### What you'll learn about

- Polynomial Inequalities
- Rational Inequalities
- Other Inequalities
- Applications

### … and why

Designing containers as well as other types of applications often require that an inequality be solved.

# 2.8 Solving Inequalities in One Variable

## Polynomial Inequalities

A polynomial inequality takes the form $f(x) > 0$, $f(x) \geq 0$, $f(x) < 0$, $f(x) \leq 0$ or $f(x) \neq 0$, where $f(x)$ is a polynomial. There is a fundamental connection between inequalities and the positive or negative sign of the corresponding expression $f(x)$:

- To solve $f(x) > 0$ is to find the values of $x$ that make $f(x)$ *positive.*
- To solve $f(x) < 0$ is to find the values of $x$ that make $f(x)$ *negative.*

If the expression $f(x)$ is a product, we can determine its sign by determining the sign of each of its factors. Example 1 illustrates that a polynomial function changes sign only at its real zeros of odd multiplicity.

### EXAMPLE 1 Finding Where a Polynomial Is Zero, Positive, or Negative

Let $f(x) = (x + 3)(x^2 + 1)(x - 4)^2$. Determine the real number values of $x$ that cause $f(x)$ to be **(a)** zero, **(b)** positive, **(c)** negative.

**SOLUTION** We begin by verbalizing our analysis of the problem:

**(a)** The real zeros of $f(x)$ are $-3$ (with multiplicity 1) and 4 (with multiplicity 2). So $f(x)$ is zero if $x = -3$ or $x = 4$.

**(b)** The factor $x^2 + 1$ is positive for all real numbers $x$. The factor $(x - 4)^2$ is positive for all real numbers $x$ except $x = 4$, which makes $(x - 4)^2 = 0$. The factor $x + 3$ is positive if and only if $x > -3$. So $f(x)$ is positive if $x > -3$ and $x \neq 4$.

**(c)** By process of elimination, $f(x)$ is negative if $x < -3$.

This verbal reasoning process is aided by the following **sign chart**, which shows the $x$-axis as a number line with the real zeros displayed as the locations of potential sign change and the factors displayed with their sign value in the corresponding interval:

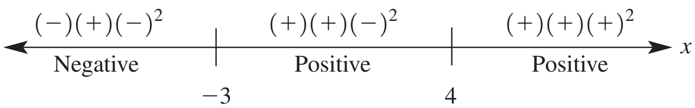
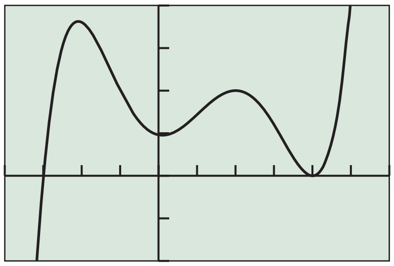



Figure 2.64 supports our findings because the graph of $f$ is above the $x$-axis for $x$ in $(-3, 4)$ or $(4, \infty)$, is on the $x$-axis for $x = -3$ or $x = 4$, and is below the $x$-axis for $x$ in $(-\infty, -3)$. ***Now try Exercise 1.***



[–4, 6] by [–100, 200]

**FIGURE 2.64** The graph of $f(x) = (x + 3)(x^2 + 1)(x - 4)^2$. (Example 1)

Our work in Example 1 allows us to report the solutions of four polynomial inequalities:

- The solution of $(x + 3)(x^2 + 1)(x - 4)^2 > 0$ is $(-3, 4) \cup (4, \infty)$.
- The solution of $(x + 3)(x^2 + 1)(x - 4)^2 \geq 0$ is $[-3, \infty)$.
- The solution of $(x + 3)(x^2 + 1)(x - 4)^2 < 0$ is $(-\infty, -3)$.
- The solution of $(x + 3)(x^2 + 1)(x - 4)^2 \leq 0$ is $(-\infty, -3] \cup \{4\}$.

Example 1 illustrates some important general characteristics of polynomial functions and polynomial inequalities. The polynomial function $f(x) = (x + 3)(x^2 + 1)(x - 4)^2$ in Example 1 and Figure 2.64—

- changes sign at its real zero of odd multiplicity $(x = -3)$;

- touches the $x$-axis but does not change sign at its real zero of even multiplicity $(x = 4)$;
- has no $x$-intercepts or sign changes at its nonreal complex zeros associated with the irreducible quadratic factor $(x^2 + 1)$.

This is consistent with what we learned about the relationships between zeros and graphs of polynomial functions in Sections 2.3–2.5. The real zeros and their multiplicity together with the end behavior of a polynomial function give us sufficient information about the polynomial to sketch its graph well enough to obtain a correct sign chart, as shown in Figure 2.65.

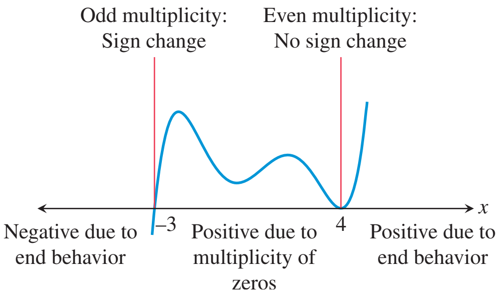



**FIGURE 2.65** The sign chart and graph of $f(x) = (x + 3)(x^2 + 1)(x - 4)^2$ overlaid.

### EXPLORATION 1 Sketching a Graph of a Polynomial from Its Sign Chart

Use your knowledge of end behavior and multiplicity of real zeros to create a sign chart and sketch the graph of the function. Check your sign chart using the factor method of Example 1. Then check your sketch using a grapher.

