Name:

Algebra 2
Semester 1 2015
Final Exam Study Guide

NC = No calculator

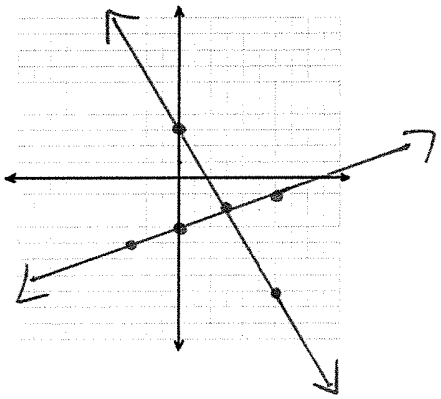
Show your work on separate sheets of paper. You may need graph paper.

UNIT 1: SYSTEMS OF EQUATIONS

1. **NC** Solve the following system of equations: $\begin{cases} y = -\frac{5}{3}x + 3 \\ y = \frac{1}{3}x - 3 \end{cases}$

   a. What method (elimination, substitution, or graphing) would be best to solve this system? Why?

$-2 = -\frac{5}{3}(3) + 3$
$-2 = -5 + 3$
$-2 = -2$ ✓
$-2 = \frac{1}{3}(3) - 3$
$-2 = 1 - 3$
$-2 = -2$ ✓



(3, -2)

   b. Solve the system by graphing.
   c. Check your solution.

2. **NC** Solve the following system of equations: $\begin{cases} y = 6x - 11 \\ -2x - 3y = -7 \end{cases}$

   a. Which method would be the best to use for this system? Why?
   b. Solve the system by **substitution**.

$-2x - 3(6x - 11) = -7$
$-2x - 18x + 33 = -7$
$\frac{-20x}{-20} = \frac{-40}{-20}$
$x = 2$
$y = 6(2) - 11$ $y = 1$

(2, 1)

3. **NC** Solve the following system of equations: $\begin{cases} 5x + y = 9 \\ 10x - 7y = -18 \end{cases}$

   a. Which method would be the best to use for this system? Why?
   b. Solve by **elimination**.

$-2(5x + y = 9)$
$-10x - 2y = -18$
$+ \quad 10x - 7y = -18$
$\frac{-9y}{-9} = \frac{-36}{-9}$ $y = 4$
$5x + 4 = 9$
$5x = 5$ $x = 1$

(1, 4)

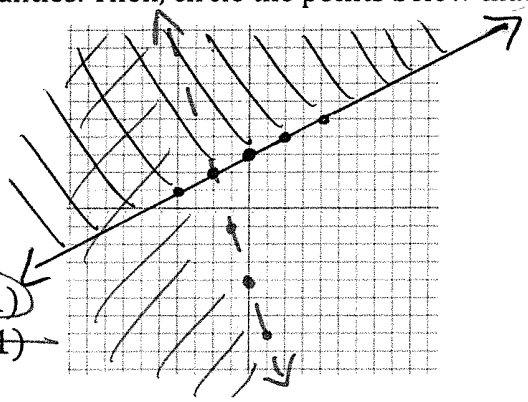
4. **NC** Solve the following systems of equations using the method of your choice.

a) $\begin{cases} y = 5x - 7 \\ -3x - 2y = -12 \end{cases}$ b) $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ c) $\begin{cases} y = \frac{1}{3}x - 3 \\ y = -x + 1 \end{cases}$

NEXT PAGE

5. **NC** Graph the following system of inequalities. Then, circle the points below that are solutions to the system.

