3.4 Properties of Logarithmic Functions

What you'll learn about

- Properties of Logarithms
- Change of Base
- Graphs of Logarithmic Functions with Base b
- Re-expressing Data

... and why

The applications of logarithms are based on their many special properties, so learn them well.

Properties of Logarithms

Logarithms have special algebraic traits that historically made them indispensable for calculations and that still make them valuable in many areas of application and modeling. In Section 3.3 we learned about the inverse relationship between exponents and logarithms and how to apply some basic properties of logarithms. We now delve deeper into the nature of logarithms to prepare for equation solving and modeling.

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$, and c any real number.

- **Product rule**: $\log_b (RS) = \log_b R + \log_b S$
- **Quotient rule**: $\log_b \dfrac{R}{S} = \log_b R - \log_b S$
- **Power rule**: $\log_b R^c = c \log_b R$

Properties of Exponents

Let b, x, and y be real numbers with $b > 0$.

1. $b^x \cdot b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$

The properties of exponents in the margin are the basis for these three properties of logarithms. For instance, the first exponent property listed in the margin is used to verify the product rule.

EXAMPLE 1 Proving the Product Rule for Logarithms

Prove $\log_b (RS) = \log_b R + \log_b S$.

SOLUTION Let $x = \log_b R$ and $y = \log_b S$. The corresponding exponential statements are $b^x = R$ and $b^y = S$. Therefore,

$$
\begin{aligned}
RS &= b^x \cdot b^y \\
&= b^{x+y} && \text{First property of exponents} \\
\log_b (RS) &= x + y && \text{Change to logarithmic form.} \\
&= \log_b R + \log_b S && \text{Use the definitions of } x \text{ and } y.
\end{aligned}
$$

Now try Exercise 37.

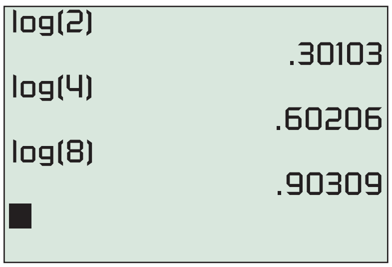


FIGURE 3.26 An arithmetic pattern of logarithms. (Exploration 1)

EXPLORATION 1 Exploring the Arithmetic of Logarithms

Use the 5-decimal-place approximations shown in Figure 3.26 to support the properties of logarithms numerically.

1. Product $\log (2 \cdot 4) = \log 2 + \log 4$
2. Quotient $\log \left(\dfrac{8}{2}\right) = \log 8 - \log 2$
3. Power $\log 2^3 = 3 \log 2$

(continued)

Now evaluate the common logs of other positive integers using the information given in Figure 3.26 and without using your calculator.

4. Use the fact that $5 = 10/2$ to evaluate log 5.
5. Use the fact that 16, 32, and 64 are powers of 2 to evaluate log 16, log 32, and log 64.
6. Evaluate log 25, log 40, and log 50.

List all of the positive integers less than 100 whose common logs can be evaluated knowing only log 2 and the properties of logarithms and without using a calculator.

When we solve equations algebraically that involve logarithms, we often have to rewrite expressions using properties of logarithms. Sometimes we need to expand as far as possible, and other times we condense as much as possible. The next three examples illustrate how properties of logarithms can be used to change the form of expressions involving logarithms.

EXAMPLE 2 Expanding the Logarithm of a Product

Assuming x and y are positive, use properties of logarithms to write $\log(8xy^4)$ as a sum of logarithms or multiples of logarithms.

SOLUTION

$$\begin{aligned} \log(8xy^4) &= \log 8 + \log x + \log y^4 && \text{Product rule} \\ &= \log 2^3 + \log x + \log y^4 && 8 = 2^3 \\ &= 3\log 2 + \log x + 4\log y && \text{Power rule} \end{aligned}$$

Now try Exercise 1.

EXAMPLE 3 Expanding the Logarithm of a Quotient

Assuming x is positive, use properties of logarithms to write $\ln(\sqrt{x^2+5}/x)$ as a sum or difference of logarithms or multiples of logarithms.