1. $f(x) = 2(x - 2)^3(x + 3)^2$
2. $f(x) = -(x + 2)^4(x + 1)(2x^2 + x + 1)$
3. $f(x) = 3(x - 2)^2(x + 4)^3(-x^2 - 2)$

So far in this section all of the polynomials have been presented in factored form and all of the inequalities have had zero on one of the sides. Examples 2 and 3 show us how to solve polynomial inequalities when they are not given in such a convenient form.

**Worth Trying**

You may wish to make a table or graph for the function $f$ in Example 2 to support the analytical approach used.

### EXAMPLE 2 Solving a Polynomial Inequality Analytically

Solve $2x^3 - 7x^2 - 10x + 24 > 0$ analytically.

**SOLUTION** Let $f(x) = 2x^3 - 7x^2 - 10x + 24$. The Rational Zeros Theorem yields several possible rational zeros of $f$ for factoring purposes:

$$\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 8,\ \pm 12,\ \pm 24,\ \pm\frac{1}{2},\ \pm\frac{3}{2}$$

A table or graph of $f$ can suggest which of these candidates to try. In this case, $x = 4$ is a rational zero of $f$, as the following synthetic division shows:

$$\begin{array}{r|rrrr} 4 & 2 & -7 & -10 & 24 \\ & & 8 & 4 & -24 \\ \hline & 2 & 1 & -6 & 0 \end{array}$$

*(continued)*

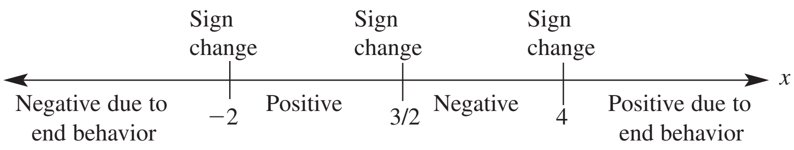
The synthetic division lets us start the factoring process, which can then be completed using basic factoring methods:

$$\begin{aligned} f(x) &= 2x^3 - 7x^2 - 10x + 24 \\ &= (x - 4)(2x^2 + x - 6) \\ &= (x - 4)(2x - 3)(x + 2) \end{aligned}$$

So the zeros of $f$ are 4, 3/2, and $-2$. They are all real and all of multiplicity 1, so each will yield a sign change in $f(x)$. Because the degree of $f$ is odd and its leading coefficient is positive, the end behavior of $f$ is given by

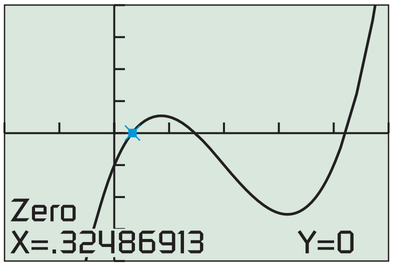
$$\lim_{x\to\infty} f(x) = \infty \quad \text{and} \quad \lim_{x\to-\infty} f(x) = -\infty.$$

Our analysis yields the following sign chart:




The solution of $2x^3 - 7x^2 - 10x + 24 > 0$ is $(-2, 3/2) \cup (4, \infty)$.

***Now try Exercise 11.***




[–2, 5] by [–8, 8]

**FIGURE 2.66** The graph of $f(x) = x^3 - 6x^2 + 8x - 2$, with one of three real zeros highlighted. (Example 3)

## EXAMPLE 3 Solving a Polynomial Inequality Graphically

Solve $x^3 - 6x^2 \le 2 - 8x$ graphically.

**SOLUTION** First we rewrite the inequality as $x^3 - 6x^2 + 8x - 2 \le 0$. Then we let $f(x) = x^3 - 6x^2 + 8x - 2$ and find the real zeros of $f$ graphically as shown in Figure 2.66. The three real zeros are approximately 0.32, 1.46, and 4.21. The solution consists of the $x$-values for which the graph is on or below the $x$-axis. So the solution of $x^3 - 6x^2 \le 2 - 8x$ is approximately $(-\infty, 0.32] \cup [1.46, 4.21]$.

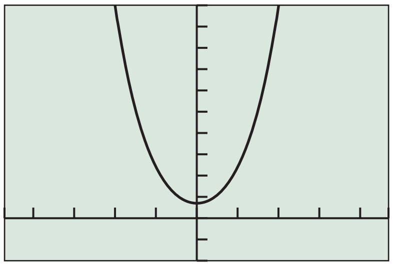
The end points of these intervals are accurate to two decimal places. We use square brackets because the zeros of the polynomial are solutions of the inequality even though we only have approximations of their values. ***Now try Exercise 13.***

When a polynomial function has no sign changes, the solutions of the associated inequalities can look a bit unusual, as illustrated in Example 4.

