What you'll learn about

- Right Triangle Trigonometry
- Two Famous Triangles
- Evaluating Trigonometric Functions with a Calculator
- Applications of Right Triangle Trigonometry

. . . and why

The many applications of right triangle trigonometry gave the subject its name.

4.2 Trigonometric Functions of Acute Angles

Right Triangle Trigonometry

Recall that geometric figures are **similar** if they have the same shape even though they may have different sizes. Having the same shape means that the angles of one are congruent to the angles of the other and their corresponding sides are proportional. Similarity is the basis for many applications, including scale drawings, maps, and **right triangle trigonometry**, which is the topic of this section.

Two triangles are similar if the angles of one are congruent to the angles of the other. For two *right* triangles we need only know that an acute angle of one is equal to an acute angle of the other for the triangles to be similar. Thus a single acute angle θ of a right triangle determines six distinct ratios of side lengths. Each ratio can be considered a function of θ as θ takes on values from 0° to 90° or from 0 radians to $\pi/2$ radians. We wish to study these functions of acute angles more closely.

To bring the power of coordinate geometry into the picture, we will often put our acute angles in **standard position** in the xy-plane, with the vertex at the origin, one ray along the positive x-axis, and the other ray extending into the first quadrant. (See Figure 4.7.)

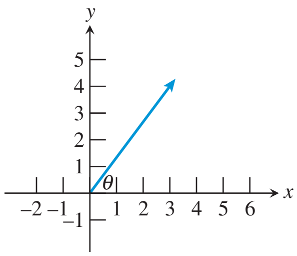

FIGURE 4.7 An acute angle θ in **standard position**, with one ray along the positive x-axis and the other extending into the first quadrant.

The six ratios of side lengths in a right triangle are the six *trigonometric functions* (often abbreviated as *trig functions*) of the acute angle θ. We will define them here with reference to the right ΔABC as labeled in Figure 4.8. The abbreviations *opp*, *adj*, and *hyp* refer to the lengths of the side opposite θ, the side adjacent to θ, and the hypotenuse, respectively.

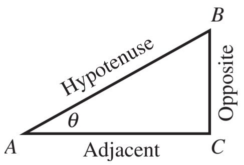

FIGURE 4.8 The triangle referenced in our definition of the trigonometric functions.

DEFINITION Trigonometric Functions

Let θ be an acute angle in the right ΔABC (Figure 4.8). Then

$$\textbf{sine}\ (\theta) = \sin\theta = \frac{opp}{hyp} \qquad \textbf{cosecant}\ (\theta) = \csc\theta = \frac{hyp}{opp}$$

$$\textbf{cosine}\ (\theta) = \cos\theta = \frac{adj}{hyp} \qquad \textbf{secant}\ (\theta) = \sec\theta = \frac{hyp}{adj}$$

$$\textbf{tangent}\ (\theta) = \tan\theta = \frac{opp}{adj} \qquad \textbf{cotangent}\ (\theta) = \cot\theta = \frac{adj}{opp}$$

Function Reminder

Both $\sin\theta$ and $\sin(\theta)$ represent a function of the variable θ. Neither notation implies multiplication by θ. The notation $\sin(\theta)$ is just like the notation $f(x)$, while the notation $\sin\theta$ is a widely accepted shorthand. The same note applies to all six trigonometric functions.

EXPLORATION 1 Exploring Trigonometric Functions

There are twice as many trigonometric functions as there are triangle sides that define them, so we can already explore some ways in which the trigonometric functions relate to each other. Doing this Exploration will help you learn which ratios are which.

1. Each of the six trig functions can be paired to another that is its reciprocal. Find the three pairs of reciprocals.
2. Which trig function can be written as the quotient $\sin\theta/\cos\theta$?
3. Which trig function can be written as the quotient $\csc\theta/\cot\theta$?
4. What is the (simplified) product of all six trig functions multiplied together?
5. Which two trig functions must be less than 1 for any acute angle θ? [*Hint:* What is always the longest side of a right triangle?]

