CHAPTER 4

Trigonometric Functions

When the motion of an object causes air molecules to vibrate, we hear a sound. We measure sound according to its pitch and loudness, which are attributes associated with the frequency and amplitude of sound waves. As we shall see, it is the branch of mathematics called trigonometry that enables us to analyze waves of all kinds; indeed, that is only one application of this powerful analytical tool. See page 393 for an application of trigonometry to sound waves.

Hipparchus of Nicaea (190–120 B.C.E.)

Hipparchus of Nicaea, the "father of trigonometry," compiled the first trigonometric tables to simplify the study of astronomy more than 2000 years ago. Today, that same mathematics enables us to store sound waves digitally on a compact disc. Hipparchus wrote during the second century B.C.E., but he was not the first mathematician to "do" trigonometry. Greek mathematicians like Hippocrates of Chois (470–410 B.C.E.) and Eratosthenes of Cyrene (276–194 B.C.E.) had paved the way for using triangle ratios in astronomy, and those same triangle ratios had been used by Egyptian and Babylonian engineers at least 4000 years earlier. The term "trigonometry" itself emerged in the 16th century, although it derives from ancient Greek roots: "tri" (three), "gonos" (side), and "metros" (measure).

Chapter 4 Overview

The trigonometric functions arose from the consideration of ratios within right triangles, the ultimate computational tool for engineers in the ancient world. As the great mysteries of civilization progressed from a flat Earth to a world of circles and spheres, trigonometry was soon seen to be the secret to understanding circular phenomena as well. Then circular motion led to harmonic motion and waves, and suddenly trigonometry was the indispensable tool for understanding everything from electrical current to modern telecommunications.

The advent of calculus made the trigonometric functions more important than ever. It turns out that every kind of periodic (recurring) behavior can be modeled to any degree of accuracy by simply combining sine functions. The modeling aspect of trigonometric functions is another focus of our study.

4.1 Angles and Their Measures

What you'll learn about

- The Problem of Angular Measure
- Degrees and Radians
- Circular Arc Length
- Angular and Linear Motion

… and why

Angles are the domain elements of the trigonometric functions.

The Problem of Angular Measure

The input variable in a trigonometric function is an angle measure; the output is a real number. Believe it or not, this poses an immediate problem for us if we choose to measure our angles in degrees (as most of us did in our geometry courses).

The problem is that degree units have no mathematical relationship whatsoever to linear units. There are 360 degrees in a circle of radius 1. What relationship does the 360 have to the 1? In what sense is it 360 times as big? Answering these questions isn't possible, because a "degree" is another unit altogether.

Consider the diagrams in Figure 4.1. The ratio of s to h in the right triangle in Figure 4.1a is independent of the size of the triangle. (You may recall this fact about similar triangles from geometry.) This valuable insight enabled early engineers to compute triangle ratios on a small scale before applying them to much larger projects. That was (and still is) trigonometry in its most basic form. For astronomers tracking celestial motion, however, the extended diagram in Figure 4.1b was more interesting. In this picture, s is half a chord in a circle of radius h, and θ is a **central angle** of the circle intercepting a circular arc of length a. If θ were 40 degrees, we might call a a "40-degree arc" because of its direct association with the central angle θ, but notice that a also has a *length* that can be measured in the *same units* as the other lengths in the picture. Over time it became natural to think of the angle being determined by the arc rather than the arc being determined by the angle, and that led to radian measure.

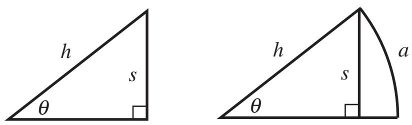
Why 360°?

The idea of dividing a circle into 360 equal pieces dates back to the sexagesimal (60-based) counting system of the ancient Sumerians. The appeal of 60 was that it was evenly divisible by so many numbers (2, 3, 4, 5, 6, 10, 12, 15, 20, and 30). Early astronomical calculations wedded the sexagesimal system to circles, and the rest is history.

Degrees and Radians

A **degree**, represented by the symbol °, is a unit of angular measure equal to 1/180th of a straight angle. In the DMS (degree-minute-second) system of angular measure, each degree is subdivided into 60 **minutes** (denoted by ′) and each minute is subdivided into 60 **seconds** (denoted by ″). (Notice that Sumerian influence again.)