## EXAMPLE 4 Solving a Polynomial Inequality with Unusual Answers

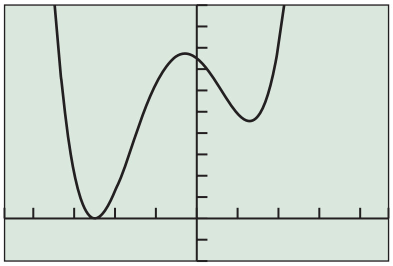
**(a)** The inequalities associated with the strictly positive polynomial function $f(x) = (x^2 + 7)(2x^2 + 1)$ have unusual solution sets. We use Figure 2.67a as a guide to solving the inequalities:

- The solution of $(x^2 + 7)(2x^2 + 1) > 0$ is $(-\infty, \infty)$, all real numbers.
- The solution of $(x^2 + 7)(2x^2 + 1) \ge 0$ is also $(-\infty, \infty)$.
- The solution set of $(x^2 + 7)(2x^2 + 1) < 0$ is empty. We say an inequality of this sort has no solution.
- The solution set of $(x^2 + 7)(2x^2 + 1) \le 0$ is also empty, so the inequality has no solution.



[–4.7, 4.7] by [–20, 100]

(a)



[–4.7, 4.7] by [–20, 100]

(b)

**FIGURE 2.67** The graphs of (a) $f(x) = (x^2 + 7)(2x^2 + 1)$ and (b) $g(x) = (x^2 - 3x + 3)(2x + 5)^2$. (Example 4)

**(b)** The inequalities associated with the nonnegative polynomial function $g(x) = (x^2 - 3x + 3)(2x + 5)^2$ also have unusual solution sets. We use Figure 2.67b as a guide to solving the inequalities:

- The solution of $(x^2 - 3x + 3)(2x + 5)^2 > 0$ is $(-\infty, -5/2) \cup (-5/2, \infty)$, all real numbers except $x = -5/2$, the lone real zero of $g$.
- The solution of $(x^2 - 3x + 3)(2x + 5)^2 \geq 0$ is $(-\infty, \infty)$, all real numbers.
- The inequality $(x^2 - 3x + 3)(2x + 5)^2 < 0$ has no solution.
- The solution of $(x^2 - 3x + 3)(2x + 5)^2 \leq 0$ is the single number $x = -5/2$.

***Now try Exercise 21.***

## Rational Inequalities

A polynomial function $p(x)$ is positive, negative, or zero for all real numbers $x$, but a rational function $r(x)$ can be positive, negative, zero, or *undefined*. In particular, a rational function is undefined at the zeros of its denominator. Thus when solving rational inequalities we modify the kind of sign chart used in Example 1 to include the real zeros of both the numerator and the denominator as locations of potential sign change.

### EXAMPLE 5 Creating a Sign Chart for a Rational Function

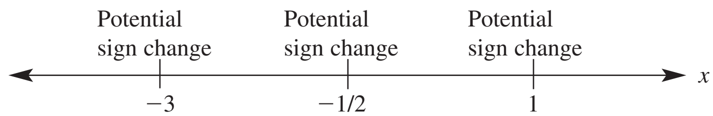
Let $r(x) = (2x + 1)/((x + 3)(x - 1))$. Determine the real number values of $x$ that cause $r(x)$ to be **(a)** zero, **(b)** undefined. Then make a sign chart to determine the real number values of $x$ that cause $r(x)$ to be **(c)** positive, **(d)** negative.

**SOLUTION**

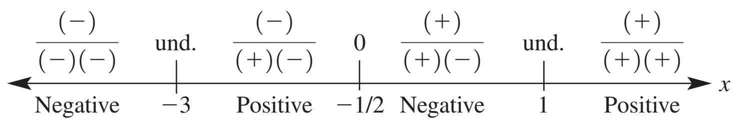
**(a)** The real zeros of $r(x)$ are the real zeros of the numerator $2x + 1$. So $r(x)$ is zero if $x = -1/2$.

**(b)** $r(x)$ is undefined when the denominator $(x + 3)(x - 1) = 0$. So $r(x)$ is undefined if $x = -3$ or $x = 1$.

These findings lead to the following sign chart, with three points of potential sign change:




Analyzing the factors of the numerator and denominator yields:

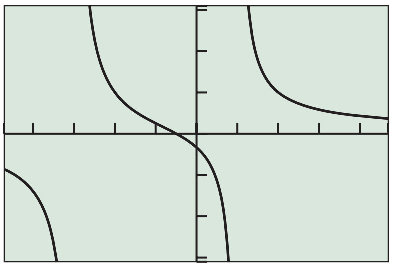



**(c)** So $r(x)$ is positive if $-3 < x < -1/2$ or $x > 1$, and the solution of $(2x + 1)/((x + 3)(x - 1)) > 0$ is $(-3, -1/2) \cup (1, \infty)$.

**(d)** Similarly, $r(x)$ is negative if $x < -3$ or $-1/2 < x < 1$, and the solution of $(2x + 1)/((x + 3)(x - 1)) < 0$ is $(-\infty, -3) \cup (-1/2, 1)$.

Figure 2.68 supports our findings because the graph of $r$ is above the $x$-axis for $x$ in $(-3, -1/2) \cup (1, \infty)$ and is below the $x$-axis for $x$ in $(-\infty, -3) \cup (-1/2, 1)$.