- $y < -3x - 4$
- $y \geq \frac{1}{2}x + 3$



Circle the solutions to the system:

~~(0, 0)~~ (-4, 3) (-7, 1)
~~(5, 3)~~ ~~(-2, -3)~~ ~~(-3, -1)~~

(4) (a)

$y = 5x - 7$

$-3x - 2y = -12$

$-3x - 2(5x-7) = -12$

$-3x - 10x + 14 = -12$

$\frac{-13x}{-13} = \frac{-26}{-13}$

$\boxed{x = 2}$

$y = 5(2) - 7$

$y = 10 - 7$

$\boxed{y = 3}$

$\boxed{2, 3}$

(b) $-4x + 9y = 9$

$3(x - 3y = -6)$

$-4x + 9y = 9$

$+ \quad 3x - 9y = -18$

$-x = -9$

$\boxed{x = 9}$

$9 - 3y = -6$

$\frac{-3y}{-3} = \frac{-15}{-3}$

$\boxed{y = 5}$

$\boxed{(9, 5)}$

(c) $y = \frac{1}{3}x - 5$

$y = -x + 1$

$\frac{1}{3}x - 3 = -x + 1$

$+x \quad +x$

$\frac{4}{3}x - 3 = 1$

$+3 \quad +3$

$\frac{3}{4}\left(\frac{4}{3}x\right) = (4)\frac{3}{4}$

$\boxed{x = 3}$

$y = \frac{1}{3}(3) - 3$

$y = 1 - 3$

$\boxed{y = -2}$

$\boxed{(3, -2)}$

---

(6) x = pizza y = breadsticks

$5x + 4y = 51$

$-2(x + 2y = 15)$

$5x + 4y = 51$

$+ \quad -2x - 4y = -30$

$\frac{3x}{3} = \frac{21}{3}$

$\boxed{x = 7}$

$7 + 2y = 15$

$\frac{2y}{2} = \frac{8}{2}$

$\boxed{y = 4}$

Pizza costs \$7

Breadsticks cost \$4

---

(7) x = bananas y = apples

$0.3x + 0.5y = 7$

$x = 3y$ .....

$0.3(3y) + 0.5y = 7$

$0.9y + 0.5y = 7$

$1.4y = 7$

$\boxed{y = 5}$

$x = 3(5)$

$\boxed{x = 15}$

He bought 15 bananas and 5 apples.

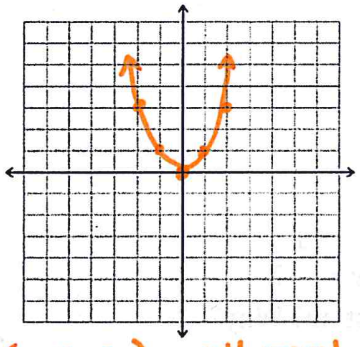
6. **NC** Maria is hosting a party. She places an order at Pizza Hut for 5 pizzas and 4 breadsticks for a total of $51. Halfway through the party, she realizes that she will need one more pizza and 2 more order of breadsticks. This time she was given a total of $15. How much does a pizza cost? How much does a breadstick cost?

7. **NC** Saleh has to buy apples and bananas. A bunch of bananas is $0.30 and an apple is $0.50. He spends a total of $7.00 . He bought 3 times as many bananas as he did apples. How many apples did he buy? How many bananas did he buy?

UNIT 2: FAMILIES OF FUNCTIONS

1. NC Given the family name, find the parent equation and graph it. Determine the domain, range, and end behavior.

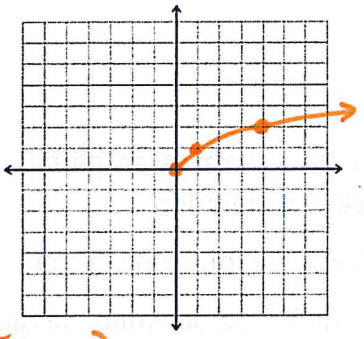
Family: Quadratic
Equation: $y = x^2$ or $f(x) = x^2$



x	y
-2	4
-1	1
0	0
1	1
2	4

Domain: $(-\infty, \infty)$ all real #s
Range: $[0, \infty)$
End behavior: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to \infty$

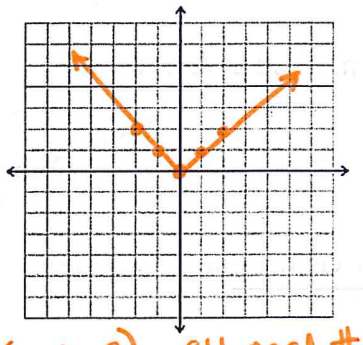
Family: Radical
Equation: $y = \sqrt{x}$ or $f(x) = \sqrt{x}$



x	y
0	0
1	1
2	
3	
4	2

Domain: $[0, \infty)$
Range: $[0, \infty)$
End behavior: $x \to \infty$ $f(x) \to \infty$
$x \to 0$ $f(x) \to 0$