Two Famous Triangles

Evaluating trigonometric functions of particular angles used to require trig tables or slide rules; now it requires only a calculator. However, the side ratios for some angles that appear in right triangles can be found *geometrically*. Every student of trigonometry should be able to find these special ratios without a calculator.

EXAMPLE 1 Evaluating Trigonometric Functions of 45°

Find the values of all six trigonometric functions for an angle of 45°.

SOLUTION A 45° angle occurs in an *isosceles right triangle*, with angles 45°–45°–90° (see Figure 4.9).

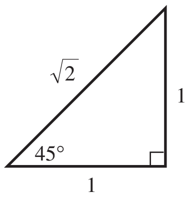

FIGURE 4.9 An isosceles right triangle. (Example 1)

Since the size of the triangle does not matter, we set the length of the two equal legs to 1. The hypotenuse, by the Pythagorean Theorem, is $\sqrt{1+1}=\sqrt{2}$. Applying the definitions, we have

$$\sin 45° = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2} \qquad \csc 45° = \frac{hyp}{opp} = \frac{\sqrt{2}}{1}$$

$$\cos 45° = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2} \qquad \sec 45° = \frac{hyp}{adj} = \frac{\sqrt{2}}{1}$$

$$\tan 45° = \frac{opp}{adj} = \frac{1}{1} = 1 \qquad \cot 45° = \frac{adj}{opp} = \frac{1}{1} = 1$$

Now try Exercise 1.

Whenever two sides of a right triangle are known, the third side can be found using the Pythagorean Theorem. All six trigonometric functions of either acute angle can then be found. We illustrate this in Example 2 with another well-known triangle.

EXAMPLE 2 Evaluating Trigonometric Functions of 30°

Find the values of all six trigonometric functions for an angle of 30°.

SOLUTION A 30° angle occurs in a **30°–60°–90°** triangle, which can be constructed from an equilateral (60°–60°–60°) triangle by constructing an altitude to any side. Since size does not matter, start with an equilateral triangle with sides 2 units long. The altitude splits it into two congruent 30°–60°–90° triangles, each with hypotenuse 2 and smaller leg 1. By the Pythagorean Theorem, the longer leg has length $\sqrt{2^2-1^2}=\sqrt{3}$. (See Figure 4.10.)

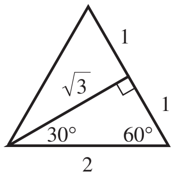

FIGURE 4.10 An altitude to any side of an equilateral triangle creates two congruent 30°–60°–90° triangles. If each side of the equilateral triangle has length 2, then the two 30°–60°–90° triangles have sides of length 2, 1, and $\sqrt{3}$. (Example 2)

We apply the definitions of the trigonometric functions to get:

$$\sin 30° = \frac{opp}{hyp} = \frac{1}{2} \qquad \csc 30° = \frac{hyp}{opp} = \frac{2}{1} = 2$$

$$\cos 30° = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} \qquad \sec 30° = \frac{hyp}{adj} = \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$\tan 30° = \frac{opp}{adj} = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3} \qquad \cot 30° = \frac{adj}{opp} = \frac{\sqrt{3}}{1}$$

Now try Exercise 3.

EXPLORATION 2 Evaluating Trigonometric Functions of 60°

1. Find the values of all six trigonometric functions for an angle of 60°. Note that most of the preliminary work has been done in Example 2.
2. Compare the six function values for 60° with the six function values for 30°. What do you notice?
3. We will eventually learn a rule that relates trigonometric functions of any angle with trigonometric functions of the *complementary* angle. (Recall from geometry that 30° and 60° are complementary because they add up to 90°.) Based on this exploration, can you predict what that rule will be? [*Hint:* The "co" in cosine, cotangent, and cosecant actually comes from "complement."]

Example 3 illustrates that knowing one trigonometric ratio in a right triangle is sufficient for finding all the others.

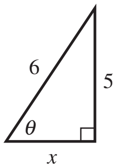

FIGURE 4.11 How to create an acute angle θ such that $\sin \theta = 5/6$. (Example 3)

A Word About Radical Fractions

There was a time when $\frac{5\sqrt{11}}{11}$ was considered "simpler" than $\frac{5}{\sqrt{11}}$ because it was easier to approximate, but today they are just equivalent expressions for the same irrational number. With technology, either form leads easily to an approximation of 1.508. We leave the answers in exact form here because we want you to practice problems of this type without a calculator.

