low it is your turn to do the following problems.

1. Use right Riemann sums to write the following integrals.

   (a) $\int_{-1}^{5} x dx$ $= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{6}{n} f\left(-1 + \frac{6}{n}k\right)$ where $f(x) = x$ or $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{6}{n} \cdot \left(-1 + \frac{6}{n}k\right)$

   (b) $\int_{1}^{2} x^2 dx$

   $= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} f\left(1 + \frac{1}{n}k\right)$, where $f(x) = x^2$ or $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \left(1 + \frac{1}{n}k\right)^2$

2. Use left Riemann sums to write the following integrals.

   (a) $\int_{-1}^{3} x dx$ $= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{4}{n} f\left(-1 + \frac{4}{n}(k-1)\right)$, where $f(x) = x$ or $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{4}{n} \left(-1 + \frac{4}{n}(k-1)\right)$

   (b) $\int_{2}^{4} x^2 dx$

   $= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{2}{n} f\left(2 + \frac{2}{n}(k-1)\right)$, where $f(x) = x^2$ or $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2}{n} \left(2 + \frac{2}{n}(k-1)\right)^2$

3. Use midpoint Riemann sum to write the following integrals.

(a) $\int_1^2 x dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} f(1+\frac{1}{n}(k-\frac{1}{2}))$ where $f(x)=x$ or $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}(1+\frac{1}{n}(k-\frac{1}{2}))$

(b) $\int_0^2 x^2 dx$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n} f(0+\frac{2}{n}(k-\frac{1}{2}))$, where $f(x)=x^2$ or $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(\frac{2}{n}(k-\frac{1}{2})\right)^2$

4. Use right Riemann sums to write the following integral. $\int_{-1}^{5} 2x^2 + 3x dx$

$\int_{-1}^{5} 2x^2+3x\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{6}{n} \cdot f(-1+\frac{6}{n}k)$ where $f(x) = 2x^2+3x$

or $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{6}{n}\left(2(-1+\frac{6}{n}k)^2 + 3(-1+\frac{6}{n}k)\right)$

we approach the problem from the other direction, namely given a limit of a Riemann sum, we will write that definite integral and compute the integral using FTC.

Evaluate $\lim_{n\to\infty} \Sigma_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^3$ by expressing it as a definite integral and then evaluating this integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^3 = \int_0^1 x^3\,dx$$

Evaluate $\lim_{n\to\infty} \Sigma_{i=1}^{n} \frac{1}{n}\left(\frac{n+i}{n}\right)^2$ by expressing it as a definite integral and then evaluating this integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{n+i}{n}\right)^2 = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(1+\frac{i}{n}\right)^2 = \int_0^1 x^2\,dx \text{ or } \int_1^2 (1+x)^2\,dx$$

$\Delta x = \frac{1}{n}$ $\quad$ $\Delta x = \frac{1}{n}$

$a = 0$ $\quad$ $a = 1$

Evaluate $\lim_{n\to\infty} \frac{4}{n}\Sigma_{i=1}^{n}\sqrt{\left(\frac{2(2i-1)}{n}\right)}$ by expressing it as a definite integral and then evaluating this integral.

$$\lim_{n\to\infty} \frac{4}{n}\sum_{i=1}^{n}\sqrt{\frac{2(2i-1)}{n}} = \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\sqrt{\frac{4(i-1/2)}{n}} = \int_0^4 \sqrt{x}\,dx$$

$\Delta x = \frac{4}{n}$

$a = 0$

2. The table shows the velocity of a remote-controlled toy car as it traveled down a hallway for 10 seconds.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (in./sec)	0	6	10	16	14	12	18	22	12	4	2

Estimate the distance traveled by the car using 10 subintervals of length 1 and the methods shown.

a) LRAM
b) RRAM

LRAM = 114 in

RRAM = 116 in

3. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table below. Find the LRAM and RRAM for the distance that she traveled during these three seconds.

t(s)	0	0.5	1.0	1.5	2.0	2.5
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4

a. Find LRAM using five rectangles

b. Find RRAM using five rectangles

c. Find MRAM using ~~four~~ 3 rectangles

a) LRAM(5) = 25 ft

b) RRAM(5) = 34.7 ft

c) MRAM(3) = 40.5 ft