LIMITS

$\lim_{x\to c} f(x)$ **"What is happening to y as x gets close to a certain number?"**
$\lim_{x\to c^-} f(x)$ **"What is happening to y as x gets close to a certain number FROM THE LEFT?"**
$\lim_{x\to c^+} f(x)$ **"What is happening to y as x gets close to a certain number FROM THE RIGHT?"**

Properties of Limits

1. $\lim_{x\to c}[f(x)+g(x)] =$ ________
2. $\lim_{x\to c}[f(x)-g(x)] =$ ________
3. $\lim_{x\to c} k\cdot f(x) =$ ________
4. $\lim_{x\to c}[f(x)\cdot g(x)] =$ ________
5. $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] =$ ________, $\lim_{x\to c} g(x) \neq$
6. $\lim_{x\to c}[f(x)]^n =$ ________
7. If $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c} f(g(x)) =$ ________

Two Special Trigonometric Limits:

$\lim_{x\to 0}\frac{\sin x}{x} =$ ________ $\lim_{x\to 0}\frac{x}{\sin x} =$ ________ $\lim_{x\to 0}\frac{\sin 6x}{x} =$ ________ $\lim_{x\to\infty}\frac{\sin x}{x} =$ ________

$\lim_{x\to 0}\frac{1-\cos x}{x} =$ ________ $\lim_{x\to 0}\frac{\cos x - 1}{x} =$ ________ $\lim_{x\to\infty}\frac{\cos x}{x} =$ ________

L'Hopital's Rule:

Given $\lim_{x\to a}\frac{f(x)}{g(x)}$ If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) =$ ____ or ____ then $\lim_{x\to a}\frac{f(x)}{g(x)} =$ ________

Limits at Infinity:

$\lim_{x\to\infty} f(x)$ the limit of $f(x)$ **as x increases without bound.**

$\lim_{x\to-\infty} f(x)$ the limit of $f(x)$ **as x decreases without bound.**

Rules for Evaluating Limits at Infinity

1. If the highest power of x appears in the denominator (bottom heavy), $\lim_{x\to\pm\infty} f(x) =$ ____
2. If the highest power of x appears in the numerator (top heavy), $\lim_{x\to\pm\infty} f(x) =$ ____ or ____
3. If the highest power of x appears both in the numerator and denominator (powers equal),

   $\lim_{x\to\pm\infty} f(x) =$ ________

Limit definition for a Horizontal Asymptote:

$\lim_{x\to\pm\infty} f(x)$

Definition of Continuity:

A function is continuous at c if __**all three**__ of the following holds true:

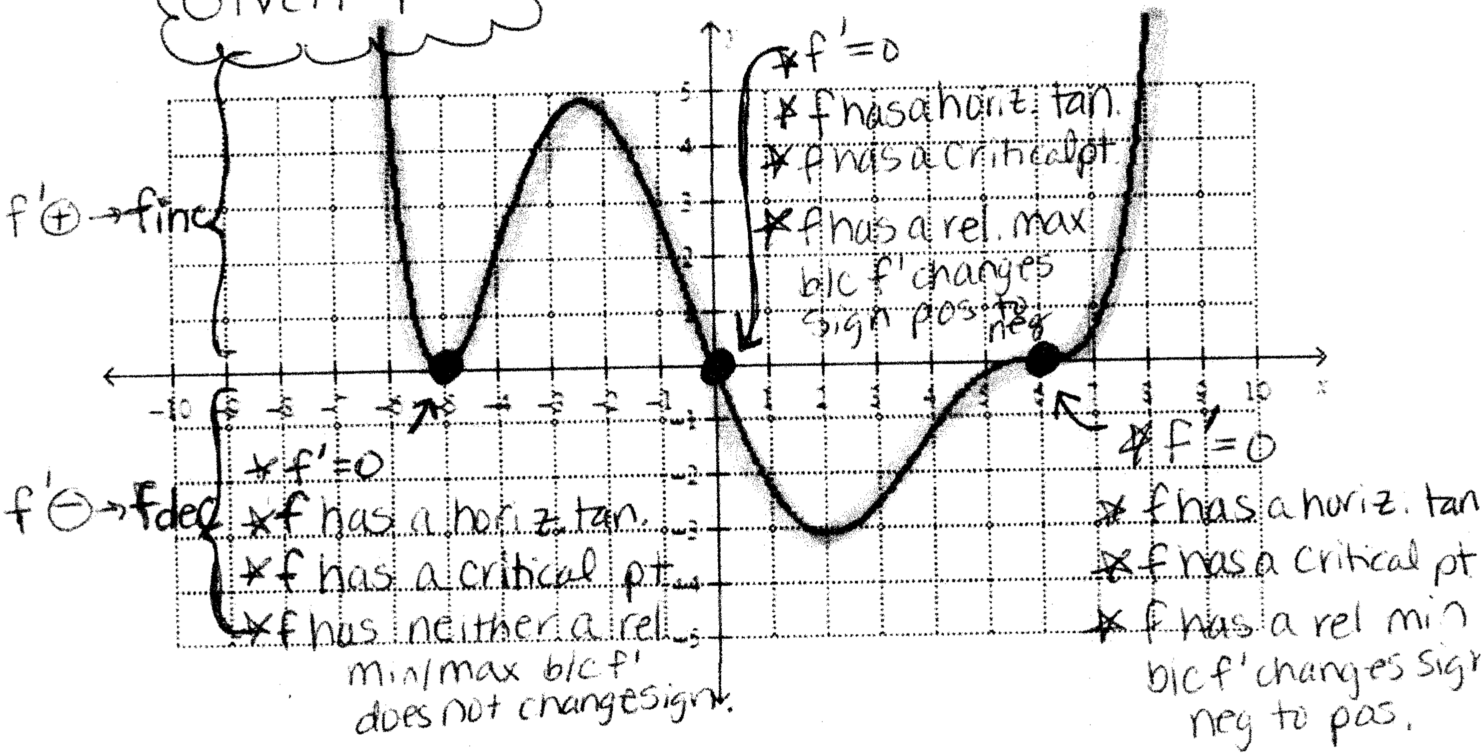
1. $\lim_{x\to c} f(x)$ ____ **THIS IMPLIES THAT** $\lim_{x\to c^+} f(x) =$ ________
2. $f(c)$ ____
3. $\lim_{x\to c} f(x) =$ ____