Example 1 illustrates how to convert from degrees in decimal form to DMS and vice versa.



(a) (b)

FIGURE 4.1 The pictures that motivated trigonometry.

Calculator Conversions

Your calculator probably has built-in functionality to convert degrees to DMS. Consult your owner's manual. Meanwhile, you should try some conversions the "long way" to get a better feel for how DMS works.

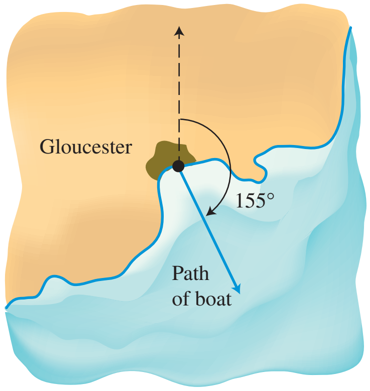


FIGURE 4.2 The course of a fishing boat bearing 155° out of Gloucester.

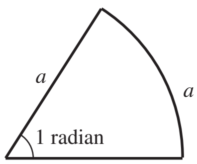


FIGURE 4.3 In a circle, a central angle of 1 radian intercepts an arc of length one radius.

EXAMPLE 1 Working with DMS Measure

(a) Convert 37.425° to DMS.

(b) Convert 42°24′36″ to degrees.

SOLUTION

(a) We need to convert the fractional part to minutes and seconds. First we convert 0.425° to minutes:

$$0.425°\left(\frac{60'}{1°}\right) = 25.5'$$

Then we convert 0.5 minute to seconds:

$$0.5'\left(\frac{60''}{1'}\right) = 30''$$

Putting it all together, we find that $37.425° = 37°25'30''$.

(b) Each minute is 1/60th of a degree, and each second is 1/3600th of a degree. Therefore,

$$42°24'36'' = 42° + \left(\frac{24}{60}\right)^{\circ} + \left(\frac{36}{3600}\right)^{\circ} = 42.41°.$$

Now try Exercises 3 and 5.

In navigation, the **course** or **bearing** of an object is sometimes given as the angle of the **line of travel** measured clockwise from due north. For example, the line of travel in Figure 4.2 has the bearing of 155°.

In this book we use degrees to measure angles in their familiar geometric contexts, especially when applying trigonometry to real-world problems in surveying, construction, and navigation, where degrees are still the accepted units of measure. When we shift our attention to the trigonometric *functions*, however, we will measure angles in *radians* so that domain and range values can be measured on comparable scales.

DEFINITION Radian

A central angle of a circle has measure 1 **radian** if it intercepts an arc with the same length as the radius. (See Figure 4.3.)

EXPLORATION 1 Constructing a 1-Radian Angle

Carefully draw a large circle on a piece of paper, either by tracing around a circular object or by using a compass. Identify the center of the circle (O) and draw a radius horizontally from O toward the right, intersecting the circle at point A. Then cut a piece of thread or string the same size as the radius. Place one end of the string at A and bend it around the circle counterclockwise, marking the point B on the circle where the other end of the string ends up. Draw the radius from O to B.

The measure of angle AOB is one radian.

1. What is the circumference of the circle, in terms of its radius r?
2. How many radians must there be in a complete circle?
3. If we cut a piece of thread 3 times as big as the radius, would it extend halfway around the circle? Why or why not?
4. How many radians are in a straight angle?

EXAMPLE 2 Working with Radian Measure

(a) How many radians are in 90 degrees?

(b) How many degrees are in $\pi/3$ radians?

(c) Find the length of an arc intercepted by a central angle of 1/2 radian in a circle of radius 5 inches.

(d) Find the radian measure of a central angle that intercepts an arc of length s in a circle of radius r.

SOLUTION

(a) Since π radians and 180° both measure a straight angle, we can use the conversion factor $(\pi \text{ radians})/(180°) = 1$ to convert degrees to radians:

$$90°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{90\pi}{180} \text{ radians} = \frac{\pi}{2} \text{ radians}$$

(b) In this case, we use the conversion factor $(180°)/(\pi \text{ radians}) = 1$ to convert radians to degrees:

$$\left(\frac{\pi}{3} \text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right) = \frac{180°}{3} = 60°$$

(c) A central angle of 1 radian intercepts an arc of length 1 radius, which is 5 inches. Therefore, a central angle of 1/2 radian intercepts an arc of length 1/2 radius, which is 2.5 inches.