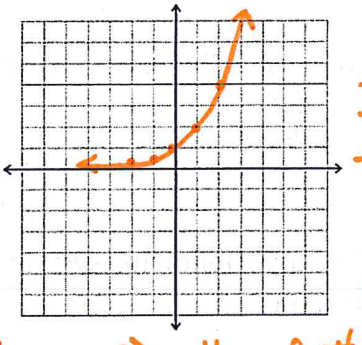
Family: Absolute Value
Equation: $y = |x|$ or $f(x) = |x|$



x	y
-2	2
-1	1
0	0
1	1
2	2

Domain: $(-\infty, \infty)$ all real #s
Range: $[0, \infty)$
End behavior: $x \to \infty$, $f(x) \to \infty$
$x \to -\infty$, $f(x) \to \infty$

Family: Exponential Growth
Equation: $y = 2^x$ $f(x) = 2^x$



x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

Domain: $(-\infty, \infty)$ all real #s
Range: $(0, \infty)$
End behavior: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$, $f(x) \to 0$

a h k

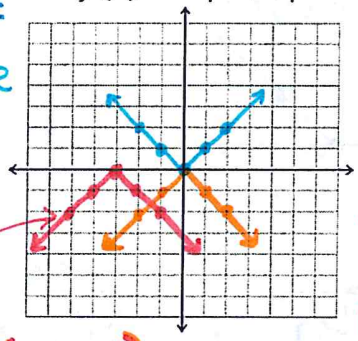
2. For the function $a * f(x-b)+c$, complete the table for the effects of each parameter. Name two examples of values for each situation and describe the effect in words.

$\lvert a\rvert > 1$ Examples: (One reflected, one not) a=2 a=-2 Effect: Vertical Stretch	$b \geq 0$ (positive) like h Examples: 2, 8 Effect: shift right	$c \leq 0$ (negative) like k Examples: -3, -9 Effect: Shift down
$0 < \lvert a\rvert < 1$ Examples: (One reflected, one not) 1/2, -1/2 Effect: vertical shrink	$b \leq 0$ (negative) Examples: -3, -16 Effect: shift left	$c \geq 0$ Examples: 8, 41 Effect: shift up
$a < 0$ Examples: -4 -16 Effect: reflection across x-axis		