EXAMPLE 3 Using One Trigonometric Ratio to Find Them All

Let θ be an acute angle such that $\sin \theta = 5/6$. Evaluate the other five trigonometric functions of θ.

SOLUTION Sketch a triangle showing an acute angle θ. Label the opposite side 5 and the hypotenuse 6. (See Figure 4.11.) Since $\sin \theta = 5/6$, this must be our angle! Now we need the other side of the triangle (labeled x in the figure).

From the Pythagorean Theorem it follows that $x^2 + 5^2 = 6^2$, so $x = \sqrt{36 - 25} = \sqrt{11}$. Applying the definitions,

$$\sin \theta = \frac{opp}{hyp} = \frac{5}{6} \qquad \csc \theta = \frac{hyp}{opp} = \frac{6}{5} = 1.2$$

$$\cos \theta = \frac{adj}{hyp} = \frac{\sqrt{11}}{6} \qquad \sec \theta = \frac{hyp}{adj} = \frac{6}{\sqrt{11}} \text{ or } \frac{6\sqrt{11}}{11}$$

$$\tan \theta = \frac{opp}{adj} = \frac{5}{\sqrt{11}} \text{ or } \frac{5\sqrt{11}}{11} \qquad \cot \theta = \frac{adj}{opp} = \frac{\sqrt{11}}{5}$$

Now try Exercise 9.

Evaluating Trigonometric Functions with a Calculator

Using a calculator for the evaluation step enables you to concentrate all your problem-solving skills on the modeling step, which is where the real trigonometry occurs. The danger is that your calculator will try to evaluate what you ask it to evaluate, even if you ask it to evaluate the wrong thing. If you make a mistake, you might be lucky and see an error message. In most cases you will unfortunately see an answer that you will assume is correct but is actually wrong. We list the most common calculator errors associated with evaluating trigonometric functions.

Common Calculator Errors When Evaluating Trig Functions

1. **Using the Calculator in the Wrong Angle Mode (Degrees/Radians)** This error is so common that everyone encounters it once in a while. You just hope to recognize it when it occurs. For example, suppose we are doing a problem in which we need to evaluate the sine of 10 degrees. Our calculator shows us this screen (Figure 4.12):

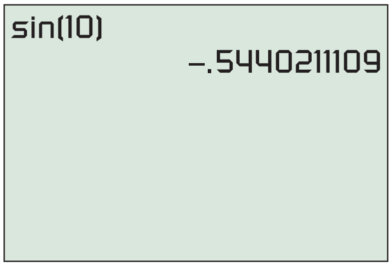

FIGURE 4.12 Wrong mode for finding sin $(10°)$.

Why is the answer negative? Our first instinct should be to check the mode. Sure enough, it is in radian mode. Changing to degrees, we get sin $(10) = 0.1736481777$, which is a reasonable answer. (That still leaves open the question of why the sine of 10 radians is negative, but that is a topic for the next section.) We will revisit the mode problem later when we look at trigonometric graphs.

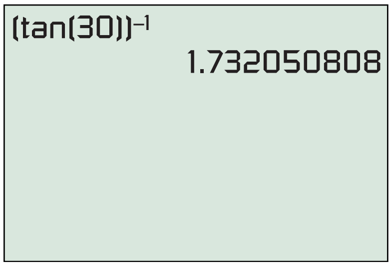

FIGURE 4.13 Finding cot $(30°)$.

2. **Using the Inverse Trig Keys to Evaluate cot, sec, and csc** There are no buttons on most calculators for cotangent, secant, and cosecant. The reason is because they can be easily evaluated by finding reciprocals of tangent, cosine, and sine, respectively. For example, Figure 4.13 shows the correct way to evaluate the cotangent of 30 degrees.