SOLUTION

$$\begin{aligned} \ln\frac{\sqrt{x^2+5}}{x} &= \ln\frac{(x^2+5)^{1/2}}{x} \\ &= \ln(x^2+5)^{1/2} - \ln x && \text{Quotient rule} \\ &= \frac{1}{2}\ln(x^2+5) - \ln x && \text{Power rule} \end{aligned}$$

Now try Exercise 9.

EXAMPLE 4 Condensing a Logarithmic Expression

Assuming x and y are positive, write $\ln x^5 - 2\ln(xy)$ as a single logarithm.

SOLUTION

$$\begin{aligned} \ln x^5 - 2\ln(xy) &= \ln x^5 - \ln(xy)^2 && \text{Power rule} \\ &= \ln x^5 - \ln(x^2y^2) \\ &= \ln\frac{x^5}{x^2y^2} && \text{Quotient rule} \\ &= \ln\frac{x^3}{y^2} \end{aligned}$$

Now try Exercise 13.

As we have seen, logarithms have some surprising properties. It is easy to overgeneralize and fall into misconceptions about logarithms. Exploration 2 should help you discern what is true and false about logarithmic relationships.

EXPLORATION 2 Discovering Relationships and Nonrelationships

Of the eight relationships suggested here, four are *true* and four are *false* (using values of x within the domains of both sides of the equations). Thinking about the properties of logarithms, make a prediction about the truth of each statement. Then test each with some specific numerical values for x. Finally, compare the graphs of the two sides of the equation.

1. $\ln(x+2) = \ln x + \ln 2$
2. $\log_3(7x) = 7\log_3 x$
3. $\log_2(5x) = \log_2 5 + \log_2 x$
4. $\ln \frac{x}{5} = \ln x - \ln 5$
5. $\log \frac{x}{4} = \frac{\log x}{\log 4}$
6. $\log_4 x^3 = 3\log_4 x$
7. $\log_5 x^2 = (\log_5 x)(\log_5 x)$
8. $\log|4x| = \log 4 + \log|x|$

Which four are true, and which four are false?

Change of Base

When working with a logarithmic expression with an undesirable base, it is possible to change the expression into a quotient of logarithms with a different base. For example, it is hard to evaluate $\log_4 7$ because 7 is not a simple power of 4 and there is no $\boxed{\log_4}$ key on a calculator or grapher.

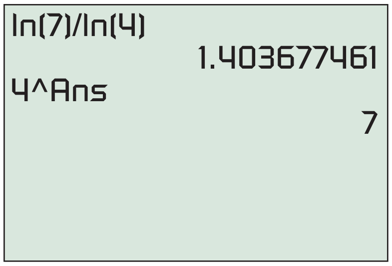


FIGURE 3.27 Evaluating and checking $\log_4 7$.

We can work around this problem with some algebraic trickery. First let $y = \log_4 7$. Then

$$\begin{aligned} 4^y &= 7 && \text{Switch to exponential form.} \\ \ln 4^y &= \ln 7 && \text{Apply ln.} \\ y\ln 4 &= \ln 7 && \text{Power rule} \\ y &= \frac{\ln 7}{\ln 4} && \text{Divide by ln 4.} \end{aligned}$$

Using a grapher (Figure 3.27), we see that

$$\log_4 7 = \frac{\ln 7}{\ln 4} = 1.4036\ldots$$

We generalize this useful trickery as the change-of-base formula:

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Calculators and graphers generally have two logarithm keys—LOG and LN—which correspond to the bases 10 and e, respectively. So we often use the change-of-base formula in one of the following two forms:

$$\log_b x = \frac{\log x}{\log b} \quad \text{or} \quad \log_b x = \frac{\ln x}{\ln b}$$

These two forms are useful in evaluating logarithms and graphing logarithmic functions.