The Definition of the Derivative:

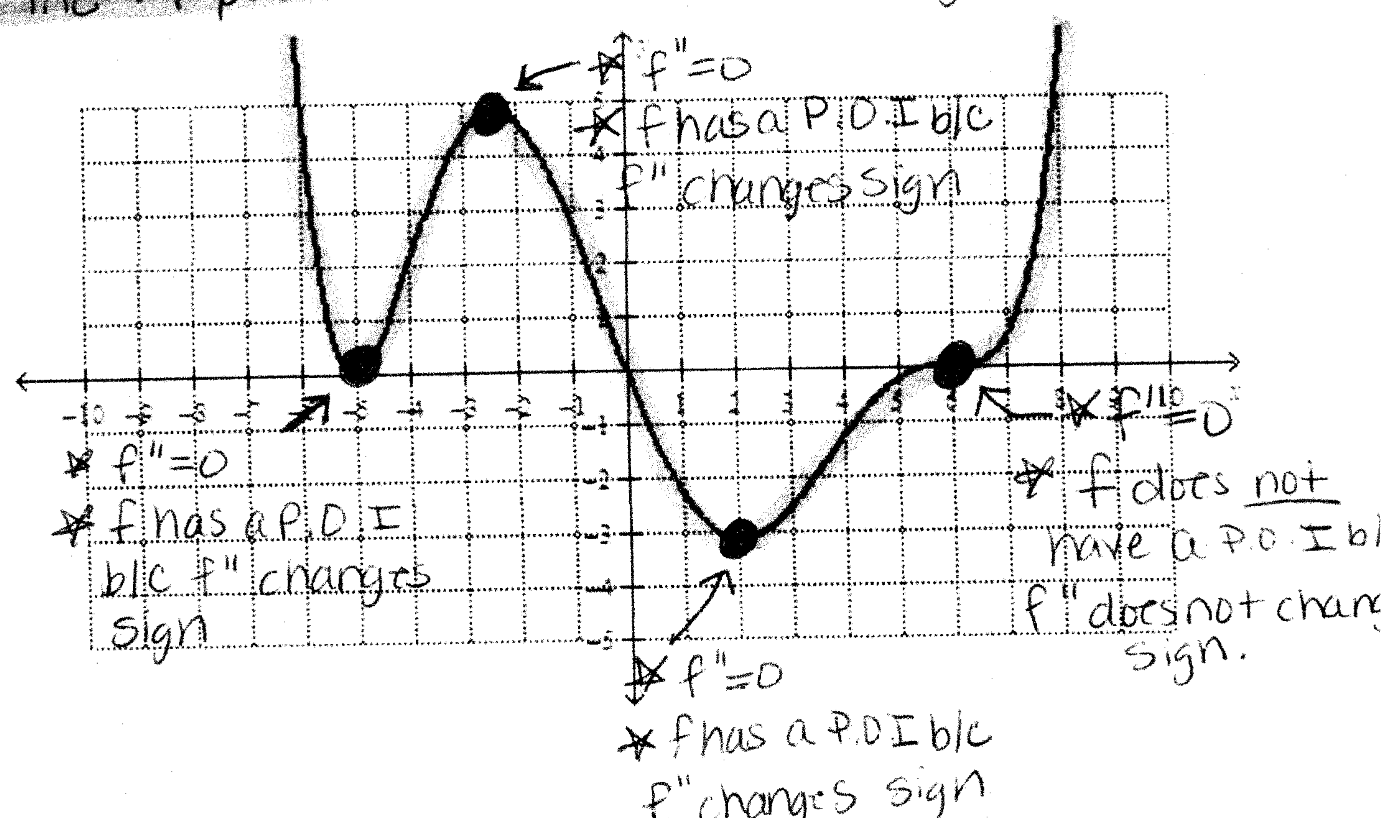
$\lim_{h\to 0}$ ________ $= f'(x)$ $\lim_{h\to 0}$ ________ $= f'(a)$ $\lim_{x\to c}$ ________ $= f'(c)$