   There is also a key on the calculator for "TAN^{-1}"—but this is *not* the cotangent function! Remember that an exponent of -1 on a *function* is *never* used to denote a reciprocal; it is always used to denote the *inverse function*. We will study the inverse trigonometric functions in a later section, but meanwhile you can see that it is a bad way to evaluate cot (30) (Figure 4.14).

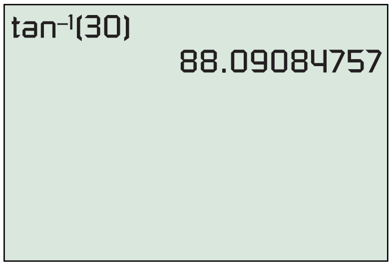

FIGURE 4.14 This is not cot $(30°)$.

3. **Using Function Shorthand That the Calculator Does Not Recognize** This error is less dangerous because it usually results in an error message. We will often abbreviate powers of trig functions, writing (for example) "$\sin^3 \theta - \cos^3 \theta$" instead of the more cumbersome "$(\sin(\theta))^3 - (\cos(\theta))^3$." The calculator does not recognize the shorthand notation and interprets it as a syntax error.

4. **Not Closing Parentheses** This general algebraic error is easy to make on calculators that automatically open a parenthesis pair whenever you type a function key. Check your calculator by pressing the SIN key. If the screen displays "sin (" instead of just "sin" then you have such a calculator. The danger arises because the calculator will automatically *close* the parenthesis pair at the end of a command if you have forgotten to do so. That is fine if you *want* the parenthesis at the end of the command, but it is bad if you want it somewhere else. For example, if you want "sin (30)" and you type "sin (30", you will get away with it. But if you want "sin $(30) + 2$" and you type "sin $(30 + 2$", you will not (Figure 4.15).

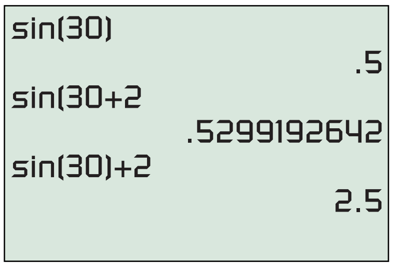

FIGURE 4.15 A correct and incorrect way to find sin $(30°) + 2$.

It is usually impossible to find an "exact" answer on a calculator, especially when evaluating trigonometric functions. The actual values are usually irrational numbers with nonterminating, nonrepeating decimal expansions. However, you can find some exact answers if you know what you are looking for, as in Example 4.

EXAMPLE 4 Getting an "Exact Answer" on a Calculator

Find the exact value of $\cos 30°$ on a calculator.

SOLUTION As you see in Figure 4.16, the calculator gives the answer 0.8660254038. However, if we recognize 30° as one of our special angles (see Example 2 in this section), we might recall that the exact answer can be written in terms of a square root. We square our answer and get 0.75, which suggests that the exact value of $\cos 30°$ is $\sqrt{3/4} = \sqrt{3}/2$. *Now try Exercise 25.*

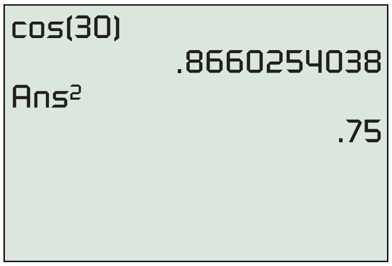

FIGURE 4.16 (Example 4)

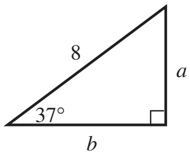

FIGURE 4.17 (Example 5)

Applications of Right Triangle Trigonometry

A triangle has six "parts," three angles and three sides, but you do not need to know all six parts to determine a triangle up to congruence. In fact, three parts are usually sufficient. The trigonometric functions take this observation a step further by giving us the means for actually *finding* the rest of the parts once we have enough parts to establish congruence. Using some of the parts of a triangle to solve for all the others is **solving a triangle**.

We will learn about solving general triangles in Sections 5.5 and 5.6, but we can already do some right triangle solving just by using the trigonometric ratios.