EXAMPLE 5 Evaluating Logarithms by Changing the Base

(a) $\log_3 16 = \dfrac{\ln 16}{\ln 3} = 2.523\ldots \approx 2.52$

(b) $\log_6 10 = \dfrac{\log 10}{\log 6} = \dfrac{1}{\log 6} = 1.285\ldots \approx 1.29$

(c) $\log_{1/2} 2 = \dfrac{\ln 2}{\ln (1/2)} = \dfrac{\ln 2}{\ln 1 - \ln 2} = \dfrac{\ln 2}{-\ln 2} = -1$ ***Now try Exercise 23.***

Graphs of Logarithmic Functions with Base *b*

Using the change-of-base formula we can rewrite any logarithmic function $g(x) = \log_b x$ as

$$g(x) = \frac{\ln x}{\ln b} = \frac{1}{\ln b} \ln x.$$

Therefore, every logarithmic function is a constant multiple of the natural logarithmic function $f(x) = \ln x$. If the base is $b > 1$, the graph of $g(x) = \log_b x$ is a vertical stretch or shrink of the graph of $f(x) = \ln x$ by the factor $1/\ln b$. If $0 < b < 1$, a reflection across the x-axis is required as well.

EXAMPLE 6 Graphing Logarithmic Functions

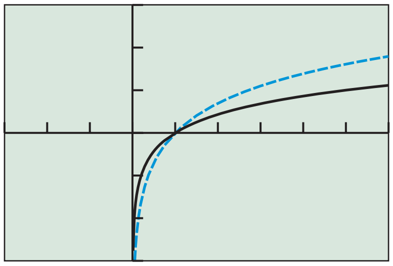
Describe how to transform the graph of $f(x) = \ln x$ into the graph of the given function. Sketch the graph by hand and support your answer with a grapher.

(a) $g(x) = \log_5 x$ **(b)** $h(x) = \log_{1/4} x$

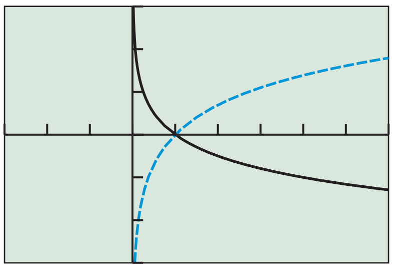
SOLUTION

(a) Because $g(x) = \log_5 x = \ln x / \ln 5$, its graph is obtained by vertically shrinking the graph of $f(x) = \ln x$ by a factor of $1/\ln 5 \approx 0.62$. See Figure 3.28a.

(b) $h(x) = \log_{1/4} x = \dfrac{\ln x}{\ln 1/4} = \dfrac{\ln x}{\ln 1 - \ln 4} = \dfrac{\ln x}{-\ln 4} = -\dfrac{1}{\ln 4} \ln x$. We can obtain the graph of h from the graph of $f(x) = \ln x$ by applying, in either order, a reflection across the x-axis and a vertical shrink by a factor of $1/\ln 4 \approx 0.72$. See Figure 3.28b. ***Now try Exercise 39.***



[–3, 6] by [–3, 3]
(a)



[–3, 6] by [–3, 3]
(b)

FIGURE 3.28 Transforming $f(x) = \ln x$ to obtain (a) $g(x) = \log_5 x$ and (b) $h(x) = \log_{1/4} x$. (Example 6)

We can generalize Example 6b in the following way: If $b > 1$, then $0 < 1/b < 1$ and

$$\log_{1/b} x = -\log_b x.$$

So when given a function like $h(x) = \log_{1/4} x$, with a base between 0 and 1, we can immediately rewrite it as $h(x) = -\log_4 x$. Because we can so readily change the base of logarithms with bases between 0 and 1, such logarithms are rarely encountered or used. Instead, we work with logarithms that have bases $b > 1$, which behave much like natural and common logarithms, as we now summarize.

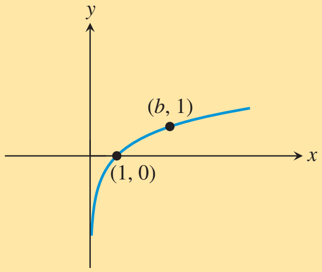


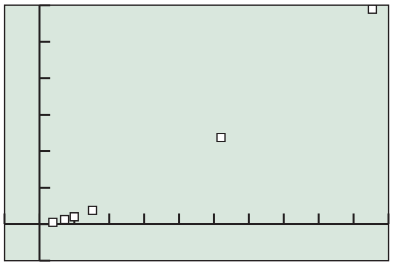
FIGURE 3.29 $f(x) = \log_b x,\, b > 1$.