(d) We can solve this problem with ratios:

$$\frac{x \text{ radians}}{s \text{ units}} = \frac{1 \text{ radian}}{r \text{ units}}$$

$$xr = s$$

$$x = \frac{s}{r}$$

Now try Exercises 11 and 19.

Degree-Radian Conversion

To convert radians to degrees, multiply by $\dfrac{180°}{\pi \text{ radians}}$.

To convert degrees to radians, multiply by $\dfrac{\pi \text{ radians}}{180°}$.

With practice, you can perform these conversions in your head. The key is to think of a straight angle equaling π radians as readily as you think of it equaling 180 degrees.

Circular Arc Length

Since a central angle of 1 radian always intercepts an arc of one radius in length, it follows that a central angle of θ radians in a circle of radius r intercepts an arc of length θr. This gives us a convenient formula for measuring arc length.

Arc Length Formula (Radian Measure)

If θ is a central angle in a circle of radius r, and if θ is measured in radians, then the length s of the intercepted arc is given by

$$s = r\theta.$$

Does a Radian Have Units?

The formula $s = r\theta$ implies an interesting fact about radians: As long as s and r are measured in the same units, the radian is unit-neutral. For example, if $r = 5$ inches and $\theta = 2$ radians, then $s = 10$ inches (not 10 "inch-radians"). This unusual situation arises from the fact that the definition of the radian is tied to the length of the radius, units and all.

A somewhat less simple formula (which incorporates the degree-radian conversion formula) applies when θ is measured in degrees.

Arc Length Formula (Degree Measure)

If θ is a central angle in a circle of radius r, and if θ is measured in degrees, then the length s of the intercepted arc is given by

$$s = \frac{\pi r\theta}{180}.$$

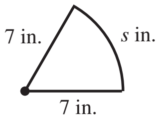


FIGURE 4.4 A 60° slice of a large pizza. (Example 3)

EXAMPLE 3 Perimeter of a Pizza Slice

Find the perimeter of a 60° slice of a large (7-in. radius) pizza.

SOLUTION The perimeter (Figure 4.4) is 7 in. + 7 in. + s in., where s is the arc length of the pizza's curved edge. By the arc length formula:

$$s = \frac{\pi(7)(60)}{180} = \frac{7\pi}{3} \approx 7.3$$

The perimeter is approximately 21.3 in. *Now try Exercise 35.*

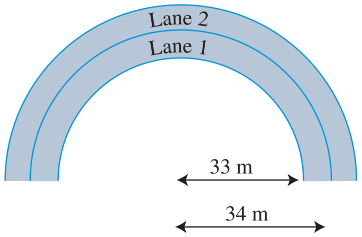


FIGURE 4.5 Two lanes of the track described in Example 4.

EXAMPLE 4 Designing a Running Track

The running lanes at the Emery Sears track at Bluffton College are 1 meter wide. The inside radius of lane 1 is 33 meters, and the inside radius of lane 2 is 34 meters. How much longer is lane 2 than lane 1 around one turn? (See Figure 4.5.)

SOLUTION We think this solution through in radians. Each lane is a semicircle with central angle $\theta = \pi$ and length $s = r\theta = r\pi$. The difference in their lengths, therefore, is $34\pi - 33\pi = \pi$. Lane 2 is about 3.14 meters longer than lane 1.
Now try Exercise 37.

Angular and Linear Motion

In applications it is sometimes necessary to connect *angular speed* (measured in units like revolutions per minute) to *linear speed* (measured in units like miles per hour). The connection is usually provided by one of the arc length formulas or by a conversion factor that equates "1 radian" of angular measure to "1 radius" of arc length.

EXAMPLE 5 Using Angular Speed

Albert Juarez's truck has wheels 36 inches in diameter. If the wheels are rotating at 630 rpm (revolutions per minute), find the truck's speed in miles per hour.