Review this !!!

Given f'



f' inc → f'' pos → f CC↑ f' dec → f'' neg → f CC↓



P.O.I = point of inflection

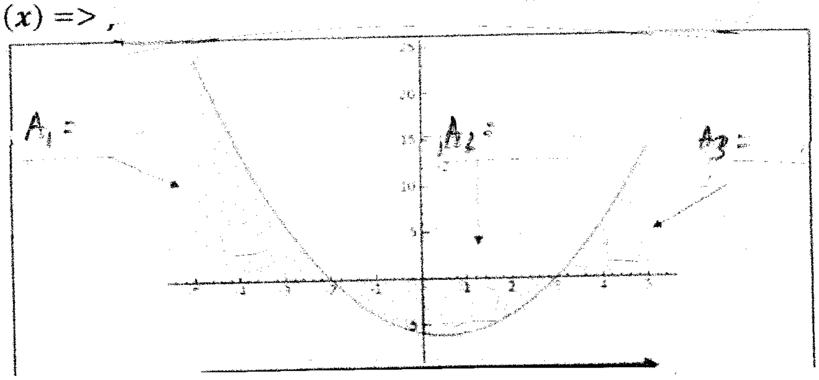
INTERPRETING GRAPHS

When Given the graph of f'

- When $f'$is positive then f is ____________
- When $f'$is negative then f is ____________
- When $f' = 0$, f has a __________ or __________
- When f'changes sign from positive to negative f has ______________
- When f'changes sign from negative to positive f has ______________
- When f'doesn't change sign then f has neither a maximum nor a minimum
- When $f'$is increasing then f is concave _____
- When $f'$is decreasing then f is concave ________
- When f' changes from increasing to decreasing or vice versa f'' ______________ and f has ____________

When given the graph of $g(x) = \int_a^x f(t)dt$

- ***This means that $g' =$***
- When f is positive then g is
- When f is negative then g is
- When $f = 0$, g has
- When $f$changes sign from positive to negative g has
- When $f$changes sign from negative to positive g has a relative or local minimum
- When $f$doesn't change sign then g has neither a maximum nor a minimum
- When $f$is increasing then g is concave
- When $f$is decreasing then g is concave
- When f changes from increasing to decreasing or vice versa g'' and g has
- ***$g(x)$ => area***
- ***$g'(x)$ =>***
- ***$g''(x)$ =>***



