.

53. $y = \sin 3x + 2 \cos 2x$

54. $y = 4 \cos 2x - 2 \cos (3x - 1)$

55. $y = 2 \sin (3x + 1) - \cos (5x - 1)$

56. $y = 3 \cos (2x - 1) - 4 \sin (3x - 2)$

In Exercises 57–62, graph f over the interval $[-4\pi, 4\pi]$. Determine whether the function is periodic and, if it is, state the period.

57. $f(x) = \left|\sin \frac{1}{2}x\right| + 2$ **58.** $f(x) = 3x + 4 \sin 2x$

59. $f(x) = x - \cos x$ **60.** $f(x) = x + \sin 2x$

61. $f(x) = \frac{1}{2}x + \cos 2x$ **62.** $f(x) = 3 - x + \sin 3x$

In Exercises 63–70, find the domain and range of the function.

63. $f(x) = 2x + \cos x$ **64.** $f(x) = 2 - x + \sin x$

65. $f(x) = |x| + \cos x$ **66.** $f(x) = -2x + |3 \sin x|$

67. $f(x) = \sqrt{\sin x}$ **68.** $f(x) = \sin |x|$

69. $f(x) = \sqrt{|\sin x|}$ **70.** $f(x) = \sqrt{\cos x}$

71. Oscillating Spring The oscillations of a spring subject to friction are modeled by the equation $y = 0.43e^{-0.55t} \cos 1.8t$.

(a) Graph y and its two damping curves in the same viewing window for $0 \le t \le 12$.

(b) Approximately how long does it take for the spring to be damped so that $-0.2 \le y \le 0.2$?

72. Predicting Economic Growth The business manager of a small manufacturing company finds that she can model the company's annual growth as roughly exponential, but with cyclical fluctuations. She uses the function $S(t) = 75(1.04)^t + 4 \sin (\pi t/3)$ to estimate sales (in millions of dollars), t years after 2005.

(a) What are the company's sales in 2005?

(b) What is the approximate annual growth rate?

(c) What does the model predict for sales in 2013?

(d) How many years are in each economic cycle for this company?

73. Writing to Learn Example 3 shows that the function $y = \sin^3 x$ is periodic. Explain whether you think that $y = \sin x^3$ is periodic and why.

74. Writing to Learn Example 4 shows that $y = |\tan x|$ is periodic. Write a convincing argument that $y = \tan |x|$ is not a periodic function.

In Exercises 75 and 76, select the one correct inequality, **(a)** or **(b)**. Give a convincing argument.

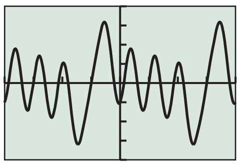
75. (a) $x - 1 \le x + \sin x \le x + 1$ for all x.

(b) $x - \sin x \le x + \sin x$ for all x.

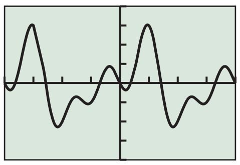
76. (a) $-x \le x \sin x \le x$ for all x.

(b) $-|x| \le x \sin x \le |x|$ for all x.

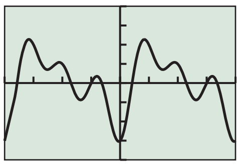
In Exercises 77–80, match the function with its graph. In each case state the viewing window.



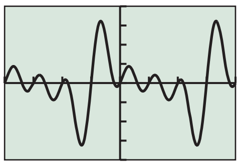
(a)



(b)



(c)



(d)

77. $y = \cos x - \sin 2x - \cos 3x + \sin 4x$

78. $y = \cos x - \sin 2x - \cos 3x + \sin 4x - \cos 5x$

79. $y = \sin x + \cos x - \cos 2x - \sin 3x$

80. $y = \sin x - \cos x - \cos 2x - \cos 3x$

Standardized Test Questions

81. True or False The function $f(x) = \sin |x|$ is periodic. Justify your answer.

82. True or False The sum of two sinusoids is a sinusoid. Justify your answer.

You may use a graphing calculator when answering these questions.

83. Multiple Choice What is the period of the function $f(x) = |\sin x|$?

(A) $\pi/2$ **(B)** π **(C)** 2π

(D) 3π **(E)** None; the function is not periodic.

84. Multiple Choice The function $f(x) = x \sin x$ is

(A) discontinuous. **(B)** bounded. **(C)** even.

(D) one-to-one. **(E)** periodic.

85. Multiple Choice The function $f(x) = x + \sin x$ is

(A) discontinuous. **(B)** bounded. **(C)** even.

(D) odd. **(E)** periodic.

86. Multiple Choice Which of the following functions is *not* a sinusoid?

(A) $2 \cos (2x)$ **(B)** $3 \sin (2x)$

(C) $3 \sin (2x) + 2 \cos (2x)$ **(D)** $3 \sin (3x) + 2 \cos (2x)$

(E) $\sin (3x + 3) + \cos (3x + 2)$

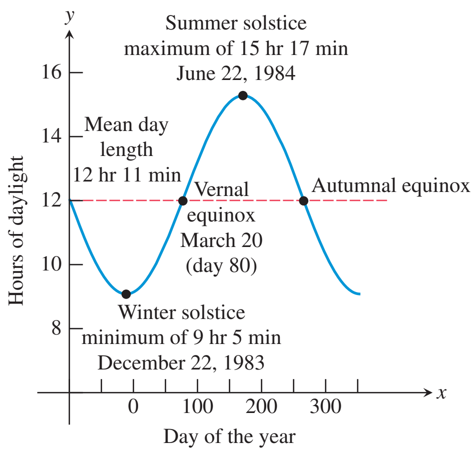
Explorations

87. Group Activity Inaccurate or Misleading Graphs

(a) Set Xmin = 0 and Xmax = 2π. Move the cursor along the x-axis. What is the distance between one pixel and the next (to the nearest hundredth)?

(b) What is the period of $f(x) = \sin 250x$? Consider that the period is the length of one full cycle of the graph. Approximately how many cycles should there be between two adjacent pixels? Can your grapher produce an accurate graph of this function between 0 and 2π?

88. Group Activity Length of Days The graph shows the number of hours of daylight in Boston as a function of the day of the year, from September 21, 1983, to December 15, 1984. Key points are labeled and other critical information is provided. Write a formula for the sinusoidal function and check it by graphing.



Extending the Ideas

In Exercises 89–96, first try to predict what the graph will look like (without too much effort, that is, just for fun). Then graph the function in one or more viewing windows to determine the main features of the graph, and draw a summary sketch. Where applicable, name the period, amplitude, domain, range, asymptotes, and zeros.

89. $f(x) = \cos e^x$

90. $g(x) = e^{\tan x}$

91. $f(x) = \sqrt{x} \sin x$

92. $g(x) = \sin \pi x + \sqrt{4 - x^2}$

93. $f(x) = \dfrac{\sin x}{x}$

94. $g(x) = \dfrac{\sin x}{x^2}$

95. $f(x) = x \sin \dfrac{1}{x}$

96. $g(x) = x^2 \sin \dfrac{1}{x}$