EXAMPLE 5 Solving a Right Triangle

A right triangle with a hypotenuse of 8 includes a 37° angle (Figure 4.17). Find the measures of the other two angles and the lengths of the other two sides.

SOLUTION Since it is a right triangle, one of the other angles is 90°. That leaves $180° - 90° - 37° = 53°$ for the third angle.

Referring to the labels in Figure 4.17, we have

$$\sin 37° = \frac{a}{8} \qquad \cos 37° = \frac{b}{8}$$

$$a = 8 \sin 37° \qquad b = 8 \cos 37°$$

$$a \approx 4.81 \qquad b \approx 6.39$$

Now try Exercise 55.

The real-world applications of triangle solving are many, reflecting the frequency with which one encounters triangular shapes in everyday life.

A Word About Rounding Answers

Notice in Example 6 that we rounded the answer to the nearest integer. In applied problems it is illogical to give answers with more decimal places of accuracy than can be guaranteed for the input values. An answer of 729.132 feet implies razor-sharp accuracy, whereas the reported height of the building (340 feet) implies a much less precise measurement. (So does the angle of 65°.) Indeed, an engineer following specific rounding criteria based on "significant digits" would probably report the answer to Example 6 as 730 feet. We will not get too picky about rounding, but we will try to be sensible.

EXAMPLE 6 Finding the Height of a Building

From a point 340 feet away from the base of the Peachtree Center Plaza in Atlanta, Georgia, the angle of elevation to the top of the building is 65°. (See Figure 4.18.) Find the height h of the building.

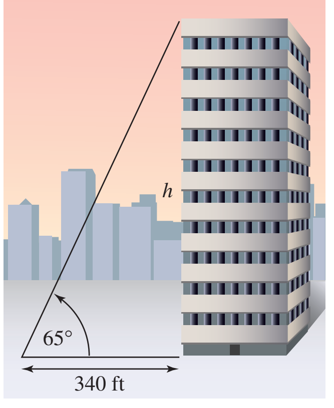

FIGURE 4.18 (Example 6)

SOLUTION We need a ratio that will relate an angle to its opposite and adjacent sides. The tangent function is the appropriate choice.

$$\tan 65° = \frac{h}{340}$$
$$h = 340 \tan 65°$$
$$h \approx 729 \text{ feet}$$

Now try Exercise 61.

QUICK REVIEW 4.2 *(For help, go to Sections P.2 and 1.7.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, use the Pythagorean Theorem to solve for x.

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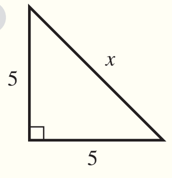

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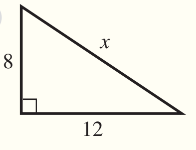

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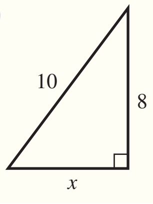

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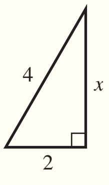

In Exercises 5 and 6, convert units.

5. 8.4 ft to inches

6. 940 ft to miles

In Exercises 7–10, solve the equation. State the correct unit.

7. $0.388 = \dfrac{a}{20.4 \text{ km}}$ **8.** $1.72 = \dfrac{23.9 \text{ ft}}{b}$

9. $\dfrac{2.4 \text{ in.}}{31.6 \text{ in.}} = \dfrac{a}{13.3}$ **10.** $\dfrac{5.9}{\beta} = \dfrac{8.66 \text{ cm}}{6.15 \text{ cm}}$

SECTION 4.2 EXERCISES

In Exercises 1–8, find the values of all six trigonometric functions of the angle θ.