SOLUTION We convert revolutions per minute to miles per hour by a series of unit conversion factors. Note that the conversion factor $\frac{18 \text{ in.}}{1 \text{ radian}}$ works for this example because the radius is 18 in.

$$\frac{630 \text{ rev}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{2\pi \text{ radians}}{1 \text{ rev}} \times \frac{18 \text{ in.}}{1 \text{ radian}} \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ mi}}{5280 \text{ ft}}$$

$$\approx 67.47 \frac{\text{mi}}{\text{hr}}$$

Now try Exercise 45.

A **nautical mile** mile (naut mi) is the length of 1 minute of arc along Earth's equator. Figure 4.6 shows, though not to scale, a central angle *AOB* of Earth that measures 1/60 of a degree. It intercepts an arc 1 naut mi long.

The arc length formula allows us to convert between nautical miles and **statute miles** (stat mi), the familiar "land mile" of 5280 feet.

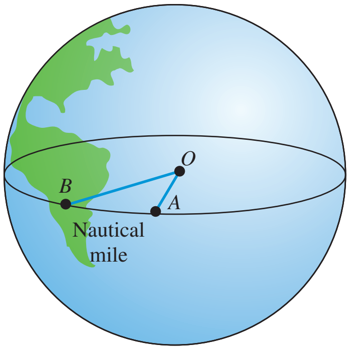


FIGURE 4.6 Although Earth is not a perfect sphere, its diameter is, on average, 7912.18 statute miles. A nautical mile is 1′ of Earth's circumference at the equator.

EXAMPLE 6 Converting to Nautical Miles

Megan McCarty, a pilot for Western Airlines, frequently pilots flights from Boston to San Francisco, a distance of 2698 stat mi. Captain McCarty's calculations of flight time are based on nautical miles. How many nautical miles is it from Boston to San Francisco?

SOLUTION The radius of the Earth at the equator is approximately 3956 stat mi. Convert 1 minute to radians:

$$1' = \left(\frac{1}{60}\right)^{\circ} \times \frac{\pi \text{ rad}}{180^{\circ}} = \frac{\pi}{10{,}800} \text{ radians}$$

Now we can apply the formula $s = r\theta$:

$$\begin{aligned} 1 \text{ naut mi} &= (3956)\left(\frac{\pi}{10{,}800}\right) \text{ stat mi} \\ &\approx 1.15 \text{ stat mi} \\ 1 \text{ stat mi} &= \left(\frac{10{,}800}{3956\pi}\right) \text{ naut mi} \\ &\approx 0.87 \text{ naut mi} \end{aligned}$$

The distance from Boston to San Francisco is

$$2698 \text{ stat mi} = \frac{2698 \cdot 10{,}800}{3956\pi} \approx 2345 \text{ naut mi}.$$

Now try Exercise 51.

Distance Conversions

1 statute mile $\approx$ 0.87 nautical mile

1 nautical mile $\approx$ 1.15 statute miles

QUICK REVIEW 4.1 *(For help, go to Section 1.7.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1 and 2, find the circumference of the circle with the given radius r. State the correct unit.

1. $r = 2.5$ in. **2.** $r = 4.6$ m

In Exercises 3 and 4, find the radius of the circle with the given circumference C.

3. $C = 12$ m **4.** $C = 8$ ft

In Exercises 5 and 6, evaluate the expression for the given values of the variables. State the correct unit.

5. $s = r\theta$

(a) $r = 9.9$ ft $\theta = 4.8$ rad

(b) $r = 4.1$ km $\theta = 9.7$ rad

6. $v = r\omega$

(a) $r = 8.7$ m $\omega = 3.0$ rad/sec

(b) $r = 6.2$ ft $\omega = 1.3$ rad/sec

In Exercises 7–10, convert from miles per hour to feet per second or from feet per second to miles per hour.

7. 60 mph **8.** 45 mph

9. 8.8 ft/sec **10.** 132 ft/sec

SECTION 4.1 EXERCISES

In Exercises 1–4, convert from DMS to decimal form.

1. 23°12′ **2.** 35°24′

3. 118°44′15″ **4.** 48°30′36″

In Exercises 5–8, convert from decimal form to degrees, minutes, seconds (DMS).

5. 21.2° **6.** 49.7°

7. 118.32° **8.** 99.37°

In Exercises 9–16, convert from DMS to radians.