***Now try Exercise 25.***



[–4.7, 4.7] by [–3.1, 3.1]

**FIGURE 2.68** The graph of $r(x) = (2x + 1)/((x + 3)(x - 1))$. (Example 5)

### EXAMPLE 6 Solving a Rational Inequality by Combining Fractions

Solve $\dfrac{5}{x+3} + \dfrac{3}{x-1} < 0.$

**SOLUTION** We combine the two fractions on the left-hand side of the inequality using the least common denominator $(x+3)(x-1)$:

$$\frac{5}{x+3} + \frac{3}{x-1} < 0 \qquad \text{Original inequality}$$

$$\frac{5(x-1)}{(x+3)(x-1)} + \frac{3(x+3)}{(x+3)(x-1)} < 0 \qquad \text{Use LCD to rewrite fractions.}$$

$$\frac{5(x-1)+3(x+3)}{(x+3)(x-1)} < 0 \qquad \text{Add fractions.}$$

$$\frac{5x-5+3x+9}{(x+3)(x-1)} < 0 \qquad \text{Distributive property}$$

$$\frac{8x+4}{(x+3)(x-1)} < 0 \qquad \text{Simplify.}$$

$$\frac{2x+1}{(x+3)(x-1)} < 0 \qquad \text{Divide both sides by 4.}$$

This inequality matches Example 5d. The solution is $(-\infty, -3) \cup (-1/2, 1)$.

***Now try Exercise 49.***

## Other Inequalities

The sign chart method can be adapted to solve other types of inequalities, and we can support our solutions graphically as needed or desired.

### EXAMPLE 7 Solving an Inequality Involving a Radical

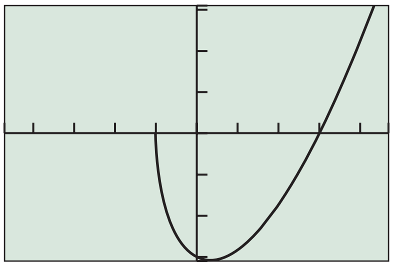
Solve $(x-3)\sqrt{x+1} \geq 0$.

**SOLUTION** Let $f(x) = (x-3)\sqrt{x+1}$. Because of the factor $\sqrt{x+1}$, $f(x)$ is undefined if $x < -1$. The zeros of $f$ are 3 and $-1$. These findings, along with a sign analysis of the two factors, lead to the following sign chart:

| | $x = -1$ | | $x = 3$ | |
|---|---|---|---|---|
| | 0 | $(-)(+)$ | 0 | $(+)(+)$ |
| Undefined | | Negative | | Positive |

The solution of $(x-3)\sqrt{x+1} \geq 0$ is $\{-1\} \cup [3, \infty)$. The graph of $f$ in Figure 2.69 supports this solution.

***Now try Exercise 43.***



[–4.7, 4.7] by [–3.1, 3.1]

**FIGURE 2.69** The graph of $f(x) = (x-3)\sqrt{x+1}$. (Example 7)

### EXAMPLE 8 Solving an Inequality Involving Absolute Value

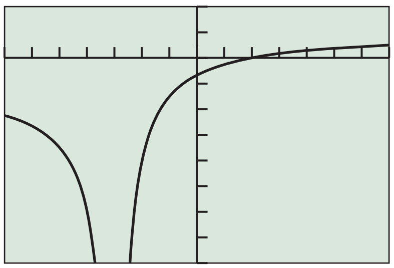
Solve $\dfrac{x-2}{|x+3|} \leq 0$.

**SOLUTION** Let $f(x) = (x-2)/|x+3|$. Because $|x+3|$ is in the denominator, $f(x)$ is undefined if $x = -3$. The only zero of $f$ is 2. These findings, along with a sign analysis of the two factors, lead to the following sign chart:

| | $x = -3$ | | $x = 2$ | |
|---|---|---|---|---|
| $\dfrac{(-)}{\lvert - \rvert}$ | und. | $\dfrac{(-)}{\lvert + \rvert}$ | 0 | $\dfrac{(+)}{\lvert + \rvert}$ |
| Negative | | Negative | | Positive |

The solution of $(x-2)/|x+3| \leq 0$ is $(-\infty, -3) \cup (-3, 2]$. The graph of $f$ in Figure 2.70 supports this solution.

***Now try Exercise 53.***



[–7, 7] by [–8, 2]

**FIGURE 2.70** The graph of $f(x) = (x-3)/|x+3|$. (Example 8)

# Applications

## EXAMPLE 9 Designing a Box—Revisited

Dixie Packaging Company has contracted with another firm to design boxes with a volume of *at least* 600 in.$^3$. Squares are still to be cut from the corners of a 20-in. by 25-in. piece of cardboard, with the flaps folded up to make an open box. What size squares should be cut from the cardboard? (See Example 9 of Section 2.3 and Figure 2.31.)

**SOLUTION**

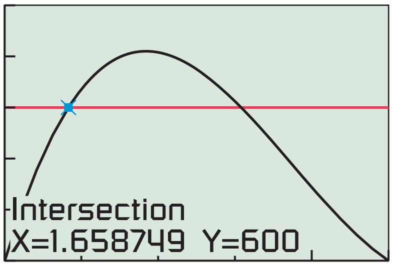
### Model

Recall that the volume $V$ of the box is given by

$$V(x) = x(25 - 2x)(20 - 2x),$$

where $x$ represents both the side length of the removed squares and the height of the box. To obtain a volume of at least 600 in.$^3$, we solve the inequality

$$x(25 - 2x)(20 - 2x) \geq 600.$$




[0, 10] by [0, 1000]

**FIGURE 2.71** The graphs of $y_1 = x(25 - 2x)(20 - 2x)$ and $y_2 = 600$. (Example 9)

### Solve Graphically

Because the width of the cardboard is 20 in., $0 \leq x \leq 10$, and we set our window accordingly. In Figure 2.71, we find the values of $x$ for which the cubic function is on or above the horizontal line. The solution is the interval $[1.66, 6.16]$.