Logarithmic Functions $f(x) = \log_b x$, with $b > 1$

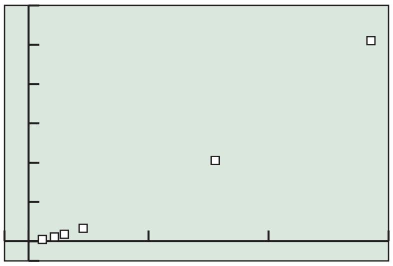
Domain: $(0, \infty)$
Range: All reals
Continuous
Increasing on its domain
No symmetry: neither even nor odd
Not bounded above or below
No local extrema
No horizontal asymptotes
Vertical asymptote: $x = 0$
End behavior: $\lim_{x \to \infty} \log_b x = \infty$

Astronomically Speaking

An astronomical unit (AU) is the average distance between the Earth and the Sun, about 149.6 million kilometers (149.6 Gm).



[–1, 10] by [–5, 30]
(a)



[–100, 1500] by [–1000, 12 000]
(b)

FIGURE 3.30 Scatter plots of the planetary data from (a) Table 3.20 and (b) Table 2.10.

Re-expressing Data

When seeking a model for a set of data, it is often helpful to transform the data by applying a function to one or both of the variables in the data set. We did this already when we treated the years 1900–2000 as 0–100. Such a transformation of a data set is a **re-expression** of the data.

Recall from Section 2.2 that Kepler's Third Law states that the square of the orbit period T for each planet is proportional to the cube of its average distance a from the Sun. If we re-express the Kepler planetary data in Table 2.10 using Earth-based units, the constant of proportion becomes 1 and the "is proportional to" in Kepler's Third Law becomes "equals." We can do this by dividing the "average distance" column by 149.6 Gm/AU and the "period of orbit" column by 365.2 days/yr. The re-expressed data are shown in Table 3.20.

Table 3.20 Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (AU)	Period of Orbit (yr)
Mercury	0.3870	0.2410
Venus	0.7233	0.6161
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.86
Saturn	9.539	29.46

Source: Re-expression of data from: Shupe, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

Notice that the pattern in the scatter plot of these re-expressed data, shown in Figure 3.30a, is essentially the same as the pattern in the plot of the original data, shown in Figure 3.30b. What we have done is to make the numerical values of the data more convenient and to guarantee that our plot contains the ordered pair (1, 1) for Earth, which could potentially simplify our model. What we have *not* done and still wish to do is to clarify the relationship between the variables a (distance from the Sun) and T (orbit period).

Logarithms can be used to re-express data and help us clarify relationships and uncover hidden patterns. For the planetary data, if we plot $(\ln a, \ln T)$ pairs instead of (a, T) pairs, the pattern is much clearer. In Example 7, we carry out this re-expression of the data and then use an algebraic *tour de force* to obtain Kepler's Third Law.

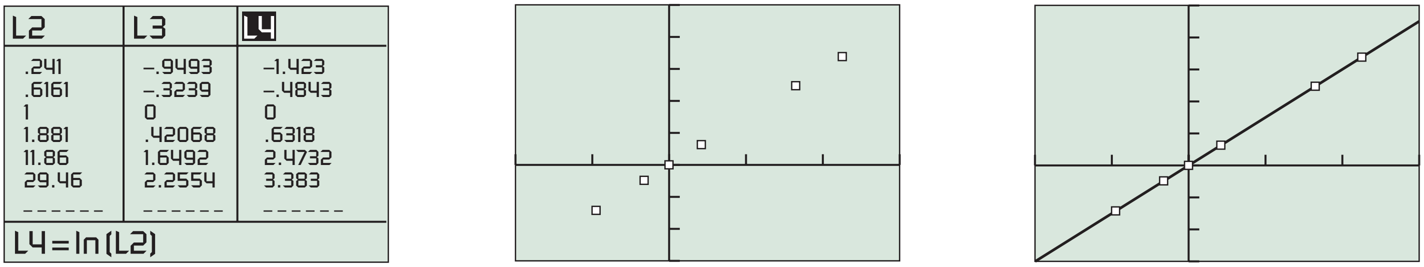
EXAMPLE 7 Establishing Kepler's Third Law Using Logarithmic Re-expression

Re-express the (a, T) data pairs in Table 3.20 as $(\ln a, \ln T)$ pairs. Find a linear regression model for the $(\ln a, \ln T)$ pairs. Rewrite the linear regression in terms of a and T, and rewrite the equation in a form with no logs or fractional exponents.