9. 60° **10.** 90°

11. 120° **12.** 150°

13. 71.72° **14.** 11.83°

15. 61°24′ **16.** 75°30′

In Exercises 17–24, convert from radians to degrees.

17. $\pi/6$ **18.** $\pi/4$

19. $\pi/10$ **20.** $3\pi/5$

21. $7\pi/9$ **22.** $13\pi/20$

23. 2 **24.** 1.3

In Exercises 25–32, use the appropriate arc length formula to find the missing information.

	s	r	θ
25.	?	2 in.	25 rad
26.	?	1 cm	70 rad
27.	1.5 ft	?	$\pi/4$ rad
28.	2.5 cm	?	$\pi/3$ rad
29.	3 m	1 m	?
30.	4 in.	7 in.	?
31.	40 cm	?	20°
32.	?	5 ft	18°

In Exercises 33 and 34, a central angle θ intercepts arcs s_1 and s_2 on two concentric circles with radii r_1 and r_2, respectively. Find the missing information.

	θ	r_1	s_1	r_2	s_2
33.	?	11 cm	9 cm	44 cm	?
34.	?	8 km	36 km	?	72 km

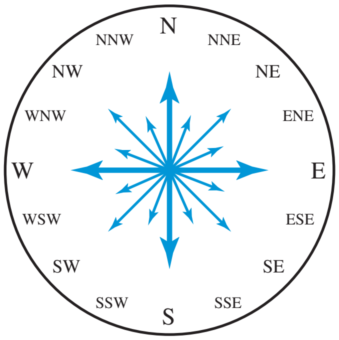
35. To the nearest inch, find the perimeter of a 10-degree sector cut from a circular disc of radius 11 inches.

36. A 100-degree arc of a circle has a length of 7 cm. To the nearest centimeter, what is the radius of the circle?

37. It takes ten identical pieces to form a circular track for a pair of toy racing cars. If the inside arc of each piece is 3.4 inches shorter than the outside arc, what is the width of the track?

38. The concentric circles on an archery target are 6 inches apart. The inner circle (red) has a perimeter of 37.7 inches. What is the perimeter of the next-largest (yellow) circle?

Exercises 39–42 refer to the 16 compass bearings shown. North corresponds to an angle of 0°, and other angles are measured clockwise from north.



39. **Compass Reading** Find the angle in degrees that describes the compass bearing.

(a) NE (northeast)

(b) NNE (north-northeast)

(c) WSW (west-southwest)

40. **Compass Reading** Find the angle in degrees that describes the compass bearing.

(a) SSW (south-southwest)

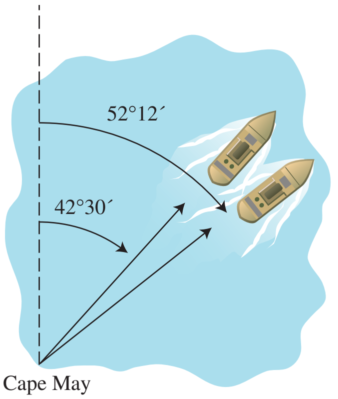
(b) WNW (west-northwest)

(c) NNW (north-northwest)

41. **Compass Reading** Which compass direction is closest to a bearing of 121°?

42. **Compass Reading** Which compass direction is closest to a bearing of 219°?

43. **Navigation** Two Coast Guard patrol boats leave Cape May at the same time. One travels with a bearing of 42°30′ and the other with a bearing of 52°12′. If they travel at the same speed, approximately how far apart will they be when they are 25 statute miles from Cape May?



44. **Automobile Design** Table 4.1 shows the size specifications for the tires that come as standard equipment on three different 2009 American vehicles.

Table 4.1 Tire Sizes for Three Vehicles

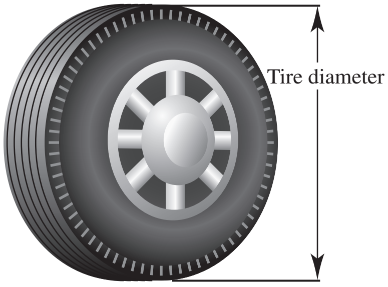
Vehicle	Tire Type	Tire Diameter
Ford Taurus	215/60–17	27.2 inches
Dodge Charger RT	225/60–18	28.6 inches
Mercury Mariner	235/70–16	29.0 inches

Source: Tirerack.com

(a) Find the speed of each vehicle in mph when the wheels are turning at 800 revolutions per minute.