### Interpret

Squares with side lengths between 1.66 in. and 6.16 in., inclusive, should be cut from the cardboard to produce a box with a volume of at least 600 in.$^3$.

*Now try Exercise 59.*

## EXAMPLE 10 Designing a Juice Can—Revisited

Stewart Cannery will package tomato juice in 2-liter (2000 cm$^3$) cylindrical cans. Find the radius and height of the cans if the cans have a surface area that is less than 1000 cm$^2$. (See Example 7 of Section 2.7 and Figure 2.63.)

**SOLUTION**

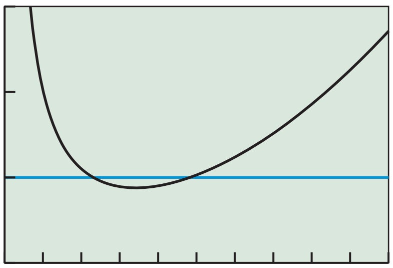
### Model

Recall that the surface area $S$ is given by

$$S(r) = 2\pi r^2 + \frac{4000}{r}.$$

The inequality to be solved is

$$2\pi r^2 + \frac{4000}{r} < 1000.$$



[0, 20] by [0, 3000]

**FIGURE 2.72** The graphs of $y_1 = 2\pi x^2 + 4000/x$ and $y_2 = 1000$. (Example 10)

### Solve Graphically

Figure 2.72 shows the graphs of $y_1 = S(r) = 2\pi r^2 + 4000/r$ and $y_2 = 1000$. Using grapher methods we find that the two curves intersect at approximately $r \approx 4.619\ldots$ and $r \approx 9.654\ldots$. (We carry all the extra decimal places for greater accuracy in a computation below.) So the surface area is less than 1000 cm$^3$ if

$$4.62 < r < 9.65.$$

The volume of a cylindrical can is $V = \pi r^2 h$ and $V = 2000$. Using substitution we see that $h = 2000/(\pi r^2)$. To find the values for $h$ we build a double inequality for $2000/(\pi r^2)$.

*(continued)*

$$\begin{aligned}
4.62 &< r < 9.65 && \text{Original inequality}\\
4.62^2 &< r^2 < 9.65^2 && 0 < a < b \Rightarrow a^2 < b^2.\\
\pi \cdot 4.62^2 &< \pi r^2 < \pi \cdot 9.65^2 && \text{Multiply by } \pi.\\
\frac{1}{\pi \cdot 4.62^2} &> \frac{1}{\pi r^2} > \frac{1}{\pi \cdot 9.65^2} && 0 < a < b \Rightarrow \frac{1}{a} > \frac{1}{b}.\\
\frac{2000}{\pi \cdot 4.62^2} &> \frac{2000}{\pi r^2} > \frac{2000}{\pi \cdot 9.65^2} && \text{Multiply by 2000.}\\
\frac{2000}{\pi(4.619\ldots)^2} &> h > \frac{2000}{\pi(9.654\ldots)^2} && \text{Use the extra decimal places now.}\\
29.83 &> h > 6.83 && \text{Compute.}
\end{aligned}$$

**Interpret**

The surface area of the can will be less than 1000 $\text{cm}^3$ if its radius is between 4.62 cm and 9.65 cm and its height is between 6.83 cm and 29.83 cm. For any particular can, $h$ must equal $2000/(\pi r^2)$. ***Now try Exercise 61.***

## QUICK REVIEW 2.8 *(For help, go to Sections A.2, A.3, and 2.3.)*

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1–4, use limits to state the end behavior of the function.

**1.** $f(x) = 2x^3 + 3x^2 - 2x + 1$

**2.** $f(x) = -3x^4 - 3x^3 + x^2 - 1$

**3.** $g(x) = \dfrac{x^3 - 2x^2 + 1}{x - 2}$

**4.** $g(x) = \dfrac{2x^2 - 3x + 1}{x + 1}$

In Exercises 5–8, combine the fractions and reduce your answer to lowest terms.

**5.** $x^2 + \dfrac{5}{x}$ **6.** $x^2 - \dfrac{3}{x}$

**7.** $\dfrac{x}{2x + 1} - \dfrac{2}{x - 3}$ **8.** $\dfrac{x}{x - 1} + \dfrac{x + 1}{3x - 4}$

In Exercises 9 and 10, **(a)** list all the possible rational zeros of the polynomial and **(b)** factor the polynomial completely.

**9.** $2x^3 + x^2 - 4x - 3$ **10.** $3x^3 - x^2 - 10x + 8$

## SECTION 2.8 EXERCISES

In Exercises 1–6, determine the $x$ values that cause the polynomial function to be **(a)** zero, **(b)** positive, and **(c)** negative.

**1.** $f(x) = (x + 2)(x + 1)(x - 5)$

**2.** $f(x) = (x - 7)(3x + 1)(x + 4)$

**3.** $f(x) = (x + 7)(x + 4)(x - 6)^2$

**4.** $f(x) = (5x + 3)(x^2 + 6)(x - 1)$

**5.** $f(x) = (2x^2 + 5)(x - 8)^2(x + 1)^3$

**6.** $f(x) = (x + 2)^3(4x^2 + 1)(x - 9)^4$

In Exercises 7–12, complete the factoring if needed, and solve the polynomial inequality using a sign chart. Support graphically.