SOLUTION

Model

We use grapher list operations to obtain the $(\ln a, \ln T)$ pairs (see Figure 3.31a). We make a scatter plot of the re-expressed data in Figure 3.31b. The $(\ln a, \ln T)$ pairs appear to lie along a straight line.



(a) (b) [–2, 3] by [–3, 5] (c) [–2, 3] by [–3, 5]

FIGURE 3.31 Scatter plot and graphs for Example 7.

We let $y = \ln T$ and $x = \ln a$. Then using linear regression, we obtain the following model:

$$y = 1.49950x + 0.00070 \approx 1.5x.$$

Figure 3.31c shows the scatter plot for the $(x, y) = (\ln a, \ln T)$ pairs together with a graph of $y = 1.5x$. You can see that the line fits the re-expressed data remarkably well.

Remodel

Returning to the original variables a and T, we obtain:

$$\ln T = 1.5 \cdot \ln a \qquad y = 1.5x$$

$$\frac{\ln T}{\ln a} = 1.5 \qquad \text{Divide by } \ln a.$$

$$\log_a T = \frac{3}{2} \qquad \text{Change of base}$$

$$T = a^{3/2} \qquad \text{Switch to exponential form.}$$

$$T^2 = a^3 \qquad \text{Square both sides.}$$

Interpret

This is Kepler's Third Law!

Now try Exercise 65.

QUICK REVIEW 3.4 *(For help, go to Sections A.1 and 3.3.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, evaluate the expression without using a calculator.

1. $\log 10^2$

2. $\ln e^3$

3. $\ln e^{-2}$

4. $\log 10^{-3}$

In Exercises 5–10, simplify the expression.

5. $\dfrac{x^5 y^{-2}}{x^2 y^{-4}}$

6. $\dfrac{u^{-3} v^7}{u^{-2} v^2}$

7. $(x^6 y^{-2})^{1/2}$

8. $(x^{-8} y^{12})^{3/4}$

9. $\dfrac{(u^2 v^{-4})^{1/2}}{(27u^6 v^{-6})^{1/3}}$

10. $\dfrac{(x^{-2} y^3)^{-2}}{(x^3 y^{-2})^{-3}}$

SECTION 3.4 EXERCISES

In Exercises 1–12, assuming x and y are positive, use properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms.

1. $\ln 8x$

2. $\ln 9y$

3. $\log \dfrac{3}{x}$

4. $\log \dfrac{2}{y}$

5. $\log_2 y^5$

6. $\log_2 x^{-2}$

7. $\log x^3 y^2$

8. $\log xy^3$

9. $\ln \dfrac{x^2}{y^3}$

10. $\log 1000x^4$

11. $\log \sqrt[4]{\dfrac{x}{y}}$

12. $\ln \dfrac{\sqrt[3]{x}}{\sqrt[3]{y}}$

In Exercises 13–22, assuming x, y, and z are positive, use properties of logarithms to write the expression as a single logarithm.

13. $\log x + \log y$

14. $\log x + \log 5$

15. $\ln y - \ln 3$

16. $\ln x - \ln y$

17. $\dfrac{1}{3} \log x$

18. $\dfrac{1}{5} \log z$

19. $2 \ln x + 3 \ln y$

20. $4 \log y - \log z$

21. $4 \log (xy) - 3 \log (yz)$

22. $3 \ln (x^3 y) + 2 \ln (yz^2)$

In Exercises 23–28, use the change-of-base formula and your calculator to evaluate the logarithm.

23. $\log_2 7$

24. $\log_5 19$

25. $\log_8 175$

26. $\log_{12} 259$

27. $\log_{0.5} 12$

28. $\log_{0.2} 29$

In Exercises 29–32, write the expression using only natural logarithms.

29. $\log_3 x$

30. $\log_7 x$

31. $\log_2 (a + b)$

32. $\log_5 (c - d)$

In Exercises 33–36, write the expression using only common logarithms.