(b) Compared to the Mercury Mariner, how many more revolutions must the tire of the Ford Taurus make in order to travel a mile?

(c) **Writing to Learn** It is unwise (and in some cases illegal) to equip a vehicle with wheels of a larger diameter than those for which it was designed. If a 2009 Ford Taurus were equipped with 29-inch tires, how would it affect the odometer (which measures mileage) and speedometer readings?



45. **Bicycle Racing** Cathy Nguyen races on a bicycle with 13-inch-radius wheels. When she is traveling at a speed of 44 ft/sec, how many revolutions per minute are her wheels making?

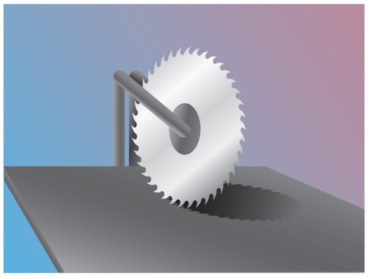
46. **Tire Sizing** The numbers in the "tire type" column in Exercise 44 give the size of the tire in the P-metric system. Each number is of the form $W/R-D$, where W is the width of the tire in millimeters, $R/100$ is the ratio of the sidewall (S) of the tire to its width W, and D is the diameter (in inches) of the wheel without the tire.

(a) Show that $S = WR/100$ millimeters $= WR/2540$ inches.

(b) The tire diameter is $D + 2S$. Derive a formula for the tire diameter that involves only the variables D, W, and R.

(c) Use the formula in part (b) to verify the tire diameters in Exercise 44. Then find the tire diameter for the 2009 Honda Ridgeline, which comes with 245/65–17 tires.

47. **Tool Design** A radial arm saw has a circular cutting blade with a diameter of 10 inches. It spins at 2000 rpm. If there are 12 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?



48. **Navigation** Sketch a diagram of a ship on the given course.

(a) 35° (b) 128° (c) 310°

49. **Navigation** The captain of the tourist boat *Julia* out of Oak Harbor follows a 38° course for 2 miles and then changes to a 47° course for the next 4 miles. Draw a sketch of this trip.

50. **Navigation** Points A and B are 257 nautical miles apart. How far apart are A and B in statute miles?

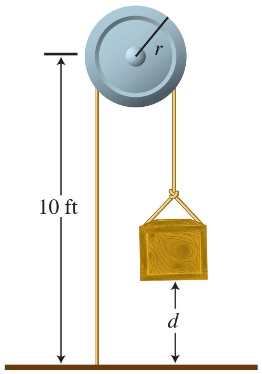
51. **Navigation** Points C and D are 895 statute miles apart. How far apart are C and D in nautical miles?

52. Designing a Sports Complex Example 4 describes how lanes 1 and 2 compare in length around one turn of a track. Find the differences in the lengths of these lanes around one turn of the same track.

(a) Lanes 5 and 6 **(b)** Lanes 1 and 6

53. Mechanical Engineering A simple pulley with the given radius r used to lift heavy objects is positioned 10 feet above ground level. Given that the pulley rotates $\theta°$, determine the height to which the object is lifted.

(a) $r = 4$ in., $\theta = 720°$ **(b)** $r = 2$ ft, $\theta = 180°$



54. Foucault Pendulum In 1851 the French physicist Jean Foucault used a pendulum to demonstrate the Earth's rotation. There are now over 30 Foucault pendulum displays in the United States. The Foucault pendulum at the Smithsonian Institution in Washington, DC, consists of a large brass ball suspended by a thin 52-foot cable. If the ball swings through an angle of 1°, how far does it travel?

55. Group Activity Air Conditioning Belt The belt on an automobile air conditioner connects metal wheels with radii $r = 4$ cm and $R = 7$ cm. The angular speed of the larger wheel is 120 rpm.

(a) What is the angular speed of the larger wheel in radians per second?

(b) What is the linear speed of the belt in centimeters per second?

(c) What is the angular speed of the smaller wheel in radians per second?