3. Graph the following functions. Find their domain and range.

a h k

$f(x) = -|x+3|$



parent: absolute value

final graph

a = -1 reflects

h = -3 shift left

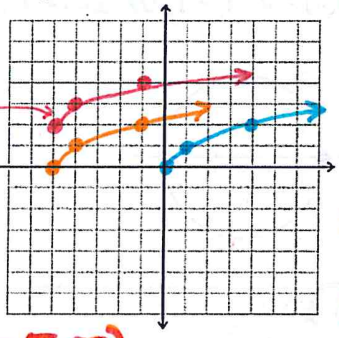
k = 0 no change

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

a h k

$f(x) = \sqrt{x+5}+2$



parent: radical

final graph

a = 1 no change

h = -5 shifts left

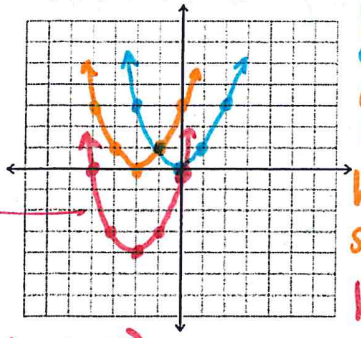
k = 2 shifts up

Domain: $[-5, \infty)$

Range: $[2, \infty)$

a h k

$f(x) = (x+2)^2 - 4$



parent: quadratic

final graph

a = 1 no change

h = -2 shift left

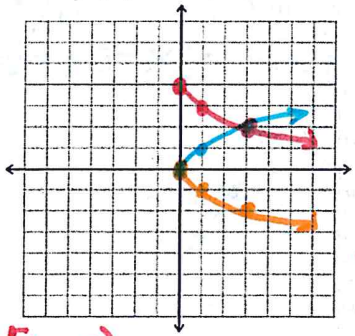
k = -4 shift down

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

a h k

$f(x) = -\sqrt{x}+4$



parent: radical

a = -1 reflects

h = 0 no change

k = 4 shift up

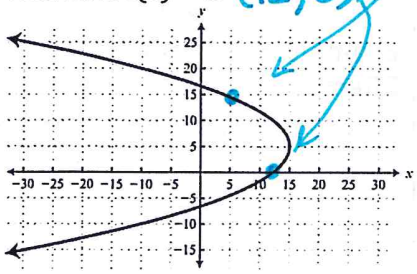
Domain: $[0, \infty)$

Range: $(-\infty, 4]$

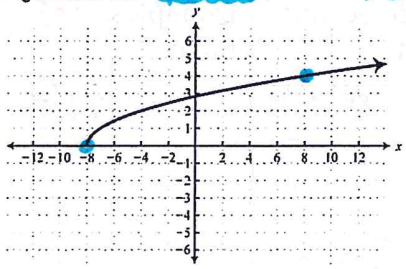
vertical line test

4. For each graph below, determine whether it is a function. Then determine its domain and range in interval notation. Identify **ONE solution to the function (point on the graph)** and **ONE solution to f(x) = 0 (x-intercept).** If asked, write the equation for the graph **(assume no vertical stretch or shrink).**

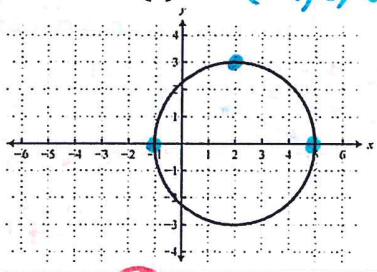
Function? YES/NO
Domain: $(-\infty, 15]$
Range: $(-\infty, \infty)$
Solution to function: $(5,15)$ (any point on graph)
Solution to f(x) = 0: $(12,0)$



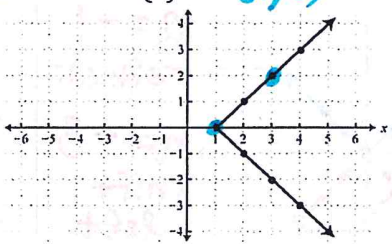
Function? YES/NO
Domain: $[-8, \infty)$
Range: $[0, \infty)$
Solution to function: $(8,4)$
Solution to f(x) = 0: $(-8,0)$
EQUATION: $f(x) = \sqrt{x+8}$



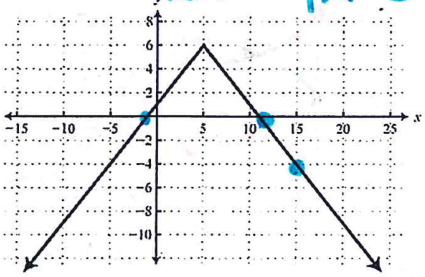
Function? YES/NO
Domain: $[-1,5]$
Range: $[-3,3]$
Solution to function: $(2,3)$
Solution to f(x) = 0: $(-1,0)$ and $(5,0)$



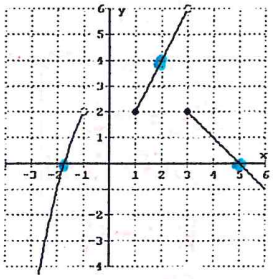
Function? YES/NO
Domain: $[1, \infty)$
Range: $(-\infty, \infty)$
Solution to function: $(3,2)$
Solution to f(x) = 0: $(1,0)$