33. $\log_2 x$

34. $\log_4 x$

35. $\log_{1/2} (x + y)$

36. $\log_{1/3} (x - y)$

37. Prove the quotient rule of logarithms.

38. Prove the power rule of logarithms.

In Exercises 39–42, describe how to transform the graph of $g(x) = \ln x$ into the graph of the given function. Sketch the graph by hand and support with a grapher.

39. $f(x) = \log_4 x$

40. $f(x) = \log_7 x$

41. $f(x) = \log_{1/3} x$

42. $f(x) = \log_{1/5} x$

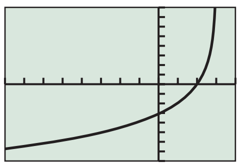
In Exercises 43–46, match the function with its graph. Identify the window dimensions, Xscl, and Yscl of the graph.

43. $f(x) = \log_4 (2 - x)$

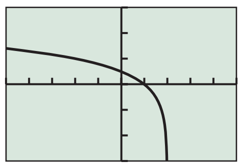
44. $f(x) = \log_6 (x - 3)$

45. $f(x) = \log_{0.5} (x - 2)$

46. $f(x) = \log_{0.7} (3 - x)$



(a)



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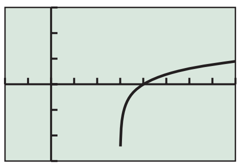

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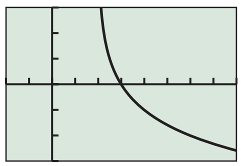

(d)

In Exercises 47–50, graph the function, and analyze it for domain, range, continuity, increasing or decreasing behavior, asymptotes, and end behavior.

47. $f(x) = \log_2 (8x)$

48. $f(x) = \log_{1/3} (9x)$

49. $f(x) = \log (x^2)$

50. $f(x) = \ln (x^3)$

51. Sound Intensity Compute the sound intensity level in decibels for each sound listed in Table 3.21.

Table 3.21 Approximate Intensities for Selected Sounds

Sound	Intensity (Watts/m^2)
(a) Hearing threshold	10^{-12}
(b) Rustling leaves	10^{-11}
(c) Conversation	10^{-6}
(d) School cafeteria	10^{-4}
(e) Jack hammer	10^{-2}
(f) Pain threshold	1

Sources: J. J. Dwyer, College Physics. Belmont, CA: Wadsworth, 1984; and E. Connally et al., Functions Modeling Change. New York: Wiley, 2000.

52. Earthquake Intensity The **Richter scale** magnitude R of an earthquake is based on the features of the associated seismic wave and is measured by

$$R = \log (a/T) + B,$$

where a is the amplitude in μm (micrometers), T is the period in seconds, and B accounts for the weakening of the seismic wave due to the distance from the epicenter. Compute the earthquake magnitude R for each set of values.

(a) $a = 250$, $T = 2$, and $B = 4.25$

(b) $a = 300$, $T = 4$, and $B = 3.5$

53. Light Intensity in Lake Erie The relationship between intensity I of light (in lumens) at a depth of x feet in Lake Erie is given by

$$\log \frac{I}{12} = -0.00235x.$$

What is the intensity at a depth of 40 ft?

54. Light Intensity in Lake Superior The relationship between intensity I of light (in lumens) at a depth of x feet in Lake Superior is given by

$$\log \frac{I}{12} = -0.0125x.$$

What is the intensity at a depth of 10 ft?

55. Writing to Learn Use the change-of-base formula to explain how we know that the graph of $f(x) = \log_3 x$ can be obtained by applying a transformation to the graph of $g(x) = \ln x$.

56. Writing to Learn Use the change-of-base formula to explain how the graph of $f(x) = \log_{0.8} x$ can be obtained by applying transformations to the graph of $g(x) = \log x$.

Standardized Test Questions

57. True or False The logarithm of the product of two positive numbers is the sum of the logarithms of the numbers. Justify your answer.

58. True or False The logarithm of a positive number is positive. Justify your answer.

In Exercises 59–62, solve the problem without using a calculator.