56. Group Activity Ship's Propeller The propellers of the *Amazon Paradise* have a radius of 1.2 m. At full throttle the propellers turn at 135 rpm.

(a) What is the angular speed of a propeller blade in radians per second?

(b) What is the linear speed of the tip of the propeller blade in meters per second?

(c) What is the linear speed (in meters per second) of a point on a blade halfway between the center of the propeller and the tip of the blade?

Standardized Test Questions

57. True or False If horse A is twice as far as horse B from the center of a merry-go-round, then horse A travels twice as fast as horse B. Justify your answer.

58. True or False The radian measure of all three angles in a triangle can be integers. Justify your answer.

You may use a graphing calculator when answering these questions.

59. Multiple Choice What is the radian measure of an angle of x degrees?

(A) πx **(B)** $x/180$
(C) $\pi x/180$ **(D)** $180x/\pi$
(E) $180/x\pi$

60. Multiple Choice If the perimeter of a sector is 4 times its radius, then the radian measure of the central angle of the sector is

(A) 2. **(B)** 4.
(C) $2/\pi$. **(D)** $4/\pi$.
(E) impossible to determine without knowing the radius.

61. Multiple Choice A bicycle with 26-inch-diameter wheels is traveling at 10 miles per hour. To the nearest whole number, how many revolutions does each wheel make per minute?

(A) 54 **(B)** 129
(C) 259 **(D)** 406
(E) 646

62. Multiple Choice A central angle in a circle of radius r has a measure of θ radians. If the same central angle were drawn in a circle of radius $2r$, its radian measure would be

(A) $\dfrac{\theta}{2}$. **(B)** $\dfrac{\theta}{2r}$.
(C) θ. **(D)** 2θ.
(E) $2r\theta$.

Explorations

Table 4.2 shows the latitude-longitude locations of several U.S. cities. Latitude is measured from the equator. Longitude is measured from the Greenwich meridian that passes north-south through London.

Table 4.2 Latitude and Longitude Locations of U.S. Cities

City	Latitude	Longitude
Atlanta	33°45′	84°23′
Chicago	41°51′	87°39′
Detroit	42°20′	83°03′
Los Angeles	34°03′	118°15′
Miami	25°46′	80°12′
Minneapolis	44°59′	93°16′
New Orleans	29°57′	90°05′
New York	40°43′	74°0′
San Diego	32°43′	117°09′
San Francisco	37°47′	122°25′
Seattle	47°36′	122°20′

Source: U.S. Department of the Interior, as reported in The World Almanac and Book of Facts 2009.

In Exercises 63–66, find the difference in longitude between the given cities.

63. Atlanta and San Francisco

64. New York and San Diego

65. Minneapolis and Chicago

66. Miami and Seattle

In Exercises 67–70, assume that the two cities have the same longitude (that is, assume that one is directly north of the other), and find the distance between them in nautical miles.

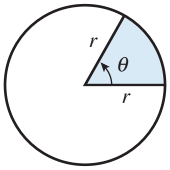
67. San Diego and Los Angeles

68. Seattle and San Francisco

69. New Orleans and Minneapolis

70. Detroit and Atlanta

71. Group Activity Area of a Sector A *sector of a circle* (shaded in the figure) is a region bounded by a central angle of a circle and its intercepted arc. Use the fact that the areas of sectors are proportional to their central angles to prove that



$$A = \frac{1}{2}r^2\theta$$

where r is the radius and θ is in radians.

Extending the Ideas

72. Area of a Sector Use the formula $A = (1/2)r^2\theta$ to determine the area of the sector with given radius r and central angle θ.

(a) $r = 5.9$ ft, $\theta = \pi/5$

(b) $r = 1.6$ km, $\theta = 3.7$

73. Navigation Control tower A is 60 miles east of control tower B. At a certain time an airplane is on bearings of 340° from tower A and 37° from tower B. Use a drawing to model the exact location of the airplane.

74. Bicycle Racing Ben Scheltz's bike wheels are 28 inches in diameter, and for high gear the pedal sprocket is 9 inches in diameter and the wheel sprocket is 3 inches in diameter. Find the angular speed in radians per second of the wheel and of both sprockets when Ben reaches his peak racing speed of 66 ft/sec in high gear.