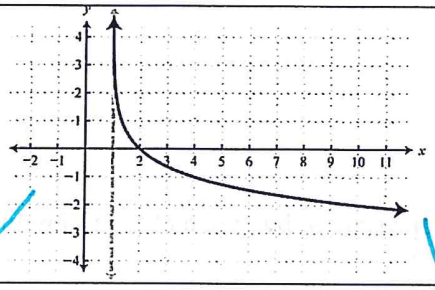
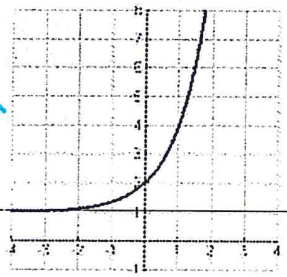
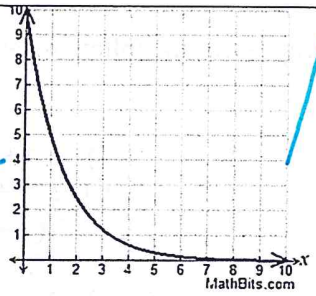
Function? YES/NO
Domain: $(-\infty, \infty)$
Range: $(-\infty, 6]$
Solution to function: $(15,-4)$
Solution to f(x) = 0: $(-1,0)$ $(11,0)$
EQUATION: $f(x) = -|x-5|+6$



Function? YES/NO (union = AND)
Domain: $(-\infty, -1] \cup [1,3) \cup [3, \infty)$
Range: $(-\infty, 6)$
Solution to function: $(2,4)$
Solution to f(x) = 0: $(-1.8,0)$ $(5,0)$



5. For each graph, draw two lines connecting it to its matching **domain** and **end behavior.**

DOMAIN		END BEHAVIOR
$\{x: all\ real\ numbers\}$		As x approaches ∞, y approaches 0. As x approaches -∞, y approaches ∞.
$\{x: 1 \leq x < \infty\}$		As x approaches 1, y approaches ∞. As x approaches ∞, y approaches - ∞.
$\{x: all\ real\ numbers\}$		As x approaches -∞, y approaches 1. As x approaches ∞, y approaches ∞.

UNIT 3: QUADRATICS

1. Which form of the quadratic equation shows the minimum or the maximum value of the function without changing the form of the equation

a) Standard form b) factored form c) vertex form

2. NC Which of the following equations shows the minimum or the maximum of h(x)?
   $h(x) = 2(x + 3)(x + 1)$ $h(x) = 2(x + 2)^2 - 2$ $h(x) = 2x^2 + 8x + 6$
   Does h(x) have a maximum or a minimum?

3. The John Deere Company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p that it charges. The revenue R is

$$R = -\frac{1}{2}p^2 + 1900p$$

What unit price p should be charged to maximize revenue? What is the maximum revenue?

4. The sum of the areas of two square plots of land is 45 square feet. The length of the side of one of the squares is 3 feet more than the length of the side of the other. What is the length of the sides of each square area?

UNIT 3: QUADRATICS

the minimum or maximum is the vertex in a Quadratic function

1. Which form of the quadratic equation shows the minimum or the maximum value of the function without changing the form of the equation

a) Standard form b) factored form c) vertex form

2. NC Which of the following equations shows the minimum or the maximum of h(x)? → vertex

$h(x) = 2(x+3)(x+1)$ $h(x) = 2(x+2)^2 - 2$ $h(x) = 2x^2 + 8x + 6$

Does h(x) have a maximum or a minimum? $a=2$, $a>0$, parabola opens up ∪ and has a minimum

3. The John Deere Company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p that it charges. The revenue R is

$$R = -\frac{1}{2}p^2 + 1900p$$

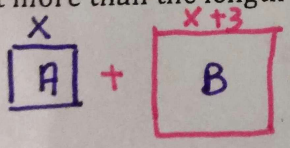
$h = \frac{-b}{2a} = \frac{-1900}{2(-\frac{1}{2})} = \frac{-1900}{-1} = 1900$

vertex (h,k) $k = -\frac{1}{2}(1900^2) + 1900(1900) = 1,805,000$

What unit price p should be charged to maximize revenue? What is the maximum revenue?

$p = 1900$ to maximize the revenue / the maximum revenue is 1,805,000.

4. The sum of the areas of two square plots of land is 45 square feet. The length of the side of one of the squares is 3 feet more than the length of the side of the other. What is the length of the sides of each square area?