**7.** $(x + 1)(x - 3)^2 > 0$

**8.** $(2x + 1)(x - 2)(3x - 4) \le 0$

**9.** $(x + 1)(x^2 - 3x + 2) < 0$

**10.** $(2x - 7)(x^2 - 4x + 4) > 0$

**11.** $2x^3 - 3x^2 - 11x + 6 \ge 0$

**12.** $x^3 - 4x^2 + x + 6 \le 0$

In Exercises 13–20, solve the polynomial inequality graphically.

**13.** $x^3 - x^2 - 2x \ge 0$

**14.** $2x^3 - 5x^2 + 3x < 0$

**15.** $2x^3 - 5x^2 - x + 6 > 0$

**16.** $x^3 - 4x^2 - x + 4 \le 0$

**17.** $3x^3 - 2x^2 - x + 6 \ge 0$

**18.** $-x^3 - 3x^2 - 9x + 4 < 0$

**19.** $2x^4 - 3x^3 - 6x^2 + 5x + 6 < 0$

**20.** $3x^4 - 5x^3 - 12x^2 + 12x + 16 \ge 0$

In Exercises 21–24, solve the following inequalities for the given function $f(x)$.

**(a)** $f(x) > 0$ **(b)** $f(x) \geq 0$ **(c)** $f(x) < 0$ **(d)** $f(x) \leq 0$

**21.** $f(x) = (x^2 + 4)(2x^2 + 3)$

**22.** $f(x) = (x^2 + 1)(-2 - 3x^2)$

**23.** $f(x) = (2x^2 - 2x + 5)(3x - 4)^2$

**24.** $f(x) = (x^2 + 4)(3 - 2x)^2$

In Exercises 25–32, determine the real values of $x$ that cause the function to be **(a)** zero, **(b)** undefined, **(c)** positive, and **(d)** negative.

**25.** $f(x) = \dfrac{x - 1}{((2x + 3)(x - 4))}$

**26.** $f(x) = \dfrac{(2x - 7)(x + 1)}{(x + 5)}$

**27.** $f(x) = x\sqrt{x + 3}$ **28.** $f(x) = x^2|2x + 9|$

**29.** $f(x) = \dfrac{\sqrt{x + 5}}{(2x + 1)(x - 1)}$

**30.** $f(x) = \dfrac{x - 1}{(x - 4)\sqrt{x + 2}}$

**31.** $f(x) = \dfrac{(2x + 5)\sqrt{x - 3}}{(x - 4)^2}$

**32.** $f(x) = \dfrac{3x - 1}{(x + 3)\sqrt{x - 5}}$

In Exercises 33–44, solve the inequality using a sign chart. Support graphically.

**33.** $\dfrac{x - 1}{x^2 - 4} < 0$ **34.** $\dfrac{x + 2}{x^2 - 9} < 0$

**35.** $\dfrac{x^2 - 1}{x^2 + 1} \leq 0$ **36.** $\dfrac{x^2 - 4}{x^2 + 4} > 0$

**37.** $\dfrac{x^2 + x - 12}{x^2 - 4x + 4} > 0$ **38.** $\dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$

**39.** $\dfrac{x^3 - x}{x^2 + 1} \geq 0$ **40.** $\dfrac{x^3 - 4x}{x^2 + 2} \leq 0$

**41.** $x|x - 2| > 0$ **42.** $\dfrac{x - 3}{|x + 2|} < 0$

**43.** $(2x - 1)\sqrt{x + 4} < 0$ **44.** $(3x - 4)\sqrt{2x + 1} \geq 0$

In Exercises 45–54, solve the inequality.

**45.** $\dfrac{x^3(x - 2)}{(x + 3)^2} < 0$ **46.** $\dfrac{(x - 5)^4}{x(x + 3)} \geq 0$

**47.** $x^2 - \dfrac{2}{x} > 0$ **48.** $x^2 + \dfrac{4}{x} \geq 0$

**49.** $\dfrac{1}{x + 1} + \dfrac{1}{x - 3} \leq 0$ **50.** $\dfrac{1}{x + 2} - \dfrac{2}{x - 1} > 0$

**51.** $(x + 3)|x - 1| \geq 0$ **52.** $(3x + 5)^2|x - 2| < 0$

**53.** $\dfrac{(x - 5)|x - 2|}{\sqrt{2x - 3}} \geq 0$ **54.** $\dfrac{x^2(x - 4)^3}{\sqrt{x + 1}} < 0$

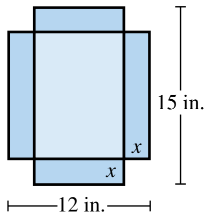
**55. Writing to Learn** Write a paragraph that explains two ways to solve the inequality $3(x - 1) + 2 \leq 5x + 6$.

**56. Company Wages** Pederson Electric charges $25 per service call plus $18 per hour for repair work. How long did an electrician work if the charge was less than $100? Assume the electrician rounds the time to the nearest quarter hour.

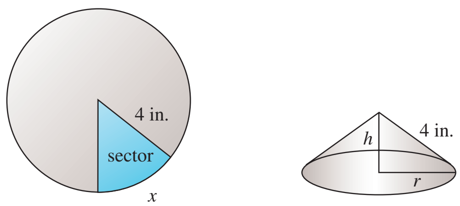
**57. Connecting Algebra and Geometry** Consider the collection of all rectangles that have lengths 2 in. less than twice their widths. Find the possible widths (in inches) of these rectangles if their perimeters are less than 200 in.