Write the following areas as definite integrals and fill in the sign

DIFFERENTIATION RULES

Power Rule: $\frac{d}{dx}[x^n] =$ ______	**With Chain Rule:** $\frac{d}{dx}\left[\left(f(x)\right)^n\right] =$ ______
Product Rule: $\frac{d}{dx}[f \cdot g] =$ ______	**Quotient Rule:** $\frac{d}{dx}\left[\frac{f}{g}\right] =$ ______
Exponential Functions:	
$\frac{d}{dx}[e^x] =$ ______	**With Chain Rule:** $\frac{d}{dx}\left[e^{f(x)}\right] =$ ______
$\frac{d}{dx}[a^x] =$ ______	**With Chain Rule:** $\frac{d}{dx}\left[a^{f(x)}\right] =$ ______
The Natural Logarithmic Function:	
$\frac{d}{dx}[\ln x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\ln f(x)] =$ ______
Trigonometric Rules:	
$\frac{d}{dx}[\sin x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\sin(f(x))] =$ ______
$\frac{d}{dx}[\cos x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\cos(f(x))] =$ ______
$\frac{d}{dx}[\tan x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\tan(f(x))] =$ ______
$\frac{d}{dx}[\csc x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\csc f(x)] =$ ______
$\frac{d}{dx}[\sec x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\sec f(x)] =$ ______
$\frac{d}{dx}[\cot x] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\cot(f(x))] =$ ______
Inverse Trigonometric Functions:	
$\frac{d}{dx}[\sin^{-1}(x)] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\sin^{-1}(f(x))] =$ ______
$\frac{d}{dx}[\cos^{-1}(x)] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\cos^{-1}(f(x))] =$ ______
$\frac{d}{dx}[\tan^{-1}(x)] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\tan^{-1}(f(x))] =$ ______
$\frac{d}{dx}[\cot^{-1}(x)] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\cot^{-1}(f(x))] =$ ______
$\frac{d}{dx}[\sec^{-1}(x)] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\sec^{-1}(f(x))] =$ ______
$\frac{d}{dx}[\csc^{-1}(x)] =$ ______	**With Chain Rule:** $\frac{d}{dx}[\csc^{-1}(f(x))] =$ ______

Inverse Functions:

If $f(x)$ and $g(x)$ are inverses then $f(g(x)) = x$ and $g\left(f(x)\right) = x$

If $f(x)$ and $g(x)$ are inverses and (a, b) is a point on $f(x)$ then $f(a) = b$ and $g(b) = a$

Derivative Rule: Given $f(x)$ and $g(x) = f^{-1}(x)$ $\quad g'(x) =$ ______

Implicit Differentiation:

The derivative of y with respect to x: $\frac{d}{dx}[y] = \frac{dy}{dx}$

Horizontal tangents occur when $\frac{dy}{dx} =$ ______ ***Vertical*** tangents occur when $\frac{dy}{dx} =$ ______

No tangent line exits when $\frac{dy}{dx} =$ ______

INTEGRATION RULES

Power Rule: $\int x^n dx =$ ______ $\int (f(x))^n f'(x)\, dx =$ ______

Trigonometric Functions:

$\int \sin x\, dx =$ ______ $\int \sin(f(x)) f'(x)dx =$ ______

$\int \cos x\, dx =$ ______ $\int \cos(f(x)) f'(x)dx =$ ______

$\int \sec^2 x\, dx =$ ______ $\int \sec^2(f(x)) f'(x)dx =$ ______

$\int \csc^2 x\, dx =$ ______ $\int \csc^2(f(x)) f'(x)dx =$ ______

$\int \sec x \tan x\, dx =$ ______ $\int \sec(f(x)) \tan(f(x)) f'(x)dx =$ ______

$\int \csc x \cot x\, dx =$ ______ $\int \csc(f(x)) \cot(f(x)) f'(x)dx =$ ______

Exponential Functions:

$\int e^x dx =$ ______ $\int e^{f(x)} f'(x)dx =$ ______

The Natural Logarithmic Function:

$\int \frac{1}{x} dx =$ ______ $\int \frac{f'(x)}{f(x)} dx =$ ______

Inverse Trigonometric Functions:

$\int \frac{1}{\sqrt{1-x^2}} dx =$ ______ $\int \frac{f'(x)dx}{\sqrt{a^2-(f(x))^2}} =$ ______

$\int \frac{1}{1+x^2} dx =$ ______ $\int \frac{f'(x)dx}{a^2+(f(x))^2} =$ ______

SOME APPLICATION OF DERIVATIVES AND INTEGRALS

Derivatives:

When you see the following – evaluate the derivative:

- Instantaneous rate of change
- Slope of the tangent line
- Slope of $f(x)$ at $x = a$

Equation of a line tangent to $f(x)$ at $x = a$: $y - f(a) = f'(a)(x - a)$

Equation of a line ***normal*** (perpendicular) to $f(x)$ at $x = a$: $y - f(a) = \frac{-1}{f'(a)}(x - a)$

Average rate of change of $f(x)$ on $[a, b]$: $\frac{f(b)-f(a)}{b-a}$

A function is not differentiable at corners, cusps, holes, asymptotes or vertical tangents.

