3/23/2020

1. $\frac{dy}{dx} = x(2y+12)$

non separable. You can only use x or ÷ to separate the variables in order to create an expression that can be integrated over one variable.

(B)

2. $\frac{dy}{dx} = e^{y-x}$

use property of exponents

$\frac{dy}{dx} = e^{y} \cdot e^{-x}$

use property of proportions

$\frac{dy}{e^{y}} = e^{-x}dx$

(D)

3. $\frac{dy}{dx} = x^2 + 12xy = x(x+12y)$

non separable

(B)

4. $\frac{dy}{dx} = xy+12x$

common factor

$\frac{dy}{dx} = x(y+12)$

use property of proportions

$\frac{dy}{y+12} = x\,dx$

(A)

5. $\frac{dy}{dx} = -2y+12$ $\frac{dy}{dx} = -2(y-6)$ common factor

use property of proportions

$\frac{dy}{y-6} = -2\,dx$

(C)

6. $\frac{dy}{dx} = \frac{x+12}{2y}$

use property of proportions (cross multiplication)

$2y\,dy = (x+12)\,dx$

(F)

7. $\frac{dy}{dx} = e^{x+y}$

use property of exponents

$\frac{dy}{dx} = e^x \cdot e^y$

use property of proportions

$\frac{dy}{e^y} = e^x\,dx$

(G)

8. $\frac{dy}{dx} = x(-)2y$

non-separable (see #1)

(B)

9. $\frac{dy}{dx} = \frac{x}{12y}$

use property of proportions (cross multiplication)

$12y\,dy = x\,dx$

(E)

Remember

Prop of exp

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m \cdot n}$
4. $(a \cdot b)^m = a^m \cdot b^n$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. $a^{-m} = \frac{1}{a^m}$

Properties of proportions $\frac{a}{b} = \frac{c}{d}$

1. $ad = b \cdot c$
2. $\frac{a}{c} = \frac{b}{d}$
3. $\frac{d}{b} = \frac{c}{a}$
4. $\frac{a+b}{b} = \frac{c+d}{d}$

Problem 1 $\frac{dy}{dx} = \frac{-xy^2}{2}$

Step 1 $\int -y^{-2}dy = \frac{x}{2}dx$ using prop of proportions separation of variables is correct

Step 2 $y^{-1} = \frac{x^2}{4} + C_1$ integration of both sides relative to each variable is correct

Step 3 $y = \frac{4}{x^2} + C_2$ reciprocal of right side is <u>not</u> correct!

Corrected work

$y = \frac{1}{\frac{x^2}{4} + C_1}$ or $y^{-1} = \frac{x^2+4C_1}{4}$

$y = \frac{4}{x^2+4C_1}$

Problem 2 $\frac{dy}{dx} = e^{x-y}$

$\frac{dy}{dx} = \frac{e^x}{e^y}$

Step 1 $\int e^{-y}dy = \int e^x dx$ The separation of variables is <u>not</u> correct.

Corrected work

using cross multiplication $e^y dy = e^x dx$

integrate both sides $\int e^y dy = \int e^x dx$

Step 2 $e^y = e^x + C_1$

Step 3 isolate y $\ln e^y = \ln(e^x + C_1)$

$y = \ln(e^x + C_1)$

Problem 3 $\frac{dy}{dx} = \frac{xy}{2}$

Step 1 $\int \frac{1}{y}\,dy = \int \frac{x}{2}\,dx$ using prop of proportions separation of variables is correct ✓

Step 2 $\ln|y| = \frac{x^2}{4}$ integration of both sides is not correct missing the constant

Correction Step 2 $\ln|y| = \frac{x^2}{4} + C_1$

Step 3 isolate y $e^{\ln|y|} = e^{\frac{x^2}{4} + C_1}$

Step 4 $|y| = e^{\frac{x^2}{4}} \cdot e^{C_1}$

Step 5 if $e^{C_1} = C_2$ $|y| = C_2 \cdot e^{\frac{x^2}{4}}$

$y = \pm C_2 \cdot e^{\frac{x^2}{4}}$

Problem 4. $\frac{dy}{dx} = \frac{-2x}{y}$

Step 1 $\int y\,dy = \int -2x\,dx$ using cross multiplication separation of variables is correct

Step 2 $\frac{y^2}{2} = -x^2 + C_1$ integration of both sides is correct

Step 3 $y^2 = -2x^2 + C_2$ where $2 \cdot C_1 = C_2$, so this step is correct

Step 4 $y = \sqrt{-2x^2 + C_2}$ Solving for y is not correct

Correction when solving a quadratic eq. by taking the square root, there are also two solutions

Step 4 $y = \pm\sqrt{-2x^2 + C_2}$

Problem 5.

$\frac{d(y)}{dt} = -2v - 32$ → there is a typo here. It should be dv

Step 1 $\int 2v\,dy = \int -32\,dt$ Separation of variables is not correct.

Correction Step 1 $\int \frac{dv}{-2v-32} = \int dt$

Step 2 $u = -2v - 32$

$du = -2\,dv$

$-\frac{1}{2}\int \frac{-2\,dv}{-2v-32} = \int dt$

$-\frac{1}{2}\int \frac{du}{u} = \int dt$

$-\frac{1}{2}\ln|u| = t + C_1$

Step 3 $\ln|-2v-32| = -2t + C_2$, where $C_2 = -2C_1$

Step 4 $e^{\ln|-2v-32|} = e^{-2t+C_2}$

Step 5 $|-2v-32| = e^{-2t}\cdot e^{C_2}$

Step 6 $|-2v-32| = C_3 e^{-2t}$ where $C_3 = e^{C_2}$

Step 7 $-2v-32 = \pm C_3 \cdot e^{-2t}$

Step 8 $v = \frac{32 \pm C_3 e^{-2t}}{-2}$

Step 9 $v = -16 \pm C_4 e^{-2t}$ where $C_4 = \frac{C_3}{-2}$