5.4 DIVIDING POLYNOMIALS

VOCABULARY

Division Algorithm for Polynomials – Divide polynomial $P(x)$ by polynomial $D(x)$ to get polynomial quotient $Q(x)$ and polynomial remainder $R(x)$.
The result is $P(x) = D(x)Q(x) + R(x)$. If $R(x) = 0$, then $P(x) = D(x)Q(x)$ and $D(x)$ and $Q(x)$ are factors of $P(x)$.

Synthetic division – the process for dividing a polynomial by a linear expression ***x – a***. A polynomial must be in standard form. By omitting all variables and exponents

Remainder Theorem – If you divide a polynomial $P(x)$ of degree $n \geq 1$ by ***x – a***, then the remainder is $P(a)$.

Polynomials and Long Division

EX #1: Divide. 496 ÷ 16

$$\begin{array}{r} 31 \\ 16 \overline{)496} \\ -48\downarrow \\ \hline 16 \\ -16 \\ \hline 0 \end{array}$$

16 and 31 are factors of 496.

EX #2: Divide. $(4x^2 + 9x + 6) \div (x + 6)$

$$4x - 15 + \frac{96}{x+6}$$

$$\begin{array}{r} x+6 \overline{)\cancel{4x^2} + 9x + 6} \\ + \; (-\cancel{4x^2} - 24x) \downarrow \\ \hline -\cancel{15x} + 6 \\ + \; (+\cancel{15x} + 90) \\ \hline 96 \end{array}$$

$x+6$ is not a factor.

Using Polynomial Long Division

EX #3: Use polynomial long division to divide $3x^2 - 29x + 56$ by $x - 7$. What is the quotient and remainder?

$$
\begin{array}{r}
3x - 8 \\
x-7 \overline{\big) 3x^2 - 29x + 56} \\
+(\cancel{3x^2} + 21x) \\
\hline
-8x + 56 \\
-(-8x + 56) \\
\hline
0
\end{array}
$$

x-7 and 3x-8 are factors.

Checking Factors

EX #4: Is $(x-2)$ a factor of $x^3 - 9$?

$$\begin{array}{r} x^2 + 2x + 4 - \frac{1}{x-2} \\ x-2 \,\big)\, \cancel{x^3} + 0x^2 + 0x - 9 \\ -(\cancel{x^3} - 2x^2) \qquad\qquad \\ \hline \cancel{2x^2} + 0x \qquad \\ -(\cancel{2x^2} - 4x) \qquad \\ \hline 4x - 9 \end{array}$$

$$\begin{array}{r} 4x - 9 \\ -(4x - 8) \\ \hline -1 \end{array}$$

$x-2$ is not a factor.

EX #5: Is $(x-4)$ a factor of $f(x) = 5x^2 - 17x - 12$? If it is, write $P(x)$ as a product of two factors.

$$\begin{array}{r} 5x + 3 \\ x-4 \,\big)\, \cancel{5x^2} - 17x - 12 \\ -(\cancel{5x^2} - 20x) \qquad \\ \hline 3x - 12 \\ -(3x - 12) \\ \hline 0 \end{array}$$

$x-4$ is a factor.

$$\boxed{P(x) = (x-4)(5x+3)}$$

Using Synthetic Division

PROCEDURE:

1. **Write polynomial in standard form, include zeros for missing powers of *x*.**
2. **Omit all variables and exponents.**
3. **For the divisor, reverse the sign, use *a*.**
4. **Add instead of subtract throughout.**

EX #6: Divide $x^3 - 57x + 56$ by $x - 7$. What is the quotient and remainder?

$x - 7 = 0$

$x = 7$

$$\begin{array}{r|rrrr} 7 & 1 & 0 & -57 & 56 \\ & \downarrow & +7 & +49 & +(-56) \\ \hline x & 1 & 7 & -8 & 0 \end{array}$$

Quad Linear Constant R

$$\boxed{x^2 + 7x - 8}$$

Using Synthetic Division to Solve a Problem

EX #7: If the polynomial $x^3 + 6x^2 + 11x + 6$ expresses the volume, in cubic inches, of a box, and the width is $(x + 1)$ inches, what are the dimensions of the box?

$V = l \cdot w \cdot h$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & \downarrow & -1 & -5 & -6 \\ \hline x & 1 & 5 & 6 & 0 \end{array}$$

Quad Linear Constant R

$x^2 + 5x + 6$

a·c

6

2 3

5

$(x+2)(x+3)$

$w = (x+1)$

$l = (x+2)$

$h = (x+3)$

Evaluating a Polynomial

EX #8: Find $P(-4)$, given that
$P(x) = x^5 - 3x^4 - 28x^3 + 5x + 20$

$P(-4) = (-4)^5 - 3(-4)^4 - 28(-4)^3 + 5(-4) + 20$

$\boxed{P(-4) = 0 \quad x-4 \text{ is a factor.}}$

$x+4$ is a factor if
$P(-4) = 0$

P. 5 Polynomial Division Continued.

EX #9: Use the Factor Theorem to determine if $(x - 3)$ is a factor of $x^4 - 81$.

$P(3) = 3^4 - 81$

$P(3) = 0$

$x-3$ is a factor