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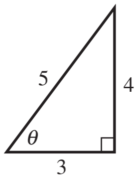

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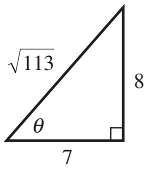

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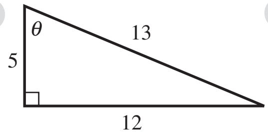

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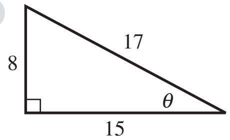

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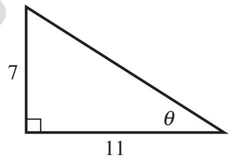

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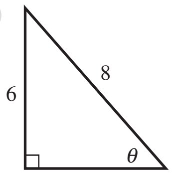

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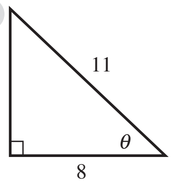

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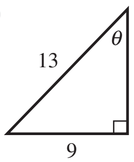

In Exercises 9–18, assume that θ is an acute angle in a right triangle satisfying the given conditions. Evaluate the remaining trigonometric functions.

9. $\sin\theta = \frac{3}{7}$ **10.** $\sin\theta = \frac{2}{3}$

11. $\cos\theta = \frac{5}{11}$ **12.** $\cos\theta = \frac{5}{8}$

13. $\tan\theta = \frac{5}{9}$ **14.** $\tan\theta = \frac{12}{13}$

15. $\cot\theta = \frac{11}{3}$ **16.** $\csc\theta = \frac{12}{5}$

17. $\csc\theta = \frac{23}{9}$ **18.** $\sec\theta = \frac{17}{5}$

In Exercises 19–24, evaluate *without* using a calculator.

19. $\sin\left(\frac{\pi}{3}\right)$ **20.** $\tan\left(\frac{\pi}{4}\right)$

21. $\cot\left(\frac{\pi}{6}\right)$ **22.** $\sec\left(\frac{\pi}{3}\right)$

23. $\cos\left(\frac{\pi}{4}\right)$ **24.** $\csc\left(\frac{\pi}{3}\right)$

In Exercises 25–28, evaluate using a calculator. Give an exact value, not an approximate answer. (See Example 4.)

25. $\sec 45°$ **26.** $\sin 60°$

27. $\csc\left(\frac{\pi}{3}\right)$

28. $\tan\left(\frac{\pi}{3}\right)$

In Exercises 29–40, evaluate using a calculator. Be sure the calculator is in the correct mode. Give answers correct to three decimal places.

29. $\sin 74°$ **30.** $\tan 8°$

31. $\cos 19°23'$ **32.** $\tan 23°42'$

33. $\tan\left(\frac{\pi}{12}\right)$ **34.** $\sin\left(\frac{\pi}{15}\right)$

35. $\sec 49°$ **36.** $\csc 19°$

37. $\cot 0.89$ **38.** $\sec 1.24$

39. $\cot\left(\frac{\pi}{8}\right)$ **40.** $\csc\left(\frac{\pi}{10}\right)$

In Exercises 41–48, find the acute angle θ that satisfies the given equation. Give θ in both degrees and radians. You should do these problems without a calculator.

41. $\sin\theta = \frac{1}{2}$ **42.** $\sin\theta = \frac{\sqrt{3}}{2}$

43. $\cot\theta = \frac{1}{\sqrt{3}}$ **44.** $\cos\theta = \frac{\sqrt{2}}{2}$

45. $\sec\theta = 2$ **46.** $\cot\theta = 1$

47. $\tan\theta = \frac{\sqrt{3}}{3}$ **48.** $\cos\theta = \frac{\sqrt{3}}{2}$

In Exercises 49–54, solve for the variable shown.

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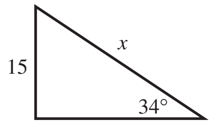

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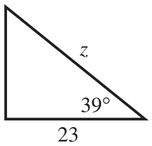

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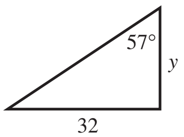
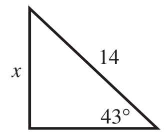


52.



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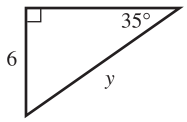

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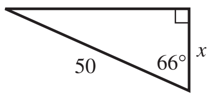
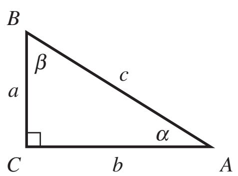


In Exercises 55–58, solve the right ΔABC for all of its unknown parts.