Area of A: $side^2 = x^2$

Area of B: $side^2 = (x+3)^2$ } sum of areas = 45

$x^2 + (x+3)^2 = 45$ → continued on the back

$x^2+(x+3)^2=45$
$x^2+(x+3)(x+3)=45$
$x^2+x^2+3x+3x+9=45$

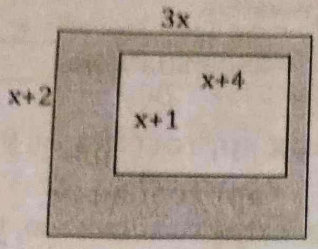
$2x^2+6x+9=45$
$-45\quad -45$
$\frac{2x^2}{2}+\frac{6x}{2}-\frac{36}{2}=0$

$2(x^2+3x-18)=0$
$a=1$
$b=3$
$c=-18$
6×-3

$(x+6)(x-3)=0$
$x+6=0 \qquad x-3=0$
$x=-6 \qquad \boxed{x=3}$
the $\downarrow$ length of the side cannot be negative

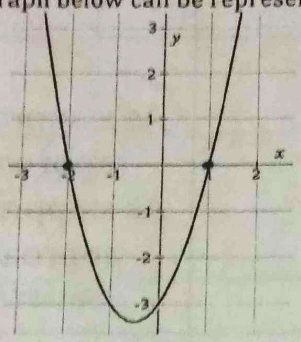
the lengths of the sides of the 2 squares are 3 ft and $3+3=6$ ft

5. Find the area of the shaded region in the picture:



Area of shaded region
= Area of Big rectangle − Area of the small rectangle
$= 3x(x+2)-(x+4)(x+1)$
$= 3x^2+6x-(x^2+x+4x+4)$
$= 3x^2+6x-x^2-5x-4$
$= 2x^2+x-4$

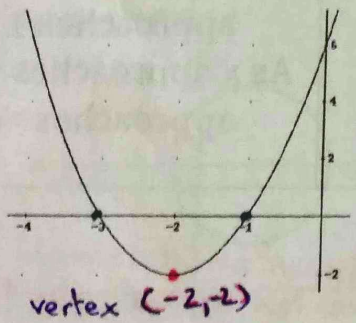
6. NC The graph below can be represented by which of the following equations:



a) $y=(x-2)(x+1)$
b) b) $y=(x-1)(x+2)$
c) c) $y=(x+1)(x+2)$
d) d) $y=(x-1)(x-2)$

the x-intercepts are $p=-2$ and $q=1$
the factored form is $(x-p)(x-q)$
opposite of x-intercepts $(x+2)(x-1)$

7. NC Select **ALL** of the functions that can represent the following graph



vertex $(-2,-2)$

x-intercepts -3 and -1
Factored form $2(x+3)(x+1)$
y-intercept $(0,6)$
Standard form $2x^2+8x+6$

a) $2x^2+4x+3$
b) $2(x+3)(x+1)$
c) $2(x+2)^2-2$
d) $2(x-3)(x-1)$
e) $2x^2+8x+6$
f) $2(x-2)^2-2$

vertex $(\overset{h}{-2},\overset{k}{-2})$
vertex form $=2(x+2)^2-2$

8. Simplify each expression. Write your answer in $a+bi$ format.

a. $-3+6i-(-5-3i)-8i$

b. $(-2-i)(4+i)$

c. $-6(4-6i)^2$

d. $(6-2i)-(11+4i)$

on separate paper

9. Solve the following equation using one of the following methods: quadratic formula, factoring, taking square roots, completing the square (don't forget to try GCF first).

