Function $f(x)$

where $f'(c)=0$ or ends of the domain:
- absolute max value = highest point in all domain
- absolute min value = lowest point in all domain

where $f'(c)=0$:
- local min value = lowest point in some open interval of the domain
- local max = highest point in some open interval of the domain

\+ $f'(c)$ does not exist

cusp, function is vertical

ex $x^{2/3}$ ex $x^{1/3}$

Critical points

First Derivative of function $f(x)$

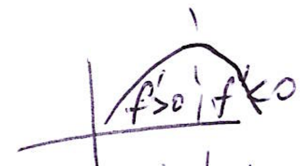
$f'(x)$

$f'=0$ or $f'=DNE \Rightarrow$ critical points

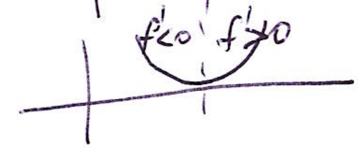
$f'>0$ function f increases

$f'<0$ function f decreases

Local max



Local min



Second derivative f''

If $f'' > 0$ concave up or f' increases

$f'' < 0$ concave down or f'' decreases

Point of inflection = where concavity changes.

Local extrema $f' = 0$ and $f'' < 0 \Rightarrow$ max

$f' = 0$ and $f'' > 0 \Rightarrow$ min

Applications

Optimization

- find the function that needs to be max or min
- if multiple variable, substitute to reduce to one variable
- Differentiate and make $f' = 0$ for max or min

Related rates

- multiple variable all changing over time
- find the relation between variables
- implicit differentiation.
- use a given condition

Approximate a function = use its value given by its linearization

$L(x) = f(a) + f'(a)(x-a)$ where $f(x) \approx L(x)$