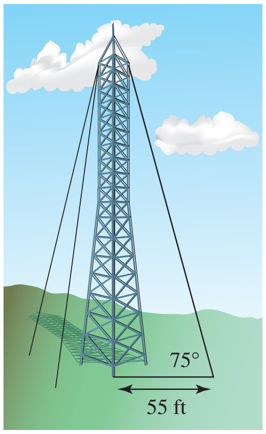
55. $\alpha = 20°$; $a = 12.3$ **56.** $\alpha = 41°$; $c = 10$

57. $\beta = 55°$; $a = 15.58$ **58.** $a = 5$; $\beta = 59°$

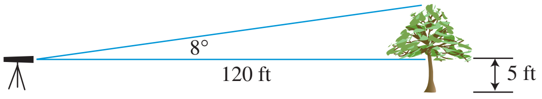
59. Writing to Learn What is $\lim_{\theta \to 0} \sin \theta$? Explain your answer in terms of right triangles in which θ gets smaller and smaller.

60. Writing to Learn What is $\lim_{\theta \to 0} \cos \theta$? Explain your answer in terms of right triangles in which θ gets smaller and smaller.

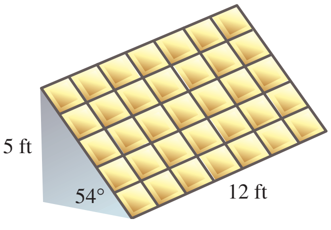
61. Height A guy wire from the top of the transmission tower at WJBC forms a 75° angle with the ground at a 55-foot distance from the base of the tower. How tall is the tower?



62. Height Kirsten places her surveyor's telescope on the top of a tripod 5 feet above the ground. She measures an 8° elevation above the horizontal to the top of a tree that is 120 feet away. How tall is the tree?

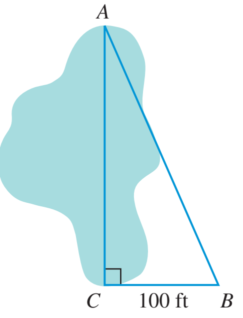


63. Group Activity Area For locations between 20° and 60° north latitude a solar collector panel should be mounted so that its angle with the horizontal is 20 greater than the local latitude. Consequently, the solar panel mounted on the roof of Solar Energy, Inc., in Atlanta (latitude 34°) forms a 54° angle with the horizontal. The bottom edge of the 12-ft-long panel is resting on the roof, and the high edge is 5 ft above the roof. What is the total area of this rectangular collector panel?

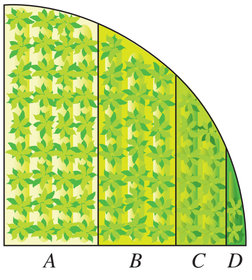


64. Height The Chrysler Building in New York City was the tallest building in the world at the time it was built. It casts a shadow approximately 130 feet long on the street when the Sun's rays form an 82.9° angle with the Earth. How tall is the building?

65. Distance DaShanda's team of surveyors had to find the distance AC across the lake at Montgomery County Park. Field assistants positioned themselves at points A and C while DaShanda set up an angle-measuring instrument at point B, 100 feet from C in a perpendicular direction. DaShanda measured $\angle ABC$ as $75°12'42''$. What is the distance AC?



66. Group Activity Garden Design Allen's garden is in the shape of a quarter-circle with radius 10 ft. He wishes to plant his garden in four parallel strips, as shown in the diagram on the left below, so that the four arcs along the circular edge of the garden are all of equal length. After measuring four equal arcs, he carefully measures the widths of the four strips and records his data in the table shown at the right below.



Strip	Width
A	3.827 ft
B	3.344 ft
C	2.068 ft
D	0.761 ft

Alicia sees Allen's data and realizes that he could have saved himself some work by figuring out the strip widths by trigonometry. By checking his data with a calculator she is able to correct two measurement errors he has made. Find Allen's two errors and correct them.

Standardized Test Questions

67. True or False If θ is an angle in any triangle, then $\tan \theta$ is the length of the side opposite θ divided by the length of the side adjacent to θ. Justify your answer.

