Multiplication Counting Principle

The number of ways to arrange **n** objects in a particular order is expressed using **factorial notation:**

"n factorial" = $n! = n*(n-1)*...*3*2*1$

Example: Ways to arrange 4 posters = 4!

$$4! = 4*3*2*1 = 24$$

$100! = 100 \cdot 99 \cdot 98 \cdots 3 \cdot 2 \cdot 1$

Practice

Calculate the following:

$3! = 3 \cdot 2 \cdot 1 = 6$

$11! = 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 39{,}916{,}800$

Calculator Shortcut

- Type in the number
- Menu -> Probability -> Factorial

Calculate:

8! = 40,320

12! = 479,001,600

Example 2: Eight schools are competing in a choral competition. How many possibilities are there for arrangements of 1st, 2nd, and 3rd place?

- 8 possibilities for 1st place.
- 7 for 2nd place
- 6 for 3rd place

$$8 \cdot 7 \cdot 6 = 336 \text{ possibilities.}$$

Permutations

To calculate the number of permutations of **n** objects using **r** of those objects, when **order matters**: *

$$nPr = \frac{n!}{(n-r)!}$$

Choral example:

$$_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!}$$

$$= \frac{8\cdot 7\cdot 6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

$$= 336$$

Calculator:

$$_nP_r(8, 3) = 336$$

Permutations

Poster example:

$$_4P_4 = \frac{4!}{(4-4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$$

Combinations

To calculate the number of combinations of **n** objects using **r** of those objects, when **order does not matter**: *

$$nCr = \frac{n!}{r!(n-r)!}$$

Example 3: There are 20 markers in a can. How many ways can I choose 5 markers?

Does order matter? No

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_{20}C_5 = \frac{20!}{5!(20-5)!} = \frac{20\cdot19\cdot18\cdot17\cdot16\cdot15\cdot14\cdots}{(5\cdot4\cdot3\cdot2\cdot1)(15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdots}$$

$$_nC_r(20,5) = 15{,}504$$