Integration:

Fundamental Theorem of Calculus:

If $F(x) = \int f(x)dx \quad \rightarrow \quad \int_a^b f(x)dx =$ ______

Fundament Theorem of Calculus PART 2 (for derivatives):

$$\frac{d}{dx}\left[\int_{h(x)}^{g(x)} f(t)dt\right] = f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$$

A few properties of Integrals:

1) $\int_a^a f(x)dx =$ ______ 2) $\int_a^b f(x)dx =$ ______ 3) $\int_a^b f(x)\, dx + \int_b^c f(x)\, dx =$ ______

Review this

RATES

Rate problems can come in any form – word problems, graphs, tables…
The first thing to analyze are the units of all the equations given. That will help you to remember where you are in the line-up. Some examples of units are given below:

DERIVE (↓)

$feet$	$inches$	$liters$	$people$
ft/min	in/sec	l/hr	ppl/hr
ft/min^2	in/sec^2	l/hr^2	ppl/hr^2

(↑) **INTEGRATE**

Given a function $f(x)$ that represents a rate
(units would be like those of the middle row in the table above)

- What I have at time $t = b : F(a) + \int_a^b f(x)dx$
- What has been accumulated between time $t = a$ and $t = b : \int_a^b f(x)dx = F(b) - F(a)$
- $f(a)$ would tell us how $F(x)$ is increasing or decreasing at $t = a$
- $f'(a)$ would tell us how $f(x)$ is increasing or decreasing at $t = a$.

Given an initial condition $(a, H(a))$, $f(x)$ that represents a rate of something being added and $g(x)$ that represents a rate of something being taken away.

($H(x)$ units would be like those of the top row in the table above while $f(x)$ and $g(x)$ units would be like those of the middle row in the table above)

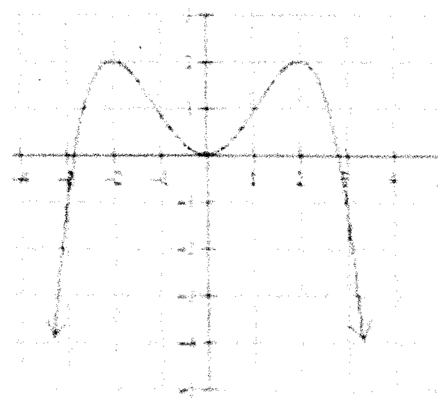
- What I have at time $t = b : H(a) + \int_a^b (f(x) - g(x))dx$
- What has been accumulated (added) between time $t = a$ and $t = b : \int_a^b f(x)dx$
- What has been taken away between time $t = a$ and $t = b : \int_a^b g(x)dx$
- $f(a) - g(a)$ would tell us how $H(x)$ is increasing or decreasing at $t = a$
- $f'(a) - g'(a)$ would tell us how $f(x) - g(x)$ is increasing or decreasing at $t = a$.

Words that would indicate a rate that would be used as an added accumulation:	**Words that would indicate a rate that would be used as an accumulation that should be subtracted:**
filling up entering pumped into piling up	leaking out decaying melting leaving removed

Practice

1. If $g'(x) = (x-3)^2(x+1)$, determine on what intervals the graph of $g(x)$ is increasing or decreasing and identify the value(s) of x at which $g(x)$ has a relative maximum or minimum. Justify your reasoning and show your work.

For exercises 2 - 4, use the graph of the function, $h(x)$, pictured to the r following. **Provide written justification.**



2. On what interval(s) is $h'(x) < 0$?

3. On what interval(s) is $h'(x) > 0$?

4. At what value(s) of x does $h'(x)$ change from positive to negative? From negative to positive?

5. For what function does $\lim_{h \to 0} \dfrac{2\sin(x+h) - 2\sin x}{h}$ give the derivative? Find the limit.

6. For what value(s) of k does the graph of $g(x) = ke^{2x} + 3x$ have a normal line whose slope is $-\frac{1}{2}$ when $x = 1$?

7. Given the curve $x^2 + y^2 = 1$ find $\frac{d^2y}{dx^2}$

8. Air is leaking out of an inflated balloon in the shape of a sphere at a rate of 230π cubic centimeters per minute. At the instant when the radius is 4 centimeters, what is the rate of change of the radius of the balloon?

9. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

a. How fast is the top of the ladder moving down the wall when the base of the ladder is 7 feet from the wall?

b. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

c. Find the rate at which the angle formed by the ladder and the wall of the house is changing when the base of the ladder is 9 feet from the wall.

10. Find the equation of the tangent line to the graph of $f(x) = 2x + \sin x + 1$ [illegible] $[0, \pi]$ at the point which is guaranteed by the mean value theorem?

11. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$ [illegible] the following questions.

a. On the axes below, sketch a slope field for the equation.

b. Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

c. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

12. A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is -9 and its position is -27.

a. Find $v(t)$, the velocity of the particle at any time t.

b. Find the net distance traveled by the particle over the interval $[0, 2]$.

c. Find the total distance traveled by the particle over the interval $[0, 2]$.

13. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.