# 7.1 EXPLORING EXPONENTIAL MODELS

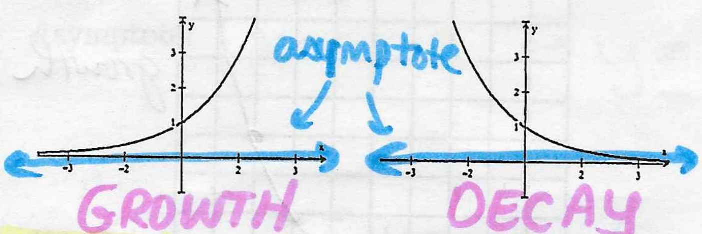
## VOCABULARY

**Exponential function** – any function written in the form $y = ab^x$, where $x$ is a real number, $a \neq 0$, $b > 0$, and $b \neq 1$. If $b > 1$, the function models exponential growth with a growth factor $b$. When $0 < b < 1$, the function models exponential decay with decay factor $b$.

$ab^x$

**Exponential growth** – any function in the form $y = ab^x$ with $b > 1$

**Exponential decay** – any function in the form $y = ab^x$ with $0 < b < 1$




**Asymptote** – a line that a graph approaches as $x$ or $y$ increases in absolute value. but never touches

x-axis $y=0$

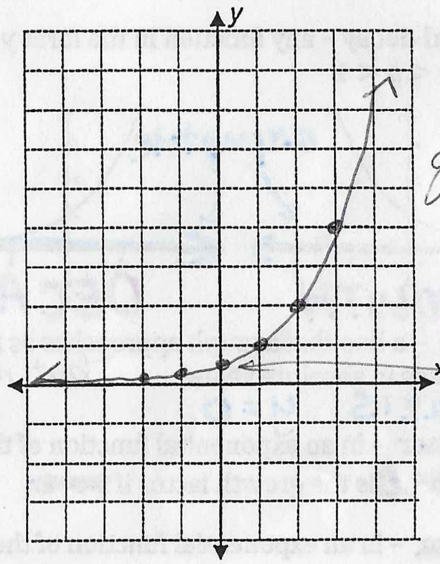
**Growth factor** – in an exponential function of the form $y = ab^x$, $b$ is the growth factor if $b > 1$.

**Decay factor** – in an exponential function of the form $y = ab^x$, $b$ is the decay factor if $0 < b < 1$.

## Graphing an Exponential Function

**EX #1:** Graph $y = \frac{1}{2}(2)^x$ growth $b=2$ yint $(0, \frac{1}{2})$

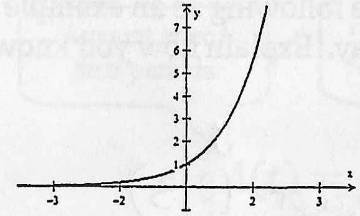
| $x$ | $\frac{1}{2}(2)^x$ | $y$ |
|---|---|---|
| −2 | $\frac{1}{2}(2)^{-2}$ | $\frac{1}{8}$ |
| −1 | $\frac{1}{2}(2)^{-1}$ | $\frac{1}{4}$ |
| 0 | $\frac{1}{2}(2)^0$ | $\frac{1}{2}$ |
| 1 | $\frac{1}{2}(2)^1$ | 1 |
| 2 | $\frac{1}{2}(2)^2$ | 2 |
| 3 | $\frac{1}{2}(2)^3$ | 4 |

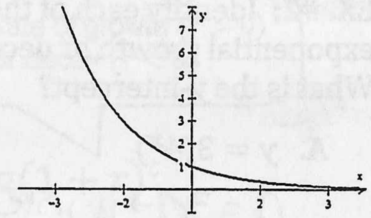



growth

yint $(0, \frac{1}{2})$

## Characteristics of Exponential Graphs




**Domain:** $(-\infty, \infty)$ all real numbers

**Range:** $(0, \infty)$ never actually 0

**Asymptote:** x-axis → $y = 0$

**Key Points:** y-intercept $(0, a)$

## Identifying Exponential Growth & Decay

**EX. #2:** Identify each of the following as an example of exponential growth or decay. Explain how you know. What is the $y$-intercept?

**A.** $y = 3(4^x)$

$a = 3 \rightarrow$ y intercept $(0,3)$

$b = 4 \rightarrow$ growth, $b > 1$

**B.** $y = 11(0.75^x)$

$a = \cancel{8}\,11 \rightarrow$ y-intercept $(0,11)$

$b = 0.75 \rightarrow$ decay, $b < 1$

**C.** You save $\underline{\$2000}$ into a college savings account for $\underline{\text{four years}}$. The account pays $\underline{6\%}$ interest annually.

a

time t

rate r

$a = 2000$

$r = 6\% = 0.06$ (growth)

$b = 1 + 0.06 = 1.06$

$A(t) = 2000(1.06)^t$

After 4 years : $2000(1.06)^4 =$

## Modeling Exponential Growth

$$A(t) = a(1 + r)^t$$

Amount after $t$ time periods

Rate of growth ($r > 0$) positive

Or decay ($r < 0$) negative

Initial Amount (starting)

Number of time periods (sec, weeks, years, etc)

**EX. #3:** Suppose you invest $\underline{\$500}$ in a savings account that $\underline{\text{pays 3.5\% annual interest}}$. How much will be in the account after $\underline{\text{five years}}$?

a

growth r (positive) t

$a = 500$

$r = 0.035$

$b = 1.035$

$A(t) = 500(1.035)^t$

After 5 years, $t = 5$:

$500(1.035)^5 =$

## Using Exponential Growth

**EX #4:** If you invest your $500 in a savings account that pays 3.5% annual interest, when will the account contain at least $1500 ?

Growth (saving/interest)

$a = 500$

$r = 3.5\% = 0.035$

$b = 1 + 0.035 = 1.035$

$\boxed{A(t) = 500(1.035)^t}$

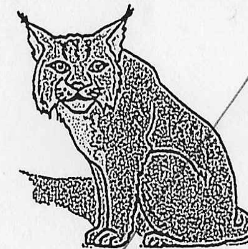
When ($t = ?$) → \$1500

→ Graph it!

Look in table for when y is greater than 1500. The x-value is the time in years.

## Writing an Exponential Function

**EX #5:** The table shows the world population of the Iberian lynx in 2003and 2004. If the population is decreasing in an exponential manner and the trend continues, how many Iberian lynx will there be in 2014?



| World Population of Iberian lynx | | |
|---|---|---|
| **Year** | **2003** | **2004** |
| Population | 150 | 120 |

Write the model as $y = ab^x = a(1+r)^x$

$r = \frac{y_2 - y_1}{y_1}$ $\quad$ $b = (1+r)$