59. Multiple Choice $\log 12 =$

(A) $3 \log 4$ **(B)** $\log 3 + \log 4$

(C) $4 \log 3$ **(D)** $\log 3 \cdot \log 4$

(E) $2 \log 6$

60. Multiple Choice $\log_9 64 =$

(A) $5 \log_3 2$ **(B)** $(\log_3 8)^2$

(C) $(\ln 64)/(\ln 9)$ **(D)** $2 \log_9 32$

(E) $(\log 64)/9$

61. Multiple Choice $\ln x^5 =$

(A) $5 \ln x$ **(B)** $2 \ln x^3$

(C) $x \ln 5$ **(D)** $3 \ln x^2$

(E) $\ln x^2 \cdot \ln x^3$

62. Multiple Choice $\log_{1/2} x^2 =$

(A) $-2 \log_2 x$ **(B)** $2 \log_2 x$

(C) $-0.5 \log_2 x$ **(D)** $0.5 \log_2 x$

(E) $-2 \log_2 |x|$

Explorations

63. (a) Compute the power regression model for the following data.

x	4	6.5	8.5	10
y	2816	31,908	122,019	275,000

(b) Predict the y-value associated with $x = 7.1$ using the power regression model.

(c) Re-express the data in terms of their natural logarithms and make a scatter plot of $(\ln x, \ln y)$.

(d) Compute the linear regression model $(\ln y) = a(\ln x) + b$ for $(\ln x, \ln y)$.

(e) Confirm that $y = e^b \cdot x^a$ is the power regression model found in (a).

64. (a) Compute the power regression model for the following data.

x	2	3	4.8	7.7
y	7.48	7.14	6.81	6.41

(b) Predict the y-value associated with $x = 9.2$ using the power regression model.

(c) Re-express the data in terms of their natural logarithms and make a scatter plot of $(\ln x, \ln y)$.

(d) Compute the linear regression model $(\ln y) = a(\ln x) + b$ for $(\ln x, \ln y)$.

(e) Confirm that $y = e^b \cdot x^a$ is the power regression model found in (a).

65. Keeping Warm—Revisited Recall from Exercise 55 of Section 2.2 that scientists have found the pulse rate r of mammals to be a power function of their body weight w.

(a) Re-express the data in Table 3.22 in terms of their *common* logarithms and make a scatter plot of $(\log w, \log r)$.

(b) Compute the linear regression model $(\log r) = a(\log w) + b$ for $(\log w, \log r)$.

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression equation to predict the pulse rate for a 450-kg horse. Is the result close to the 38 beats/min reported by A. J. Clark in 1927?

(e) Writing to Learn Why can we use either common or natural logarithms to re-express data that fit a power regression model?

Table 3.22 Weight and Pulse Rate of Selected Mammals

Mammal	Body Weight (kg)	Pulse Rate (beats/min)
Rat	0.2	420
Guinea pig	0.3	300
Rabbit	2	205
Small dog	5	120
Large dog	30	85
Sheep	50	70
Human	70	72

Source: A. J. Clark, Comparative Physiology of the Heart. New York: Macmillan, 1927.

66. Let $a = \log 2$ and $b = \log 3$. Then, for example, $\log 6 = a + b$ and $\log 15 = 1 - a + b$. List all of the positive integers less than 100 whose common logs can be written as expressions involving a or b or both. (*Hint*: See Exploration 1 on page 283.)

Extending the Ideas

67. Solve $\ln x > \sqrt[3]{x}$.

68. Solve $1.2^x \le \log_{1.2} x$.

69. Group Activity Work in groups of three. Have each group member graph and compare the domains for one pair of functions.

(a) $f(x) = 2 \ln x + \ln (x - 3)$ and $g(x) = \ln x^2(x - 3)$

(b) $f(x) = \ln (x + 5) - \ln (x - 5)$ and $g(x) = \ln \dfrac{x + 5}{x - 5}$

(c) $f(x) = \log (x + 3)^2$ and $g(x) = 2 \log (x + 3)$

Writing to Learn After discussing your findings, write a brief group report that includes your overall conclusions and insights.

70. Prove the change-of-base formula for logarithms.

71. Prove that $f(x) = \log x/\ln x$ is a constant function with restricted domain by finding the exact value of the constant $\log x/\ln x$ expressed as a common logarithm.

72. Graph $f(x) = \ln (\ln (x))$, and analyze it for domain, range, continuity, increasing or decreasing behavior, symmetry, asymptotes, end behavior, and invertibility.