68. True or False If A and B are angles of a triangle such that $A > B$, then $\cos A > \cos B$. Justify your answer.

You should answer these questions without using a calculator.

69. Multiple Choice Which of the following expressions does not represent a real number?

(A) sin 30° **(B)** tan 45° **(C)** cos 90°

(D) csc 90° **(E)** sec 90°

70. Multiple Choice If θ is the smallest angle in a 3–4–5 right triangle, then $\sin\theta =$

(A) $\frac{3}{5}$. **(B)** $\frac{3}{4}$. **(C)** $\frac{4}{5}$.

(D) $\frac{5}{4}$. **(E)** $\frac{5}{3}$.

71. Multiple Choice If a nonhorizontal line has slope $\sin\theta$, it will be perpendicular to a line with slope

(A) $\cos\theta$. **(B)** $-\cos\theta$. **(C)** $\csc\theta$.

(D) $-\csc\theta$. **(E)** $-\sin\theta$.

72. Multiple Choice Which of the following trigonometric ratios could *not* be π?

(A) $\tan\theta$ **(B)** $\cos\theta$ **(C)** $\cot\theta$

(D) $\sec\theta$ **(E)** $\csc\theta$

73. Trig Tables Before calculators became common classroom tools, students used trig tables to find trigonometric ratios. Below is a simplified trig table for angles between 40° and 50°. *Without using a calculator*, can you determine which column gives sine values, which gives cosine values, and which gives tangent values?

Trig Tables for Sine, Cosine, and Tangent

Angle	?	?	?
40°	0.8391	0.6428	0.7660
42°	0.9004	0.6691	0.7431
44°	0.9657	0.6947	0.7193
46°	1.0355	0.7193	0.6947
48°	1.1106	0.7431	0.6691
50°	1.1917	0.7660	0.6428

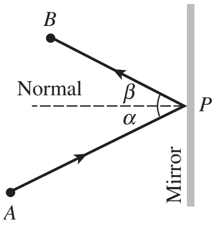
74. Trig Tables Below is a simplified trig table for angles between 30° and 40°. *Without using a calculator*, can you determine which column gives cotangent values, which gives secant values, and which gives cosecant values?

Trig Tables for Cotangent, Secant, and Cosecant

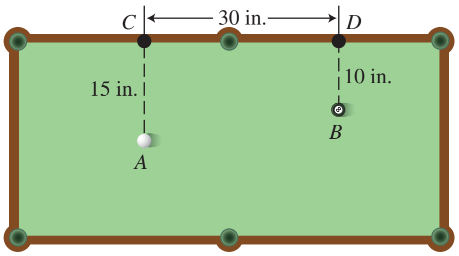
Angle	?	?	?
30°	1.1547	1.7321	2.0000
32°	1.1792	1.6003	1.8871
34°	1.2062	1.4826	1.7883
36°	1.2361	1.3764	1.7013
38°	1.2690	1.2799	1.6243
40°	1.3054	1.1918	1.5557

Explorations

75. Mirrors In the figure, a light ray shining from point A to point P on the mirror will bounce to point B in such a way that the *angle of incidence* α will equal the *angle of reflection* β. This is the *law of reflection* derived from physical experiments. Both angles are measured from the *normal line*, which is perpendicular to the mirror at the point of reflection P. If A is 2 m farther from the mirror than is B, and if $\alpha = 30°$ and $AP = 5$ m, what is the length PB?

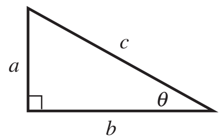


76. Pool On the pool table shown in the figure, where along the portion CD of the railing should you direct ball A so that it will bounce off CD and strike ball B? Assume that A obeys the law of reflection relative to rail CD.



Extending the Ideas

77. Using the labeling of the triangle below, prove that if θ is an acute angle in any right triangle, $(\sin\theta)^2 + (\cos\theta)^2 = 1$.



78. Using the labeling of the triangle below, prove that the area of the triangle is equal to $(1/2)\,ab\sin\theta$. [*Hint:* Start by drawing the altitude to side b and finding its length.]