Radical Equations with Extraneous Solutions

A proposed solution that is not a solution of the original equation it is called an **extraneous solution**.

Radical equations with square roots often have extraneous solutions because through the process of solving these equations we must square both sides of the equation. However, the process of squaring both sides is not a "reversible" operation. For instance, $(-2)^2 = 4$, but $\sqrt{4} = 2$. We can't get back to -2. This is why it is so important to check proposed solutions in the original equation to verify the solution is not an extraneous solution.

Steps for Solving a Radical Equation:

Step 1 . Isolate the radical expression on one side of the equation.

Step 2. Raise each side of the equation to a power equal to the index of the radical and simplify. If the index is not written, it is understood to be 2.

Step 3. Solve the resulting equation.

Step 4. Check all proposed solutions in the original equations.

Example 1: Solve the equation $\sqrt{5-x} - 1 = x$

Solution: We must first isolate the radical expression by adding one to each side. Then we will square both sides to remove the radical.

$$\sqrt{5-x} - 1 = x$$
$$+1 \quad +1$$
$$\left(\sqrt{5-x}\right)^2 = (x+1)^2$$
$$5 - x = x^2 + 2x + 1$$
$$0 = x^2 + 3x - 4$$
$$0 = (x+4)(x-1)$$
$$x = -4 \text{ or } x = 1$$

Check the proposed solutions:

$x = -4$: $\sqrt{5-(-4)} - 1 = (-4)$
$\sqrt{9} - 1 = -4$
$2 = -4$ FALSE

$x = 1$: $\sqrt{5-(1)} - 1 = (1)$
$\sqrt{4} - 1 = 1$
$1 = 1$ TRUE

$x = -4$ is an extraneous solution.

$x = 1$ is a solution of the equation

The solution set of the equation $\sqrt{5-x} = x + 1$ is $\{1\}$.

Example 2: Solve the equation $\sqrt{4-2t-t^2} = t+2$.

Solution: The radical expression is isolated to one side, so we may square both sides to remove the radical.

$$\left(\sqrt{4-2t-t^2}\right)^2 = (t+2)^2$$
$$4-2t-t^2 = t^2+4t+4$$
$$0 = 2t^2+6t$$
$$0 = 2t(t+3)$$
$$t = 0 \text{ or } t = -3$$

Check the proposed solutions:

$t = 0$: $\sqrt{4-2(0)-(0)^2} = (0)+2$

$\sqrt{4} = 2$

$2 = 2$ TRUE

$t = 0$ is a solution of the equation.

$t = -3$: $\sqrt{4-2(-3)-(-3)^2} = (-3)+2$

$\sqrt{1} = -1$

$1 = -1$ FALSE

$t = -3$ is an extraneous solution.

The solution set of the equation $\sqrt{4-2t-t^2} = t+2$ is $\{0\}$.

Practice Exercises: Solve each radical equation.

1. $\sqrt{x-3}+6 = 5$
2. $\sqrt{x-1}+7 = 2$
3. $\sqrt{x+7} = x+5$
4. $\sqrt{2x+5} = x-5$
5. $\sqrt{3x+7}-x = 3$
6. $\sqrt{3x-2}+4 = x$

Solutions:

1. $\emptyset$
2. $\emptyset$
3. $\{-3\}$
4. $\{10\}$
5. $\{-2,-1\}$
6. $\{9\}$