**58. Planning for Profit** The Grovenor Candy Co. finds that the cost of making a certain candy bar is $0.13 per bar. Fixed costs amount to $2000 per week. If each bar sells for $0.35, find the minimum number of candy bars that will earn the company a profit.

**59. Designing a Cardboard Box** Picaro's Packaging Plant wishes to design boxes with a volume of *not more than* 100 $\text{in.}^3$. Squares are to be cut from the corners of a 12-in. by 15-in. piece of cardboard (see figure), with the flaps folded up to make an open box. What size squares should be cut from the cardboard?




**60. Cone Problem** Beginning with a circular piece of paper with a 4-inch radius, as shown in (a), cut out a sector with an arc of length $x$. Join the two radial edges of the remaining portion of the paper to form a cone with radius $r$ and height $h$, as shown in (b). What length of arc will produce a cone with a volume greater than 21 $\text{in.}^3$?



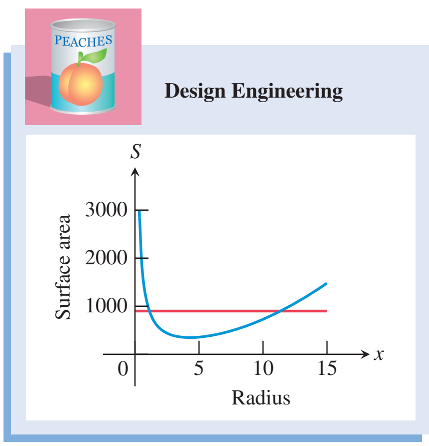

(a) (b)

**61. Design a Juice Can** Flannery Cannery packs peaches in 0.5-L cylindrical cans.

**(a)** Express the surface area $S$ of the can as a function of the radius $x$ (in cm).

**(b)** Find the dimensions of the can if the surface is less than 900 $\text{cm}^2$.

**(c)** Find the least possible surface area of the can.



**62. Resistors** The total electrical resistance $R$ of two resistors connected in parallel with resistances $R_1$ and $R_2$ is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

One resistor has a resistance of 2.3 ohms. Let $x$ be the resistance of the second resistor.

**(a)** Express the total resistance $R$ as a function of $x$.

**(b)** Find the resistance in the second resistor if the total resistance of the pair is at least 1.7 ohms.

**63. The Growing of America** Table 2.22 shows the midyear (July 1) U.S. population estimates in millions of persons for the years 2001 through 2008. Let $x$ be the number of years since July 1, 2000.

**Table 2.22 U.S. Population (in millions)**

| Year | Population |
|---|---|
| 2001 | 285.0 |
| 2002 | 287.7 |
| 2003 | 290.2 |
| 2004 | 292.9 |
| 2005 | 295.6 |
| 2006 | 298.4 |
| 2007 | 301.3 |
| 2008 | 304.1 |

*Source: U.S. Census Bureau, www.census.gov (June 2009).*

**(a)** Find the linear regression model for the U.S. population in millions since the middle of 2000.

**(b)** Use the model to predict when the U.S. population will reach 315 million.

**64. Single-Family House Cost** The midyear median sales prices of new, privately owned one-family houses sold in the United States are given for selected years in Table 2.23. Let $x$ be the number of years since July 1, 2000.

**(a)** Find the quadratic regression model for the data.

**(b)** Use the model to predict when the median cost of a new home returned to \$200,000.

**Table 2.23 Median Sales Price of a New House**

| Year | Price (dollars) |
|---|---|
| 2003 | 195,000 |
| 2004 | 221,000 |
| 2005 | 240,900 |
| 2006 | 246,500 |
| 2007 | 247,900 |
| 2008 | 232,100 |

*Source: U.S. Census Bureau, www.census.gov (June 2009)*

## Standardized Test Questions

**65. True or False** The graph of $f(x) = x^4(x + 3)^2(x - 1)^3$ changes sign at $x = 0$. Justify your answer.

**66. True or False** The graph $r(x) = \dfrac{2x - 1}{(x + 2)(x - 1)}$ changes sign at $x = -2$. Justify your answer.

In Exercises 67–70, solve the problem without using a calculator.

**67. Multiple Choice** Which of the following is the solution to $x^2 < x$?

**(A)** $(0, \infty)$ **(B)** $(1, \infty)$ **(C)** $(0, 1)$
**(D)** $(-\infty, 1)$ **(E)** $(-\infty, 0) \cup (1, \infty)$

**68. Multiple Choice** Which of the following is the solution to $\dfrac{1}{(x + 2)^2} \geq 0$?

**(A)** $(-2, \infty)$ **(B)** All $x \neq -2$ **(C)** All $x \neq 2$
**(D)** All real numbers **(E)** There are no solutions.

**69. Multiple Choice** Which of the following is the solution to $\dfrac{x^2}{x - 3} < 0$?

**(A)** $(-\infty, 3)$ **(B)** $(-\infty, 3]$ **(C)** $(-\infty, 0] \cup (0, 3)$
**(D)** $(-\infty, 0) \cup (0, 3)$ **(E)** There are no solutions.

**70. Multiple Choice** Which of the following is the solution to $(x^2 - 1)^2 \leq 0$?

**(A)** $\{-1, 1\}$ **(B)** $\{1\}$ **(C)** $[-1, 1]$
**(D)** $[0, 1]$ **(E)** There are no solutions.

## Explorations

In Exercises 71 and 72, find the vertical asymptotes and intercepts of the rational function. Then use a sign chart and a table of values to sketch the function by hand. Support your result using a grapher. (*Hint:* You may need to graph the function in more than one window to see different parts of the overall graph.)