a. $\sqrt{(p-6)^2}=\sqrt{9}$

b. $x^2-11x+19=-5$

c. $n^2+8n=3n$

d. $x^2+4x+6=0$

e. $4x^2-2x=5$

f. $3x^2+6=12$

on separate paper

10. How many times does each of the following functions intersect the x-axis?

a. $y=3x^2+\frac{2}{3}x-\frac{1}{3}$ — 2 x-intercepts

b. $f(x)=\frac{4}{3}x^2-4x+3$ — 1 x-intercept

c. $y=2x^2-\frac{1}{2}x+\frac{3}{2}$ — No x-intercept

Study guide key

Unit 3 Quadratics

#8 Simplify each expression. write your answer in a+bi format

a) $-3+6i-(-5-3i)-8i$

$= -3+6i+5+3i-8i$

$= 2+1i$

b) $(-2-i)(4+i)$ $\quad i^2=-1$

$= -8-2i-4i-i^2$

$= -8-6i+1$

$= -7-6i$

c) $-6(4-6i)^2$ $\quad i^2=-1$

$= -6(4-6i)(4-6i)$

$= -6(16-24i-24i+36i^2)$

$= -6(16-48i-36)$

$= -96+288i+216$

$= 120+288i$

d) $(6-2i)-(11+4i)$

$= 6-2i-11-4i$

$= -5-6i$

#9 Solve the following equations.

a) $\sqrt{(p-6)^2} = \sqrt{9}$ ← Solve by taking the square root.

$p-6 = \pm 3$

$p = 6 \pm 3$ → $p = 6+3 = 9$

↓ $p = 6-3 = 3$

b) $x^2-11x+19 = -5$ ← Solve by factoring.

$+5 \quad +5$

$x^2-11x+24 = 0$

24

-8 × -3

-11

$a=1$

$b=-11$

$c=24$

$(x-8)(x-3)=0$

$x-8=0$ OR $x-3=0$

$\boxed{x=8}$ $\boxed{x=3}$

c) $n^2+8n = 3n$

$-3n \quad -3n$

$n^2+5n = 0$ ← factor by GCF

$n(n+5) = 0$

$\boxed{n=0}$ OR $n+5=0$

$\boxed{n=-5}$

d) $x^2+4x+6=0$ ← Solve by completing the square

$b=4$ / $\frac{b}{2}=\frac{4}{2}=2$ / $2^2=4$

$x^2+4x = -6$

$+4 \quad +4$

$x^2+4x+4=-2$

Factor $(x+\frac{b}{2})^2$ ↓ $\sqrt{(x+2)^2}=\pm\sqrt{-2}$

$x+2=\pm\sqrt{2}\,i$

$$\boxed{x=-2\pm\sqrt{2}\,i}$$

e) $4x^2-2x=5$ ← Solve by using the Quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$-5 \quad -5$

$4x^2-2x-5=0$

$a=4 \quad b=-2 \quad c=-5$

$$x=\frac{2\pm\sqrt{(-2)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{2\pm\sqrt{4+80}}{8}=\frac{2\pm\sqrt{84}}{8}=\frac{2\pm\sqrt{4\cdot 21}}{8}=\frac{2\pm 2\sqrt{21}}{8}$$ (÷2 ÷2)

$$\boxed{x=\frac{1\pm\sqrt{21}}{4}}$$

f) $3x^2+6=12$

$-6 \quad -6$

$\frac{3x^2}{3}=\frac{6}{3}$

$x^2=2^3$

$\sqrt{x^2}=\sqrt{2}$

$$\boxed{x=\pm\sqrt{2}}$$

UNIT 4: EXPONENTIAL FUNCTIONS

1. Which of the following functions would be exponential functions? Give a rationale **for OR against** for each problem.
   a) The total amount paid for gas over x number of weeks if Sara puts $25 of gas in her car each week.
   NO 25x → linear
   b) The value of a computer after x years if it depreciates 12% each year.
   YES exponential rate is 12%
   c) The total cost of a wedding for x people if you pay $50/person.
   NO Linear 50x
   d) The population of Jonesville after x years if the population decreases by 20 people each year.
   NO -20x linear
2. Write an exponential model for the following situation: Hussein bought a house for $115,000. The value of the house appreciates by 3% each year. Write a model for the value of the house after x years.
   b = 1 + 0.03 = 1.03 $y = 115,000(1.03)^x$
3. Write an exponential model for the following situation: Laurie bought some office furniture. For tax purposes, the value of the furniture depreciates by 13% each year. Write a model for the value of the furniture after x years.
   b = 1 - 0.13 = $y = a(0.87)^x$ no a?
4. Joe took a job for $30,000 and gets an $800 raise each year. Nora took a job for $28,000 and gets a 3% raise each year. If this is a career choice and they are planning on staying in this job for the long term, who took the better deal? Explain.
   Joe - 30,000 + 800x Nora: $28000(1.03)^x$ graph it!
5. The population of Berkeley can be modeled by the function $P(t) = 115{,}000(1.013)^t$ where t is the number of years after 2015.
   a) What is the population of Berkeley in 2015?
   a = 115,000 people
   b) Is Berkeley getting bigger or smaller? Explain.
   bigger b > 1
   c) What is the rate of increase/decrease each year?
   r = 1.013 - 1 = 0.013 x 100% = 1.3%