**71.** $f(x) = \dfrac{(x - 1)(x + 2)^2}{(x - 3)(x + 1)}$ **72.** $g(x) = \dfrac{(x - 3)^4}{x^2 + 4x}$

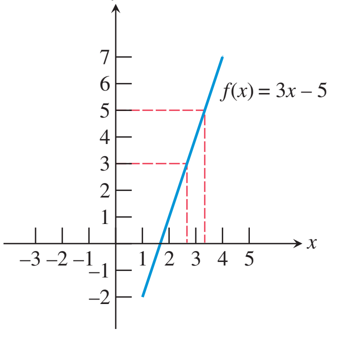
## Extending the Ideas

**73. Group Activity Looking Ahead to Calculus** Let $f(x) = 3x - 5$.

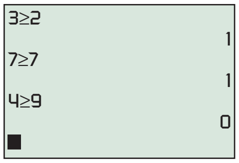
**(a)** Assume $x$ is in the interval defined by $|x - 3| < 1/3$. Give a convincing argument that $|f(x) - 4| < 1$.

**(b) Writing to Learn** Explain how (a) is modeled by the figure below.

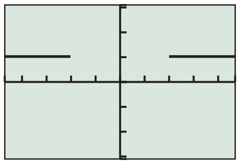
**(c)** Show how the algebra used in (a) can be modified to show that if $|x - 3| < 0.01$, then $|f(x) - 4| < 0.03$. How would the figure below change to reflect these inequalities?



**74. Writing to Learn Boolean Operators** The Test menu of many graphers contains inequality symbols that can be used to construct inequality statements, as shown in (a). An answer of 1 indicates the statement is true, and 0 indicates the statement is false. In (b), the graph of $Y_1 = (x^2 - 4 \geq 0)$ is shown using Dot mode and the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$. Experiment with the Test menu, and then write a paragraph explaining how to interpret the graph in(b).




(a)



[–4.7, 4.7] by [–3.1, 3.1]

(b)

In Exercises 75 and 76, use the properties of inequality from Chapter P to prove the statement.

**75.** If $0 < a < b$, then $a^2 < b^2$.

**76.** If $0 < a < b$, then $\frac{1}{a} > \frac{1}{b}$.

## CHAPTER 2 Key Ideas

### Properties, Theorems, and Formulas

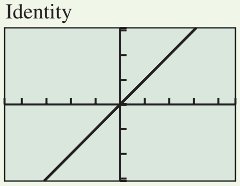
Properties of the Correlation Coefficient, $r$ 162
Vertex Form of a Quadratic Function 165
Vertical Free-Fall Motion 167
Local Extrema and Zeros of Polynomial Functions 187
Leading Term Test for Polynomial End Behavior 188
Zeros of Odd and Even Multiplicity 190
Intermediate Value Theorem 190
Division Algorithm for Polynomials 197
Remainder Theorem 198
Factor Theorem 199
Fundamental Connections for Polynomial Functions 199
Rational Zeros Theorem 201
Upper and Lower Bound Test for Real Zeros 202
Fundamental Theorem of Algebra 210
Linear Factorization Theorem 210
Fundamental Polynomial Connections in the Complex Case 211
Complex Conjugate Zeros Theorem 211
Factors of a Polynomial with Real Coefficients 214
Polynomial Function of Odd Degree 214
Graph of a Rational Function 221

### Procedures

Regression Analysis 163
Polynomial Long Division 197
Synthetic Division Example 3 200–201
Solving Inequalities Using Sign Charts 236, 237

### Gallery of Functions

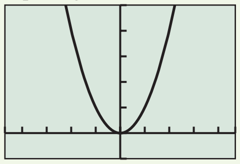
Identity



[–4.7, 4.7] by [–3.1, 3.1]

$f(x) = x$

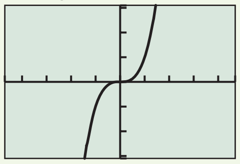
Squaring



[–4.7, 4.7] by [–1, 5]

$f(x) = x^2$

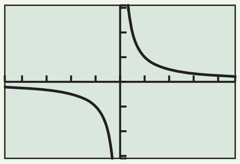
Cubing



[–4.7, 4.7] by [–3.1, 3.1]

$f(x) = x^3$

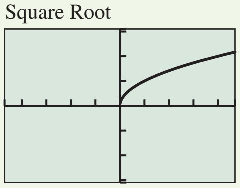
Reciprocal



[–4.7, 4.7] by [–3.1, 3.1]

$f(x) = 1/x = x^{-1}$

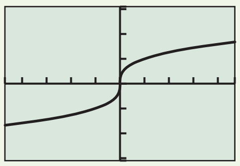
Square Root



[–4.7, 4.7] by [–3.1, 3.1]

$f(x) = \sqrt{x} = x^{1/2}$

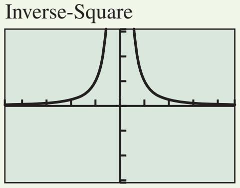
Cube Root



[–4.7, 4.7] by [–3.1, 3.1]

$f(x) = \sqrt[3]{x} = x^{1/3}$

Inverse-Square



[–4.7, 4.7] by [–3.1, 3.1]

$f(x) = 1/x^2 = x^{-2}$