SLOT: RATIONAL FUNCTIONS

1. Simplify the following rational expression:
   $$\frac{5p+12}{3p+5}+\frac{13p+18}{3p+5}$$
2. Solve the following rational equation, show your steps and indicate any extraneous solutions.
   $$\frac{4}{x}=\frac{-3}{x+8}$$
3. Solve the following rational equation, show your steps and indicate any extraneous solutions.
   $$\frac{3}{x}+\frac{4}{3x}=\frac{1}{3}$$

SLOT: RADICAL FUNCTIONS

1. Solve each following radical equation. Determine whether each solution is extraneous.

a. $\sqrt{3x-5}-5=2$
b. $\sqrt{90-x}=x$
c. $\sqrt{x-5}=\sqrt{10-2x}$

SLOT: RATIONAL FUNCTIONS

① Simplify. $\frac{5p+12}{3p+5} + \frac{13p+18}{3p+5} = \frac{5p+12+13p+18}{3p+5}$

Same denominator so add the numerators.

$= \frac{18p+30}{3p+5}$ GCF: 6

$= \frac{6(3p+5)}{3p+5} = \boxed{6}$

② Solve. $\frac{4}{x} = \frac{-3}{x+8}$

Cross-multiply.

$4(x+8) = -3(x)$

$4x + 32 = -3x$

$-4x \qquad -4x$

$\frac{32}{-7} = \frac{-7x}{-7}$

$\boxed{x = -\frac{32}{7} \approx -4.57}$

Check if it makes denominator zero → it does not. Therefore not extraneous.

③ Solve. $\frac{3}{x} + \frac{4}{3x} = \frac{1}{3}$ (multiply by $3x$)

Multiply by LCM of denominators.

$9 + 4 = x$

$\boxed{x = 13}$

↳ Does not make denominators zero → not extraneous.

SLOT: Radical Functions

1. Solve and check if extraneous.

a. $\sqrt{3x-5} - 5 = 2$

$+5 \quad +5$

$(\sqrt{3x-5})^2 = (7)^2$

$3x - 5 = 49$

$+5 \quad +5$

$3x = 54$

$\boxed{x = 18}$

CHECK:

$\sqrt{3(18)-5} - 5 = 2$

$\sqrt{54-5} - 5 = 2$

$\sqrt{49} - 5 = 2$

$7 - 5 = 2$

$2 = 2$ ✓

NOT extraneous

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2. $(\sqrt{90-x})^2 = (x)^2$

$90 - x = x^2$

$-90 \quad +x \qquad -90 + x$

$0 = x^2 + x - 90$

$0 = (x+10)(x-9)$

$10 \times -9 = -90$, $10 + (-9) = 1$

$x + 10 = 0$ ✓ $\qquad x - 9 = 0$

$x = -10 \qquad \boxed{x = 9}$

EXTRANEOUS $\qquad$ Solution

CHECK: $x = -10$

$\sqrt{90-(-10)} = -10$

$\sqrt{100} = -10$

$10 = -10$ ✗

FALSE: $x = -10$ IS extraneous

CHECK: $x = 9$

$\sqrt{90-9} = 9$

$\sqrt{81} = 9$

$9 = 9$ ✓

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3. $(\sqrt{x-5})^2 = (\sqrt{10-2x})^2$

$x - 5 = 10 - 2x$

$+2x \qquad +2x$

$3x - 5 = 10$

$+5 \quad +5$

$3x = 15$

$\boxed{x = 5}$

CHECK:

$\sqrt{5-5} = \sqrt{10-2(5)}$

$\sqrt{0} = \sqrt{0}$ ✓

TRUE